Unit 19 · Symmetry and Tessellation

STUDY NOTES

Key terms

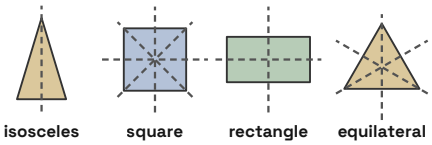
Symmetric figure: can be folded so that one half lies **exactly** on the other.

Line of symmetry (**axis of symmetry**): the fold line, drawn dotted.

Mirror image: each half of a symmetric figure is the other's; a mirror on the line shows the whole figure.

Tessellation: shapes covering a surface with **no gaps** and **no overlaps**; the shapes are its **tiles**.

Lines of symmetry



Every line of symmetry, dashed.

- isosceles triangle **1**: corner between the equal sides → middle of the base
- equilateral triangle **3**: each corner → middle of the side opposite
- square **4**: across, up and down, both diagonals
- rectangle **2**: across, up and down (**not** the diagonals)
- regular pentagon 5
- regular hexagon 6
- five-pointed star 5
- heart, arrow, arch, leaf: 1
- **0**
  - parallelogram (not a rectangle or rhombus)
  - right-angled triangle
  - lightning bolt
- test: **fold**; two equal halves are not enough (a rectangle's diagonal)

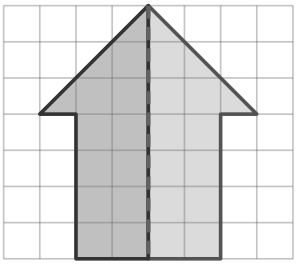
Capital letters (plain)

Line	Letters
up and down	A, M, T, U, V, W, Y
across	B, C, D, E, K
both	H, I, O, X
none	F, G, J, L, N, P, Q, R, S, Z

- N, S, Z look the same after **half a turn**, but no fold works: no line of symmetry
- a curly B or K may lose its line: test plain letters

Completing a symmetric figure

1. take one corner at a time
2. its image: the **same distance** from the line, on the **other side**
3. along a line crossing the dotted line at **90**°
4. join the images in the same order

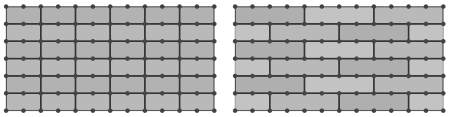


Each corner's image: as far on the other side of the dotted line.

- upright dotted line: 3 squares left → 3 squares right, same height
- slanting line through the grid's corners: up and right swap (up 5 → right 5)
- signs: arrows all going round one way, or stripes all slanting one way → not symmetric

Tessellation

- squares and rectangles of one size always tessellate: in rows and columns, or like **bricks** (each row moved along half a brick)
- also triangles (pointing up and down in turns) and hexagons (a honeycomb)



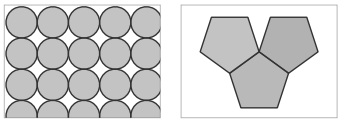
Rectangles on dots: in rows, or like bricks.

- the same tessellation can be colored many ways; like a chessboard: no two tiles sharing a side the same color

Why some shapes tessellate

- the angles round a point add to **360**°; the corners meeting there must make exactly 360°

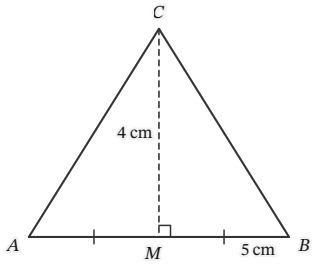
Tile	Corner	At a point
square, rectangle	90°	$4 \times 90° = 360°$ ✓
equilateral triangle	60°	$6 \times 60° = 360°$ ✓
regular hexagon	120°	$3 \times 120° = 360°$ ✓
regular pentagon	108°	$3 \to 324°$, gap 36°; $4 \to 432°$ ✗



Circles (no corners) always leave gaps; three regular pentagons leave a 36° gap.

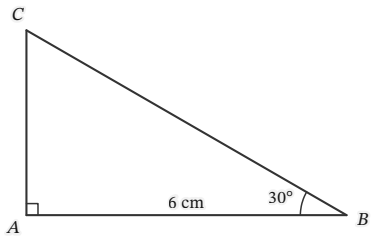
What a construction makes

- C on the perpendicular bisector of $AB \to CA = CB$: **isosceles** triangle



$AB = 5$ cm, C 4 cm up its perpendicular bisector: $CA = CB$.

- 60° at both ends of $AB \to$ **equilateral** triangle
- 90° at A, 30° at $B \to$ third angle 60°: **right-angled scalene**



90° at A, 30° at B: the arms meet at 60°.

- 120° at P and Q, same side, $RS \parallel PQ \to$ **parallelogram**; 80° at both, leaning in $\to$ **trapezium**
- top of an isosceles triangle joined to the midpoint of its base $\to$ two equal **right-angled** triangles

COMMON MISTAKES

✗ a rectangle's diagonal called a line of symmetry
✓ folded, the corners stick out: 2 lines only.

✗ N, S, Z given a line of symmetry
✓ they match after half a turn, not after a fold.

✗ an image point drawn the same distance but on the same side
✓ on the **other** side of the line.