Unit 2 · Whole Number

STUDY NOTES

Key terms

Square number (**perfect square**): a number times itself: $8 \times 8 = 8^2 = 64$.

Square root, $\sqrt{\ }$: one of the two equal factors: $\sqrt{64} = 8$.

Cube number (**perfect cube**): the product of three equal numbers: $3^3 = 27$, read "3 cubed".

Cube root, $\sqrt[3]{\ }$: one of the three equal factors: $\sqrt[3]{8} = 2$.

HCF (highest common factor): the greatest number that divides all the given numbers exactly.

LCM (lowest common multiple): the smallest number that all the given numbers divide exactly.

Formulas

$\sqrt{a^2} = a$	squaring and the root undo each other **Use:** square root
$\sqrt[3]{a^3} = a$	cubing and the cube root undo each other **Use:** cube root
$\sqrt{a \times b} = \sqrt{a} \times \sqrt{b}$	root of each factor, multiplied **Use:** $\sqrt{25 \times 36} = 5 \times 6 = 30$
$\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$	root of the top, root of the bottom **Use:** root of a fraction
$A = l^2, l = \sqrt{A}$	side of a square from its area **Use:** square floor, field
$V = l^3, l = \sqrt[3]{V}$	side of a cube from its volume **Use:** cube-shaped tank

Squares and square roots

- a square number = the sum of the first odd numbers: $64 - 1 - 3 - 5 - \ldots - 15 = 0$ in **8** steps $\rightarrow \sqrt{64} = 8$ (L-shaped layers)
- squares end only in **0, 1, 4, 5, 6 or 9**: 2,047 and 5,438 can't be squares; the test works one way only (19 ends in 9)
- a square ends in an **even** number of zeros: 100, 3,600 can be; 1,000 never

Prime factorization method

1. find the prime factors
2. group the same factors in **pairs** under $\sqrt{\ }$
3. one factor from each pair, multiplied = the root

- $\sqrt{1225} = \sqrt{5^2 \times 7^2} = 5 \times 7 = 35$; square hall of 625 $m^2 \rightarrow$ side $\sqrt{625} =$ **25 m**
- a factor **without a partner** $\rightarrow$ not a square: $12{,}675 = 3 \times 5^2 \times 13^2 \rightarrow$ divide by **3** ($4{,}225 = 65^2$), or multiply by 3 (195^2)
- **divide** instead: $7{,}200 = 2^5 \times 3^2 \times 5^2$: one 2 left over $\rightarrow \div$ **2** $= 3{,}600 = 60^2$
- each gave as many rupees as there are students: total = students^2 $\rightarrow$ students = $\sqrt{2500} =$ **50**
- fraction: top and bottom apart: $\sqrt{144} = 12$, $\sqrt{169} = 13 \rightarrow$ root of 144/169 = **12/13**

Division method (big numbers)

1. mark the digits in **pairs from the right**: 17 | 64 (odd count: a single digit first)
2. first pair: biggest square not more than it ($16 = 4^2$); 4 in the quotient and as divisor; subtract: 1
3. bring down the next pair: 164
4. **double** the quotient so far: 8_; find the biggest digit d with $8d \times d \leq 164$: $82 \times 2 = 164$
5. write d in the quotient; subtract; repeat until no pairs are left: $\sqrt{1764} =$ **42**

- why double: $42^2 = 40 \times 40 +$ **two strips** $40 \times 2 +$ corner $2 \times 2 \rightarrow (80 + 2) \times 2 = 164$
- digits of the root = number of pairs: 95,481 $\rightarrow$ 3 pairs $\rightarrow$ 3 digits
- a divisor too big to go even once $\rightarrow$ write **0** in the quotient: $\sqrt{95481} =$ **309**, not 39
- check by squaring: $309 \times 309 = 95{,}481$
- smallest number to **take away** to make a square = what is left over at the end: 18,227 $\rightarrow$ 135, 2 left $\rightarrow$ $18{,}227 - 2 = 135^2$
- smallest number to **add**: the next square − the number: 1,021 $\rightarrow 32^2 = 1{,}024 \rightarrow$ add **3**

Cubes and cube roots

- cubes 1–10: 1, 8, 27, 64, 125, 216, 343, 512, 729, 1,000
- a cube of side 3 holds $3 \times 3 \times 3 = 27$ unit cubes (Rubik's cube); a pit 6 ft each way: $6^3 =$ **216 ft^3**
- cube root: as the square root, with prime factors in **threes**
- $512 = 2^9 \rightarrow$ three groups of $2^3 \rightarrow$ $\sqrt[3]{512} = 2 \times 2 \times 2 = 8$

