Unit 12 · Algebraic Expression

STUDY NOTES

Key terms

Constant: one fixed value: 100, 7.

Variable: can take more than one value: the number of rounds x.

Term: a number, a letter, or numbers and letters multiplied: 100, $10x$, $50y$.

Coefficient: the number in front of the letters: 3 in $3xy$.

Like terms: the same letters, each to the same power: $3a^2$, $7a^2$.

Unlike terms: letters or powers differ: $7x^3$, $9x^2$.

Algebraic expression: variables and constants joined by $+$, $-$, $\times$, $\div$.

Monomial: one term ($3y$); **binomial**: two ($x + 3$); **trinomial**: three ($a + 2b + 3c$).

Like or unlike?

- cover the numbers: are the letters **and** their powers exactly the same?
- $2x$, $5x$: like; $3x$, $4y$: unlike
- $7x^3$, $9x^2$: unlike (same letter, different power)
- $3a^2b$, $3b^2a$: unlike: $a \times a \times b$ against $a \times b \times b$
- $4x^2y$, $7x^2yz$: unlike (extra z)
- order of letters doesn't matter: $3b^2a = 3ab^2$
- $5x^2 + 6x + 7$: three unlike terms, a trinomial

Words into algebra

- x marbles and 3 more: $x + 3$; y pencils, 7 given away: $y - 7$
- y books and twice as many more: $y + 2y = 3y$; x shared between two: $x/2$
- park: Rs 100 entry + Rs 10 a round for x rounds: Rs $(100 + 10x)$
- a line in parts: x cm, $2x$ cm, $3x$ cm $\rightarrow$ total $6x$ cm

Adding and subtracting

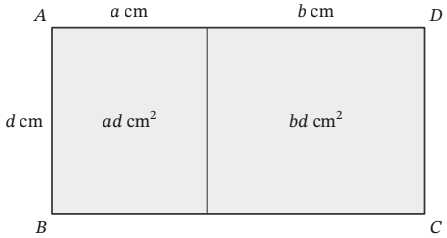
- add or subtract the **coefficients** of like terms; the letters stay: $3x + 4x = 7x$
- unlike terms stay apart: $2x + 3y$ stays $2x + 3y$ (2 books and 3 pens aren't 5 of anything)
- bring like terms together first: $2x + 3y + 7x = 9x + 3y$
- $x = 1x$: $x + 4x + 5x = 10x$
- $2a^2b + 3a^2b + 6a^2b = 11a^2b$
- $12x^2y + 15x^2y + 5x^2yz = 27x^2y + 5x^2yz$
- $(2a + 3) + (3a + 5) = 5a + 8$

Subtracting

- "**subtract A from B**" $= B - A$: write the one after "from" first
- a **minus** before a bracket changes the sign of **every** term inside; a plus changes nothing
- $8x - x = 7x$; $5x - (x + 4x) = 0$
- $6x - (2x - 3y) = 6x - 2x + 3y = 4x + 3y$
- $5a^3b - (9a^3b - 7a^2b) = -4a^3b + 7a^2b$
- $5p^2qr - (2p^2q + 11p^2q) = 5p^2qr - 13p^2q$
- $(6a + 4b) - (a + b) = 5a + 3b$

Multiplying

- **monomial × monomial**
  - multiply the numbers
  - number in front
  - multiply the letters
- $3a \times 2b = 6ab$: six small rectangles of ab in a $3a$ by $2b$ rectangle
- $2x \times 3x = 6x^2$
- $3y \times 4y \times 5y = 60y^3$
- $5c \times 7d = 35cd$
- $2/3\ a \times 6b \times c/4$: numbers $2/3 \times 6 \times 1/4 = 1 \rightarrow abc$ (a 1 isn't written)
- add or multiply? $2x + 3x = 5x$, but $2x \times 3x = 6x^2$
- **monomial × binomial**: multiply **each** term in the bracket, then add: $(a + b) \times d = ad + bd$
- $3x(4y + 5z) = 12xy + 15xz$; $8a(2a + 9ac) = 16a^2 + 72a^2c$
- a product = the area of a rectangle with those two sides
- rectangle $3x$ by $2x$: area $6x^2$, perimeter $2(3x + 2x) = 10x$; $x = 7$ cm $\rightarrow$ 294 cm^2, 70 cm
- cuboid: $V = l \times b \times h$: $3x \times 2y \times 4z = 24xyz$



$(a + b) \times d = ad + bd$: the rectangle in two parts.

Value of an expression

1. put the numbers in place of the letters
2. powers, then × and ÷, then + and −

- $x = 2, y = 3$: $x^2 + y^2 = 13$; $x^2 + 2xy + y^2 = 25$
- $x^3 - 2x^2y + y^2 = 8 - 24 + 9 = 17 - 24 = \mathbf{-7}$ (below zero)
- $a = 3$: $a(a + 4) = 3 \times 7 = 21$
- $a = -2$: $3a = -6$; $(-2)^2 = 4$

Dividing

- **monomial ÷ monomial**: divide the numbers, cancel letters in both: $4xy \div 2x = 2y$
- write the factors out
  - $12ab \div 4a = 3b$
  - $15p^2q \div 5p = 3pq$
  - $18a^2b \div 3ab = 6a$
- **binomial ÷ monomial**: divide **each** term: $(6x^2 + 9x) \div 3x = 2x + 3$
- $(21x^3y^2 - 56x^2y^3) \div 7x^2y^2 = 3x - 8y$
- area ÷ one side = the other side; **check** by multiplying back: $3x(2x + 3) = 6x^2 + 9x$
- $9x^2y$ cm², one side $3xy$ cm → other side $3x$ cm
- a diagonal halves a rectangle: triangle = area ÷ 2: $(4x^2 + 2xy) \div 2 = 2x^2 + xy$
- rectangle $(2x + y)$ by $2x$ → a square: cut the length by y; square $4x^2$

The stationery shop

- bill = $40x + 15y$ (x books at Rs 40, y pens at Rs 15)
- twice as many of each: $2(40x + 15y) = 80x + 30y$
- together: $(40x + 15y) + (80x + 30y) = 120x + 45y$
- $x = 2, y = 1$: $120 \times 2 + 45 =$ Rs $285 = 95 + 190$ ✓
- shared equally: $(120x + 45y) \div 2 = 60x + 22.5y$

COMMON MISTAKES

✗ $2x + 3y = 5xy$
✓ unlike terms stay apart: $2x + 3y$.

✗ $7x^3$ and $9x^2$ called like
✓ same letter, different power: unlike.

✗ subtract $2x - 3y$ from $6x$ as $(2x - 3y) - 6x$
✓ after "from" first: $6x - (2x - 3y) = 4x + 3y$.

✗ $6x - (2x - 3y) = 4x - 3y$
✓ a minus before a bracket changes every sign: $+3y$.

✗ $4a \times 5a = 9a$
✓ that is the sum; product $20a^2$.

✗ $(6x^2 + 9x) \div 3x = 2x + 9x$
✓ divide every term: $2x + 3$.

✗ $3x(4y + 5z) = 12xy + 5z$
✓ multiply each term: $12xy + 15xz$.

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-6-math/algebraic-expression