# Unit 1 · Set

STUDY NOTES

## Key terms

**Set**: a well-defined collection; for anything at all, you can say for certain whether it belongs.

**Empty set** (**null**, **void** set): no element at all; { } or ∅ (read "phi" or "the empty set").

**Finite set**: the counting of its elements ends: a fixed number of elements.

**Infinite set**: the elements never run out: $N = \{1, 2, 3, \ldots\}$.

**Equivalent sets**, $A \sim B$: the same **number** of elements.

**Equal sets**, $A = B$: exactly the same **elements**, in any order.

**Universal set** $U$: holds every element under discussion; every other set is made from it.

**Subset**: $A$ is a subset of $B$ if **every** element of $A$ is in $B$.

**Proper subset**, $B \subset A$: some elements of $A$, at least one left out.

**Improper subset**, $B \subseteq A$: all the elements of $A$, so $B = A$.

## Writing a set

- **describing**: $V = \{\text{vowels of the English alphabet}\}$
- **listing**: $V = \{a, e, i, o, u\}$
- **set-builder**: $V = \{x : x \text{ is a vowel of the English alphabet}\}$
- $a \in V$ (is an element of); $b \notin V$

## Empty set

- all ∅
  - the months with 32 days (English calendar)
  - boys in a Kanya school
  - natural numbers between 1 and 2
- still a set: the rule is well defined; a box with nothing in it is still a box
- {0} is **not** empty: one element, 0; {∅} also has one element
- odd numbers divisible by 2 → ∅
- even primes → {2}
- whole numbers less than 1 → {0}
- empty or not can depend on **when** you ask: women Prime Ministers of Nepal got a first element on 12 September 2025

## Finite or infinite?

- infinite: list the first few, then "…" (for ever): $E = \{2, 4, 6, 8, \ldots\}$
- **infinite** (no last one)
  - natural numbers
  - whole numbers
  - even numbers
  - odd numbers
  - multiples of 1,000
- {whole numbers less than 30} = {0, 1, …, 29}: finite, 30 elements; factors of 24: 8 elements
- **very big is still finite**: the counting would end
  - people of Nepal
  - rice in a sack
  - sand on a beach

## Equivalent and equal

- test equivalence by **pairing off**: both run out together → equivalent; one left over → not
- $\{a, b, c\} \sim \{1, 2, 3\}$; {g, o, a, t} and {d, o, g}: not equivalent (4 and 3)
- nothing in common needed: {Sunday, Monday} ∼ {tea, milk}
- $\{1, 2, 3\} = \{3, 1, 2\}$; $\{1, 2, 3\} \sim \{x, y, z\}$ but ≠
- **equal** ⇒ **equivalent**; equivalent ⇏ equal
- to make $\{a, b, c, d\}$ and $\{a, b, c, d, e\}$ equal: add e to the first, or take e out of the second
- repeats don't count: letters of "seen" = {s, e, n}, 3 elements

### Testing two sets

1. count: different numbers → neither
2. same number → equivalent
3. exactly the same elements → equal too

- {l, i, n, e} and {f, i, l, e}: ∼, not = (n and f)
- pairing works for infinite sets: 1 ↔ 2, 2 ↔ 4, 3 ↔ 6, … → natural numbers ∼ even numbers (Hilbert's hotel)

## Universal set

- drawn as a **rectangle**; each set taken from it a **ring** inside
- $U$ decides what a question is about: $U = \{1, 2, \ldots, 10\} \to$ even numbers $= \{2, 4, 6, 8, 10\}$; 12 is not in the discussion
- $U =$ {natural numbers up to 20}: multiples of 6 $= \{6, 12, 18\}$ (no 24, no 0)
- **a** universal set: more than one can fit; for $\{2, 4, \ldots, 14\}$ and $\{1, 3, \ldots, 9\}$: $\{1, 2, \ldots, 14\}$ or any bigger set
- first list $U$, then pick from it

## Subsets

- one element of $A$ outside $B \to A$ is **not** a subset of $B$
- a subset is a ring **inside** another ring
- $P = \{1, 2, 3\}$: $\{1, 2\} \subset P$; $\{3, 1, 2\} \subseteq P$
- **empty set**: a proper subset of every non-empty set (it can't have an element outside)
- **equal sets**: improper subsets of each other
- every set has exactly **one** improper subset: itself
- read like $<$ and $\leq$: the extra line in $\subseteq$ allows "or the same"
- some books write $\subset$ for every subset: say the kind in words too

| Elements | Subsets | Proper | Improper |
|---|---|---|---|
| 1: {1} | { }, {1}: 2 | 1 | 1 |
| 2: {1, 2} | { }, {1}, {2}, {1, 2}: 4 | 3 | 1 |
| 3: {1, 2, 3} | 8 | 7 | 1 |
| 4 | 16 | 15 | 1 |

- each new element **doubles** the count: old subsets without it + the same with it
- listing all
  - none
  - one at a time
  - two at a time, ...
  - all of them

### Element or subset?

- $2 \in \{1, 2, 3\}$: an element takes $\in$
- $\{2\} \subset \{1, 2, 3\}$: a set takes $\subset$

## Back to the shoes

- $U = 240$ school shoes $\to S =$ size 38 (56) $\to K =$ black (18) $\to$ under Rs 1,000: $\emptyset$
- each filter: a **proper subset** of the list before; "No items found" = the empty set; all finite

### COMMON MISTAKES

✗ {0} is the empty set
✓ it has one element, 0; empty is { } or $\emptyset$.

✗ writing $\{\emptyset\}$ for "empty"
✓ { } or $\emptyset$, never both together.

✗ the sand on a beach is infinite
✓ enormous, but the counting would end: finite.

✗ $\{l, i, n, e\} = \{f, i, l, e\}$ (same size)
✓ same number only: $\sim$, not $=$.

✗ $\{4, 3, 2, 1\} \subset \{1, 2, 3, 4\}$
✓ all the elements: improper, $\subseteq$ .

✗ the empty set is not a subset
✓ it is a subset of every set.

✗ $2 \subset \{1, 2, 3\}$ or $\{2\} \in \{1, 2, 3\}$
✓ $2 \in \{1, 2, 3\}$; $\{2\} \subset \{1, 2, 3\}$.

✗ multiples of 6 in $U = \{1, \ldots, 20\} = \{6, 12, 18, 24, \ldots\}$
✓ only those in $U$: {6, 12, 18}.