Unit 13 · Equation, Inequality and Graph

STUDY NOTES

Key terms

Mathematical statement: a sentence about numbers, in words or signs; true or false.

Open statement: has a variable; can't be called true or false until its value is known: x is a factor of 18.

Equation: an open statement with $=$ between two expressions: $x + 2 = 5$.

Solution: the value that makes the equation true ($x = 3$); finding it is **solving**.

Axiom: a rule taken as true from the start.

Inequality: $a > b$ or $a < b$; **equality**: $a = b$.

Signs

Sign	Read	In words
$<$	is less than	less than 20: $x < 20$
$>$	is greater than	
$\leq$	less than or equal to	at most 12: $x \leq 12$
$\geq$	greater than or equal to	at least 3: $x \geq 3$
$\neq$	is not equal to	$y \neq 3$

- "not less than 7" $= 7$ or more: $y \geq 7$
- the point faces the smaller number: $3 < 8$ and $8 > 3$ say the same

True, false or open

- true: $3 + 5 = 8$; false: product of 3 and 5 is 8 (it is 15)
- open: x is a factor of 18 (true for 1, 2, 3, 6, 9, 18; false for 4); $y + 2 = 9$; odd x between 9 and 12 $\rightarrow$ 11

Equality axioms

- do the same to both sides (never $\div 0$) $\rightarrow$ still equal
  - **addition**
  - **multiplication**
  - **subtraction**
  - **division**
- a balance stays level when the same is added to or taken off both pans: $x + 2 = 7 \rightarrow x = 5$

Solving

1. undo what was done to x, the **last step first**
2. add to undo $-$; subtract to undo $+$; multiply to undo $\div$; divide to undo $\times$
3. do it to the **whole** of both sides
4. **check**: put the answer back into the equation

Equation	Step	Solution
$x - 3 = 5$	$+ 3$ both sides	$x = 8$
$x + 2 = 9$	$- 2$ both sides	$x = 7$
$x/3 = 3$	$\times 3$ both sides	$x = 9$
$7x = 49$	$\div 7$ both sides	$x = 7$
$5x + 3 = 18$	$- 3$, then $\div 5$	$x = 3$
$2x + 5 = 21$	$- 5$, then $\div 2$	$x = 8$

- words: 4 added to 3 times a number is 22: $3x + 4 = 22 \rightarrow 3x = 18 \rightarrow$ **6** (check $3 \times 6 + 4 = 22$ ✓)
- $2 - x = -4$: add x: $2 = x - 4 \rightarrow x = 6$
- $9 - 2x = 13$: $-4 = 2x \rightarrow x = -2$
- $x/3 + 7 = 25 \rightarrow 54$; income $-$ spending $=$ saving: $x - 16{,}500 = 12{,}500 \rightarrow$ Rs 29,000; parts of a line add up: $x + 2x = 18 \rightarrow 6$ cm

Trichotomy law

- for any two numbers a and b, **exactly one** is true
  - $a > b$
  - $a = b$
  - $a < b$
- on the number line, the number further **right** is greater, below zero too
  - $-6 < 2$
  - $11 < 15 - 2$
  - $-5 > -7$
  - $-6 < 2 - 5\ (= -3)$
  - $-14 < -13$
  - $-8 + 1 = -3 - 4$

Inequalities on a number line

- a thick line from the number, with an arrow; **filled dot**: the number itself included ($\leq$, $\geq$)
- **empty dot**: not included ($<$, $>$)
- $x \geq -2$: -2 and every number to its right; $x > -2$: to its right, not -2

The vote

- vote: $a \geq 18$ (completed years): filled dot at 18, line to the right; 17 can't
- House of Representatives: $a \geq 25$; at 23: $23 \geq 18$ true, $23 \geq 25$ false

COMMON MISTAKES

✗ $x + 2 = 9$, so $x = 9$ (2 taken off the left only)
✓ both sides: $x = 7$; check $9 + 2 \neq 9$.

✗ $5x + 3 = 18$: $\div 5$ first
✓ undo the last step first: $- 3$, then $\div 5$.

✗ $x \geq 18$ drawn with an empty dot
✓ 18 is included: filled dot.

✗ "at least 3" written $x > 3$
✓ 3 or more: $x \geq 3$.

✗ $-5 < -7$
✓ -5 is further right: $-5 > -7$.