Unit 5 · Fraction and Decimal

STUDY NOTES

Order of operations (BODMAS)

1. **B**rackets: small ( ), then middle { }, then big [ ]
2. **O**f: "2/3 of 12" = 2/3 × 12
3. **D**ivision and **M**ultiplication, from left to right
4. **A**ddition and **S**ubtraction, from left to right

- the same order for whole numbers, fractions and decimals
- fractions: make the denominators equal before adding, subtracting or comparing
- a number right before a bracket means **multiply**: 2.8(2.1) = 2.8 × 2.1, and the product belongs together

Fractions: the basics

- add: same denominator first: 1/5 + 1/7 = 7/35 + 5/35 = **12/35**
- mixed numbers: wholes and fractions apart: 5 5/6 − 1 2/3 = 5 5/6 − 1 4/6 = **4 1/6**
- whole × fraction: multiply the top: 6 × 2/5 = 12/5 = **2 2/5**
- ÷ a fraction = × its reciprocal: 12 ÷ 2/5 = 12 × 5/2 = **30** (30 two-fifths in 12)
- 2/3 + 1/2 × 4/5 − 1/6 ÷ 1/3: × and ÷ first → 2/3 + 2/5 − 1/2 = 20/30 + 12/30 − 15/30 = **17/30**

Parts of the same whole

- parts of the **same** whole: add the fractions
- Sadikshya, Rs 24,000: 1/3 studies + 1/2 food = 2/6 + 3/6 = **5/6** → 5/6 × 24,000 = **Rs 20,000**
- check another way: 8,000 + 12,000 = 20,000; 20,000/24,000 = 5/6
- compare: same denominator (the LCM): 1/2, 1/3, 1/6 → 3/6, 2/6, 1/6 → 1/2 is the most; together 1 (the whole cake)

A fraction of the rest

1. find what is left: 1 − the first part
2. the second fraction × **what is left**, never × the whole

- Rs 20,000: 1/4 on rice → 3/4 left = Rs 15,000; 2/5 **of the rest** on oil = 2/5 × 15,000 = **Rs 6,000**; left **Rs 9,000**
- as parts of the whole: oil 2/5 × 3/4 = **3/10**; left 9/20; check 1/4 + 3/10 + 9/20 = (5 + 6 + 9)/20 = 1 ✓
- adding 1/4 + 2/5 at once is right only when both are of the same whole
- **the whole from a part**: write the part as a fraction of the whole, then 1 part → all
  - 1/4 spent, 2/5 of the rest lost = Rs 900 → 2/5 × 3/4 = 3/10 of all = 900 → 1/10 = 300 → **Rs 3,000**
  - 4/5 boys; 1/4 of the girls + 1/3 of the boys went = 190 → 1/20 + 4/15 = 19/60 of all → 1/60 = 10 → **600**

Model drawings

1. cut the bar into as many equal parts as the denominator
2. find one part
3. then any number of parts, and the rest

- 48 students, 2/3 girls: 3 parts = 48 → 1 part = 16 → girls 2 × 16 = **32**, boys **16**
- 2/5 boys, 90 girls: girls = the other **3** parts → 1 part = 30 → boys **60**, all **150** (check 60/150 = 2/5)

Decimals: the basics

- **add, subtract**: line up the points (fill in zeros): 8.97 + 23.20 = **32.17**; 7.7 − 2.8 = 4.9
- **multiply**: as whole numbers, then as many decimal places as the two numbers have between them: 36 × 58 = 2,088 → 3.6 × 5.8 = **20.88**
- **divide by a whole number**: the point in the answer right above the point: 17.40 ÷ 4 = **4.35**
- **divide by a decimal**: move both points until the divisor is whole: 54 ÷ 4.5 = 540 ÷ 45 = **12** (check 12 × 4.5 = 54)
- shapes with decimals: rectangle $A = l \times b$, $P = 2(l + b)$; breadth = $A \div l$; square side = $P \div 4$ (24.4 ft → 6.1 ft); triangle $A = \frac{1}{2} \times b \times h$
- table 5.25 ft by 3.5 ft: perimeter 2(5.25 + 3.5) = 2 × 8.75 = **17.5 ft** (bracket first, then × 2)

Simplifying decimals

- $7.5 + \{6.72 \div 2.8(3.59 - 1.49)\} = 7.5 + \{6.72 \div 2.8(2.1)\} = 7.5 + \{6.72 \div 5.88\}$
- $6.72 \div 5.88 = 8/7 = 1.142857\ldots$ (recurring) $\rightarrow 7.5 + 1.142857\ldots \approx$ **8.64** (exactly 8 9/14)
- when in doubt write the $\times$ and a bracket: $6.72 \div [2.8 \times (3.59 - 1.49)]$; $6.72 \div 2.8 \times 2.1$ from the left gives 5.04: another answer
- $a \times a - b \times b = (a + b)(a - b)$: $4.5 \times 4.5 - 2.1 \times 2.1 = 6.6 \times 2.4 = 15.84$

Check by estimating

- round, then work out roughly: $22.66 \times 15.65 \approx 23 \times 16 = 368 \rightarrow$ 354.629 believable; 35.4629 or 3,546.29 not
- $4.9 \times 3.1 \approx 5 \times 3 = 15$; exactly **15.19**
- $39.6 \div 7.92 \approx 40 \div 8 = 5$; exactly $3960 \div 792 =$ **5**

Back to the remittance

- 1,500 riyals $\times$ Rs 41.16 = **Rs 61,740**
- savings 1/4: Rs 15,435 $\rightarrow$ rest Rs 46,305
- house 2/5 **of the rest**: Rs 18,522 $\rightarrow$ left **Rs 27,783**
- of the whole: $1/4 + 3/10 = 11/20$ used, **9/20 left**: a little less than half
- $1/4 + 2/5 = 13/20$ of the whole would leave only Rs 21,609: "of the rest" makes Rs 6,174 difference

COMMON MISTAKES

✗ 2/5 of the rest = 2/5 × the whole
✓ 2/5 × what is left: $2/5 \times 15{,}000 = 6{,}000$.

✗ 1/4 + 2/5 used for "a quarter, then two fifths of the rest"
✓ of the whole: $1/4 + 3/10 = 11/20$.

✗ $1/6 \div 1/3 = 1/18$
✓ turn over the divisor: $1/6 \times 3 = 1/2$.

✗ $2/3 + 1/2 \times 4/5$ worked from the left
✓ × before +: $2/3 + 2/5$.

✗ $3.6 \times 5.8 = 208.8$
✓ 2 decimal places in all: 20.88.

✗ 8.97 + 23.2 = 11.29 (digits lined up at the right)
✓ line up the points: 8.97 + 23.20 = 32.17.

✗ $6.72 \div 2.8(2.1) = 5.04$
✓ the product $2.8 \times 2.1 = 5.88$ goes together: 1.14.

✗ 2/5 boys, 90 girls: 90 = 2 parts
✓ girls are the other 3 parts: 1 part = 30.