- cube tank of 4,096 m^3: $4096 = 2^{12} \rightarrow \sqrt[3]{4096} = 16$ → **16 m** high
- $1{,}323 = 3^3 \times 7^2$: only two 7s → not a cube; × **7** → $9{,}261 = 21^3$
- or **divide** by what is left over: $2{,}916 = 2^2 \times 3^6 \rightarrow \div 4 = 729 = 9^3$; $3{,}993 = 3 \times 11^3 \rightarrow \div 3 = 11^3$
- odd number → odd cube; even number → even cube (a factor 2 → a factor 8)
- 1 m^3 = 1,000 l: a tank's capacity in liters = its volume in m^3 × 1,000

HCF

- **prime factors**: multiply the primes common to **all**, each as often as it is in every one
- $30 = 2 \times 3 \times 5$, $40 = 2 \times 2 \times 2 \times 5$ → HCF 2×5 = **10**: 30 apples, 40 bananas → 10 people, 3 + 4 each
- 100, 125, 200: 125 has no 2 → HCF 5×5 = **25**

Division method (Euclid)

1. divide the larger by the smaller
2. divide the divisor by the remainder; repeat until the remainder is 0
3. the last divisor is the HCF; then divide the next number by it

- 125 ÷ 100 → rem 25; 100 ÷ 25 → rem 0 → HCF 25; 200 ÷ 25 → rem 0 → HCF of all three **25**
- why: a number dividing 100 and 125 also divides $125 - 100 = 25$
- 125 oranges, 150 sweet lemons, 225 guavas → HCF 25 students; 5, 6, 9 each
- greatest number that, **less 10**, divides 588, 700, 840: $x - 10$ = HCF 28 → x = **38**; **plus 1**, divides 225, 375, 525: $x + 1 = 75$ → **74**

LCM

- 10, 20, 30 cm rods end together first at **60 cm** = LCM
- **prime factors**: each prime as many times as the number that has it **most**
- $18 = 2 \times 3^2$, $24 = 2^3 \times 3$, $36 = 2^2 \times 3^2$ → LCM = $2^3 \times 3^2$ = **72**

Division method

1. write the numbers in a row
2. divide by a prime that divides **at least two**; carry the others down unchanged
3. stop when no prime divides two of them
4. LCM = all the divisors × the last row

- 24, 36, 56 → $2 \times 2 \times 2 \times 3 \times 3 \times 7$ = **504**; the prime taken first doesn't matter
- groups of 3, 5, 6 with no one left → at least **30** students
- bells every 45 and 60 min, together at 10:00 → LCM 180 min = 3 h → **1:00 p.m.**
- smallest number that, **less 5**, divides by 18, 24, 36: $x - 5 = 72$ → **77**; **plus 5**, by 32, 64, 192: $x + 5 = 192$ → **187**

HCF	LCM
each prime the **fewest** times	each prime the **most** times
never more than the smallest number	never less than the largest
most people, biggest equal piece	next time together, smallest group size
18, 24, 36: 6	18, 24, 36: 72

Back to the ropani

- 1 ropani = 5,476 $ft^2 = 2^2 \times 37^2 \rightarrow \sqrt{5476}$ = **74 ft** square
- 1 aana = 5,476 ÷ 16 = 342.25 ft^2 = 18.5 ft square (4 × 4 aana = 1 ropani; 4 × 18.5 = 74)
- 1 bigha = 72,900 $ft^2 = 2^2 \times 3^6 \times 5^2$ → **270 ft** square; about 13⅓ ropani
- 1 kattha = 3,645 $ft^2 = 3^6 \times 5$: the 5 has no partner → not a square of whole feet

COMMON MISTAKES

✗ $\sqrt{95481} = 39$
✓ a divisor that can't go in gives 0 in the quotient: 309 (3 pairs, 3 digits).

✗ 95,481 paired from the left: 95 | 48 | 1
✓ from the right: 9 | 54 | 81.

✗ next divisor = the quotient so far
✓ double it, then write the new digit after it.

✗ 12,675 is a square (it ends in 5)
✓ 3 has no partner: not a square.

✗ cube root by pairs
✓ groups of three.

✗ HCF of 100, 125, 200 = $2 \times 5 \times 5$
✓ 125 has no 2: HCF 25.

✗ LCM = the primes common to all
✓ each prime the most times it appears: 72 for 18, 24, 36.

✗ "less 10" question: answer the HCF 28
✓ $x - 10 = 28$, so 38.