Unit 14 · Lines and Angles

STUDY NOTES

Key terms

Line: goes on for ever both ways (arrowheads); no length.

Line segment: part of a line with two **end points**; has a length: $AB = 4$ cm.

Intersecting lines: cross at **one point**, the **point of intersection**.

Perpendicular lines: intersect at **90°**: $AB \perp CD$; a small square marks the right angle.

Parallel lines: never meet, however far made longer: $AB \parallel CD$; little arrowheads mark them.

Set squares: the **45° set square** (90°, 45°, 45°) and the **30°–60° set square** (90°, 60°, 30°).

Bisect: cut into two equal parts; **midpoint**: the middle of a segment.

Perpendicular bisector of AB: cuts AB into two equal parts at 90°.

Vertex, **arms**: the corner and the two sides of an angle; $\angle ABC$: vertex B (the middle letter), arms BA and BC.

Bisector of an angle: cuts it into two equal angles.

Pairs of lines

- perpendicular lines are intersecting lines too: at all **four** angles round the crossing, 90°
- tilted lines can be perpendicular: the angle counts, not the direction
- parallel: the distance straight across (at 90° to both) is the **same everywhere**; test it at both ends
- 3 cm apart at one end, 5 cm at the other → not parallel → intersecting
- short segments that don't touch can still be intersecting: think of the whole lines
- a door: side edges ||; top edge ⊥ each side
- in letters
  - H: two || sides, the bar ⊥ both
  - N: two || sides, the slant cuts both
  - T, L: ⊥
  - X: intersecting, not at 90°

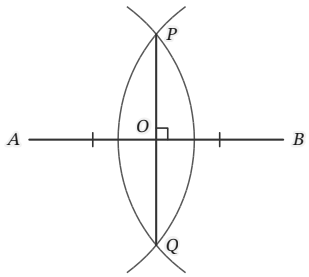
Drawing with set squares

- **parallel**: one set square along AB; a ruler or the other set square firmly against it; slide, draw along the **same edge** each time → lines $\parallel AB$
- **perpendicular**: ruler along AB; a set square's square corner on the ruler's edge; draw along its upright edge → $PQ \perp AB$
- two lines ⊥ the same line are || to each other
- hold the guide still, pressed down with spread fingers

Perpendicular bisector

- with a ruler and set square: mark the midpoint ($AB = 10$ cm → 5 cm), set square's square corner there
- with a protractor: midpoint C, then 90° at C
- with a **compass** (no measuring of the midpoint):

1. draw AB
2. needle on A, radius **more than half of** AB: long arcs above and below
3. **same radius**, needle on B: arcs cutting the first two at P and Q
4. join PQ; it cuts AB at O: $AO = OB$, $\angle AOP = 90°$



Arcs of the same radius from A and B cross at P and Q.

- $AB = 10$ cm
  - 4.5 cm arcs miss
  - 5 cm only touch (one point)
  - 6.5 cm cross twice
- $AB = 8$ cm → radius more than 4 cm (3 cm and 4 cm won't do)
- why: P and Q are each the same distance from A as from B; all such points lie on the perpendicular bisector

Measuring and drawing with a protractor

- its **center** on the vertex, its **zero line** along one arm; count up from **that** zero (inner or outer scale) to the other arm
- to draw: base line, center on its end, mark the degree, join
- a compass: needle (pointer), pencil, pencil holder

Kinds of angles

- **acute**: more than 0°, less than 90°; **right**: exactly 90°
- **obtuse**: more than 90°, less than 180°
- **straight**: exactly 180°, arms in one straight line
- **reflex**: more than 180°, less than 360°
- test with a square corner (edge along one arm)
  - other arm inside → acute

- along → right
- outside → obtuse

- 89° acute, not right
- 170° obtuse
- 200° reflex
- full turn **360°**; the angles on the two sides of the arms add to 360°
- clock
  - 3:00 → 90° and 270°
  - 6:00 → 180° both sides
  - 8:00 → 120° and 240°
  - each hour mark 30°
- 1:00 → 30°
- 4:00 → 120°
- 10:45 → 52½° (acute), other side 307½°
- parts of a right angle add to 90°: $x = 90 - 38 = 52$; on a straight line to 180°: $x = 180 - 65 = 115$

