Unit 11 · Indices

STUDY NOTES

Key terms

Base: the number or letter that is multiplied: x in x^4.

Index (**power**, **exponent**): how many times the base is multiplied: 4 in x^4; plural **indices**.

Index form: x^4; **expanded form**: $x \times x \times x \times x$.

a **squared**: $a^2 = a \times a$, the area of a square of side a.

a **cubed**: $a^3 = a \times a \times a$, the volume of a cube of side a.

Formulas

$A = a^2$	a side **Use:** area of a square
$V = a^3$	a side **Use:** volume of a cube
$V = l \times b \times h$	all three equal b: a cube, $V = b^3$ **Use:** volume of a cuboid

Reading a power

- x^4 read "x to the power 4"
- $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$: Diya's 5 folds, 32 layers
- the index counts the folds; the layers double: $2^6 = 64$, $2^7 = 128$
- $3^4 = 3 \times 3 \times 3 \times 3 = 81$
- $5^2 = 25$: a 5 cm square has area 25 cm^2
- $3^3 = 27$ small cubes in a 3 cm cube; $4^3 = 64$
- $cm^2 = cm \times cm$; $cm^3 = cm \times cm \times cm$: units are powers too

Index form and expanded form

1. count how many times each base is multiplied
2. that count is its index
3. different bases keep their own indices

- $3 \times 3 \times 3 \times 3 = 3^4$
- $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2 \ (= 8 \times 9 = 72)$
- $5 \times 5 \times 5 \times 7 \times 7 = 5^3 \times 7^2$
- $c^2 = c \times c$; $y^5 = y \times y \times y \times y \times y$
- $x^2\,y^2 = x \times x \times y \times y$; $p^3\,q^2 = p \times p \times p \times q \times q$

Multiplying powers of the same base

- count all the factors: $a^2 \times a = a \times a \times a = a^3$; $x^3 \times x^2 = x^5$
- a number in front is just another factor and stays in front: $2x \times x = 2x^2$; $3y \times y \times y = 3y^3$
- numbers in front multiply too: $3x^3 \times 6x^3 = 3 \times 6 \times x^6 = 18x^6$
- the index belongs only to what it touches: $2x^2 = 2 \times x \times x$
- with letters: rectangle x by y: area xy; square of side y: y^2; box a, b, c: $V = abc$
- box $8a$ by $4a$ by a: $V = 8 \times 4 \times a \times a \times a = 32a^3$
- $x = 3$: $2x^2 = 2 \times 9 = 18$; $(2x)^2 = 4x^2 = 36$
- Grade 8: "add the indices"

Powers grow fast

- 32 layers of 0.1 mm paper ≈ 3 mm; 7 folds ≈ 13 mm
- record: 12 folds, $2^{12} = 4096$ layers (Britney Gallivan, 2002)
- about 15 folds would reach a 3 m ceiling: $2^{15} = 32{,}768$ layers ≈ 3.3 m
- chessboard: the 64th square needs 2^{63} grains, a 19-digit number

Where it is used

- land: 1 ropani = 16 aana = 64 paisa = 256 daam: 4^2, 4^3, 4^4
- computers: a byte of 8 bits has $2^8 = 256$ patterns; memory of 64, 128, 256 GB
- paper: A4 is A0 (1 m^2) folded in half 4 times: $1 \div 2^4 = 1/16$ m^2

COMMON MISTAKES

✗ $2^3 = 2 \times 3 = 6$
✓ $2 \times 2 \times 2 = 8$.

✗ $3^4 = 12$
✓ four 3s multiplied: 81.

✗ $x^2 = 2x$
✓ $x \times x$; if $x = 5$: 25, not 10.

✗ 3^4 given as 64
✓ base and index swapped: 64 is 4^3.

✗ $2x^2$ with $x = 3 = 36$
✓ only x is squared: 18.

✗ $2^3 \times 3^2 = 6^5$
✓ different bases keep their own indices.