Unit 3 · Integer

STUDY NOTES

Key terms

Integers: whole numbers and their negatives, $Z = \{..., -2, -1, 0, 1, 2, ...\}$; 0 is the **origin**.

Zero pair: a green +1 and a red −1 counter, worth 0; put in or take away freely.

Additive inverse (**opposite**): a and $-a$, same distance from 0, opposite sides; $a + (-a) = 0$.

Additive identity: 0, since $a + 0 = a$.

Absolute value $|a|$: how far a is from 0, never negative: $|-4| = |+4| = 4$, $|0| = 0$.

Multiplicative identity: 1, since $a \times 1 = a$.

Rules

$a - b = a + (-b)$	add the opposite **Use:** every subtraction
$a \times (b + c) = ab + ac$	also over − **Use:** distributive
$a + (-a) = 0$, $a + 0 = a$	inverse, identity **Use:** adding
$a \times 1 = a$, $a \times 0 = 0$	identity, zero **Use:** multiplying

Adding

- number line
  - ■ a positive → **right**
  - ■ a negative → **left**
  - start at 0, move, move again from there
- counters: cancel zero pairs; what is left is the answer: $(-5) + (+3)$: 3 pairs go, **−2** left
- **same signs**: add the sizes, keep the sign: $(-3) + (-5) = \mathbf{-8}$
- **different signs**: bigger size − smaller size, the bigger one's sign: $(+2) + (-7) = \mathbf{-5}$
- $(+5) + (-2)$ is written $5 - 2$; $-3 - 4$ means start at −3, go 4 more left: **−7**, not −1
- 4 units left of −3: $(-3) + (-4) = -7$
- $(-12) + (+12) + (-4)$: inverses make 0 → **−4**

Subtracting

- pattern: $5 - 2 = 3$, $5 - 1 = 4$, $5 - 0 = 5$ → $5 - (-1) = 6$, $5 - (-2) = \mathbf{7} = 5 + 2$
- counters: no reds to take → put in zero pairs, take the reds; the greens stay
- taking away a positive → **left**; taking away a negative → **right**
- $(-4) - (-5) = -4 + 5 = \mathbf{1}$; $(-2) - (+5) = -2 - 5 = \mathbf{-7}$
- **a minus before a bracket with a negative inside → plus**: $6 - (-4) = \mathbf{10}$
- long sums: take off the brackets, then from the left: $250 - 275 - 148 + 207 - 175 = \mathbf{-141}$
- check: added 457, taken 598 → $598 - 457 = 141$ more taken → −141
- Ujwal: +20 km east, back 13 km: $(+20) - (+13) = \mathbf{+7}$, 7 km east

Absolute value

- A 4 km east, B 4 km west of a bus park: **8 km** apart $(4 + 4)$, not $4 + (-4) = 0$
- the sign gives the **direction**, the number **how far**; a distance is never negative
- $|+a| = |-a| = a$; opposites have the same absolute value
- **0 is its own opposite** $(0 + 0 = 0)$

Properties

Property	Addition	Multiplication
closure	$a + b$ an integer	$a \times b$ an integer
commutative	$a + b = b + a$	$a \times b = b \times a$
associative	$(a + b) + c = a + (b + c)$	$(a \times b) \times c = a \times (b \times c)$
identity	0	1

- subtraction **is** closed for integers $(2 - 3 = -1)$, not for whole numbers
- division is **not** closed: $7 \div 2 = 3.5$
- $[(-7) + (-2)] + 6 = -3 = (-7) + [(-2) + 6]$

Multiplying and dividing

Signs	× and ÷
+ and +	+
− and −	+
+ and −	−
− and +	−

- multiply (divide) the sizes, then the sign: $(-4) \times (-5) = +20$; $(-36) \div 4 = -9$
- $3 \times (-4) = (-4) + (-4) + (-4) = -12$ (repeated adding)
- **why − × − = +**:
  - times table: down the −2 column the products rise by 2: 0, 2, 4, 6 → $(-3) \times (-2) = 6$
  - number line: $(-2) \times 3$ lands on −6; the minus of −3 means **turn round** → +6
  - counters: take out 2 groups of 3 reds from 6 zero pairs → 6 greens left: +6
  - bus going west at −40 km/h, 3 hours ago (−3 h): $(-40) \times (-3) =$ **+120**, 120 km east
- multiplying by −1 flips to the opposite: $(-1) \times (-1) = 1$
- **odd** number of negatives → negative; **even** → positive: $(-1)(-2)(-3)(-4) =$ **+24**; 7 negatives → negative
- $(+12) \times (-8) \times (+2) =$ **−192** (one negative)
- $(-5) \times [(+24) - (-6)] = -120 - 30 =$ **−150** $= (-5) \times 30$
- division undoes ×: $32 \div (-8) = -4$ because $(-4) \times (-8) = 32$
- $0 \div 5 = 0$; **5 ÷ 0 has no answer**: never divide by 0

Order of working

1. brackets, innermost first
2. multiply and divide (× and ÷), from the left
3. add and subtract (+ and −), from the left

- $(+12) + (-5) + (+25) \div (-5) - (-6) \times (+7) = 12 - 5 - 5 + 42 =$ **44**
- $[(-3) + 9] \times (-2) - (-10) = -12 + 10 =$ **−2**
- a bracket round **one** signed number, (−5), only keeps the sign with it: not a step

Quiz scores

- right +5, wrong −2, left out 0
  - 8 right 3 wrong = 40 − 6 = **34**
  - 7 right 4 left = **35** (wins)
  - 3 right 8 wrong = 15 − 16 = **−1**
- a score can fall below 0
- 4 options, blind guessing: 1 right 3 wrong in 4 → 5 − 6 = −1: staying quiet (0) is better; two options ruled out → 5 − 2 = +3: guess
- wrong answers from a total: 25 right at +5, total 104, −3 each wrong: (125 − 104) ÷ 3 = **7** wrong
- missing number: call it x and undo
  - $(-25) - x = -20$ → −20 is 5 right of −25 → $x =$ **−5**
  - product +63, one is +7 → 63 ÷ 7 = **+9**
  - first = 5 × second + 2 = 77 → (77 − 2) ÷ 5 = **15**
- two taps: change each minute (+25) + (−15) = +10 l → 6 min: 500 + 60 = **560 l**
- Jumla: −5 °C to 14 °C → rise 14 − (−5) = **19** degrees

COMMON MISTAKES

✗ $-3 - 4 = -1$
✓ both moves left: −7.

✗ $6 - (-4) = 2$
✓ taking away a negative adds: 6 + 4 = 10.

✗ $(-3) \times (-4) = -12$ (as for adding)
✓ two negatives multiplied: +12; added: $(-3) + (-4) = -7$.

✗ A and B 4 km either side: 4 + (−4) = 0 km apart
✓ a distance is never negative: 4 + 4 = 8 km.

✗ zero has no opposite
✓ 0 is its own opposite.

✗ $|-4| = -4$
✓ absolute value is never negative: 4.

✗ $5 \div 0 = 0$
✓ no answer; only $0 \div 5 = 0$.

✗ 12 − 5 − 5 + 42 worked from the right
✓ + and − from the left.

✗ $(-1)(-2)(-3) = +6$
✓ three negatives, odd: −6.