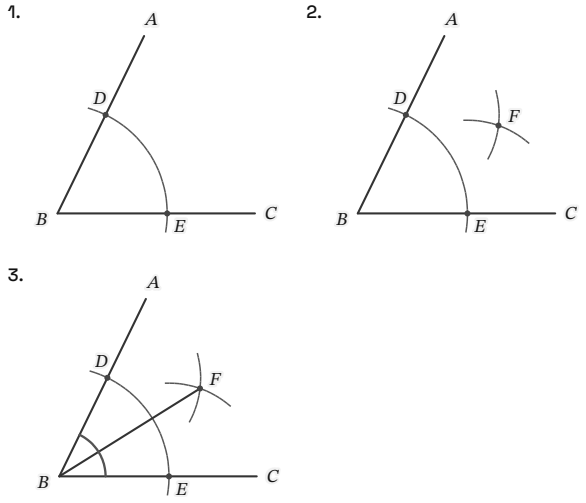
Measuring a reflex angle

- **way 1**: extend one arm past the vertex (straight angle, 180°), measure the extra: 180° + 50° = 230°
- **way 2**: measure the angle on the other side: 360° − 130° = 230°

Bisecting an angle

- **fold**: one arm onto the other through the vertex; the crease bisects
- **protractor**: measure, halve (70° ÷ 2 = 35°), draw that angle from one arm
- **compass**:

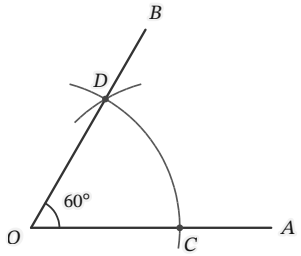
1. needle on the vertex B: an arc cutting BA at D and BC at E
2. needle on D, then on E: arcs of the **same radius** inside the angle, crossing at F
3. join BF: $\angle ABF = \angle CBF$



Bisecting $\angle ABC$ in three steps: 1. arc from B cuts at D, E; 2. equal arcs from D and E meet at F; 3. join BF.

- each half
  - 140° → 70°
  - 130° → 65°
  - 50°, 60°, 100° → 25°, 30°, 50°

Constructing angles with a compass



One step of the radius round the arc gives 60°: $OC = CD = OD$.

Angle	How
60°	arc from O cutting OA at C; same radius from $C \to D$; draw OD
120°	step the same radius round the arc **twice**
30°	bisect 60°
90°	bisect the angle between 60° and 120°
45°	bisect 90°; or halfway between 30° and 60°

- why 60°: OC, CD, OD all one radius → a triangle with three equal sides, each angle 60°
- 15°: bisect 30°
- 75°: bisect between 60° and 90°
- 20°: not possible with compass and ruler

With set squares

- 30°, 45°, 60°, 90°: one corner of a set square
- corners side by side
  - 120° = 90° + 30°
  - 150° = 90° + 60°
  - 135° = 90° + 45°
- folding: two folds at 90°; the square corner folded in half → 45°

The carpenter

- square corner: both pieces lie exactly along the set square's edges; a gap → not 90°
- parallel shelves: both drawn ⊥ one upright strip → ||; check the gap at both ends
- big corners: sides 3, 4 and 5 units → right angle between the 3 and the 4

COMMON MISTAKES

✗ arcs from A of radius exactly half AB
✓ more than half, or the arcs only touch.

✗ the compass opened wider for the arc from B
✓ the same radius from both ends.

✗ a reflex angle read straight off the protractor
✓ 180° + extra, or 360° − the other side.

✗ 89° called a right angle
✓ less than 90°: acute.

✗ lines 3 cm and 5 cm apart called parallel
✓ the gap must be the same everywhere.