Unit 4 · Fraction

STUDY NOTES

Key terms

Numerator: how many equal parts are taken (3 in 3/4).

Denominator: how many equal parts the whole is cut into (4 in 3/4).

Like fractions: same denominator, parts the same size: 2/7, 5/7.

Unlike fractions: different denominators: 3/4, 5/8.

Mixed number: a whole number and a fraction: 2 1/4 = 9/4 as an **improper fraction**.

Equivalent fractions: the same amount with different numbers: 1/2 = 2/4 = 4/8.

Lowest terms: no number but 1 divides both numerator and denominator.

Reciprocal: the number turned upside down: 2 → 1/2, 3/4 → 4/3.

Formulas

$\frac{a}{b} = \frac{a \times n}{b \times n}$	the same n top and bottom, not 0 **Use:** equivalent fraction
$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$	tops × tops, bottoms × bottoms **Use:** multiplying
$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$	turn over the divisor only **Use:** dividing
$\frac{a}{b} \times \frac{b}{a} = 1$	a number × its reciprocal **Use:** checking a reciprocal

Equivalent fractions and lowest terms

- multiply (or divide) top and bottom by the **same** number
- folding a strip: 1/2 = 2/4 = 4/8; the colored part never moves
- 1/2 = 2/4 = 3/6 = 4/8 = 5/10; 3/4 = 6/8 = 9/12 = 12/16 = 15/20 (endlessly many)
- chosen denominator: 3/4 with 12 at the bottom: 4 × 3 = 12 → 9/12
- lowest terms: 24/36 = 12/18 = 6/9 = **2/3** (or ÷ 12 at once); 18/24 = **3/4** (÷ 6)
- give every final answer in lowest terms
- **adding** the same number to top and bottom: not equivalent: 1/2 → 2/3 is more than a half
- 1 ropani = 16 aana
  - 8 aana = 8/16 = 1/2 ropani
  - 4 aana = 1/4
  - 2 aana = 1/8

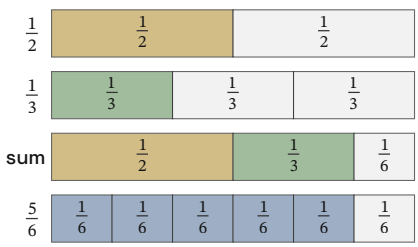
Comparing and ordering

- like fractions: bigger numerator → bigger fraction: 5/7 > 2/7
- unlike: make the denominators the same (the **LCM** of the denominators), then compare numerators
- LCM: the first number in both lists of multiples
- one denominator goes into the other: change only one: 3/5 = 6/10 and 5/10
- 2/3 and 3/4: LCM 12 → 8/12 < 9/12
- 3/5 or 4/7: 21/35 > 20/35 (too close to judge by eye)
- ascending: 2/3, 3/5, 7/10 → 20/30, 18/30, 21/30 → **3/5, 2/3, 7/10**
- mixed or improper: 2 1/4 = 9/4 > 9/5 (quarters bigger than fifths; 9/5 = 1 4/5 < 2)
- **bigger denominator → smaller parts**: 1/8 < 1/4

Adding and subtracting

1. find the LCM of the denominators
2. change each fraction to that denominator
3. add or subtract the numerators; keep the denominator
4. lowest terms; an improper answer also as a mixed number

- 1/2 + 2/5 = 5/10 + 4/10 = **9/10**
- 9/10 + 1/6 = 27/30 + 5/30 = 32/30 = 16/15 = 1 1/15
- LCM from primes: 10 = 2 × 5 (needs × 3), 6 = 2 × 3 (needs × 5) → 30
- 11/24 − 3/8 = 11/24 − 9/24 = 2/24 = **1/12**
- **never add the denominators**: 1/2 + 1/3 = 3/6 + 2/6 = 5/6, not 2/5
- two fractions add up to 6 1/6, one is 2 1/3 → the other = 6 1/6 − 2 1/3 = 3 5/6



Strips: a half and a third cover 5 sixths.

Mixed numbers

- wholes and fractions apart, or improper fractions first
- 2 1/3 + 1 3/4: 3 wholes + 13/12 = 3 + 1 1/12 = 4 1/12 (carry a whole)
- 5 1/4 − 2 2/3 = 5 3/12 − 2 8/12: borrow a whole → 4 15/12 − 2 8/12 = 2 7/12
- or 21/4 − 8/3 = 63/12 − 32/12 = 31/12 = 2 7/12 (no borrowing)

Multiplying

- whole number × fraction: **multiply the numerator**, keep the denominator: 4 × 3/5 = 12/5 = 2 2/5
- 5 × 3/5 = 15/5 = 3
- "**of**" means ×: 7/8 of 56 = 56 × 7/8 = 49 (one eighth 56 ÷ 8 = 7, then × 7: divide first, small numbers)
- 1/3 of 6 oranges = 2; 3/4 of an hour = 60 × 3/4 = 45 min (a quarter to eight, पौने आठ, is 7:45)
- fraction × fraction: 3/4 × 1/2 = 3/8 (a square cut 4 one way, 2 the other: 8 parts, overlap 3)
- **cancel** a factor shared by a top and a bottom first: 3/5 × 5/6 = 1/2
- 3/4 of 1/2 kg = 3/8 kg = 3,000/8 g = 375 g
- **what is left**: the whole − the part: Rs 1,000, 3/5 lent = Rs 600 → Rs 400 left (the other 2/5)
- glass 3/4 full, 2/3 of it drunk: 2/3 × 3/4 = 1/2 glass; left 3/4 − 1/2 = 1/4
- mixed numbers → improper first: 2 1/2 × 1 1/3 = 5/2 × 4/3 = 20/6 = 10/3 = 3 1/3
- a fraction less than 1 × a number → **less** than the number
- recipe for 4 needs 3/4 cup: for 2, 1/2 × 3/4 = 3/8 cup; for 6, 3/2 × 3/4 = 1 1/8 cups

Dividing

- dividing asks **how many fit**
- change **÷ into ×** and turn the **divisor** into its reciprocal
- ÷ 2 = taking half = × 1/2: 4/5 ÷ 2 = 4/5 × 1/2 = 2/5; 1/3 ÷ 2 = 1/6
- 1 2/5 ÷ 6 = 7/5 × 1/6 = 7/30
- 1/3 liter paints 3/5 door → 1 liter: 3/5 ÷ 1/3 = 3/5 × 3 = 9/5 = 1 4/5 doors
- 3 5/9 ÷ 2 2/3 = 32/9 × 3/8 = 96/72 = 4/3 = 1 1/3; 6 ÷ 1 1/5 = 6 × 5/6 = 5
- 3 liters, 3/4 liter a door: 3 ÷ 3/4 = 3 × 4/3 = **4 doors**
- reciprocal of a mixed number: improper first: 2 1/3 = 7/3 → **3/7**
- dividing by a fraction less than 1 → **bigger** answer (smaller pieces, more fit)

Fields in ropani and aana

- 3/4 = 6/8 > 5/8: Pemba's family has 1/8 ropani more; together 11/8 = 1 3/8 ropani
- seed bags of 1/8 ropani: 3/4 ÷ 1/8 = 3/4 × 8 = 6 bags
- in aana: 12 and 10 aana; together 22 aana = 1 ropani 6 aana: sixteenths are a common denominator for halves, quarters, eighths
- thirds don't fit sixteenths (3 doesn't divide 16): then the LCM is needed
- hills
  - 1 ropani = 16 aana
  - 1 aana = 4 paisa
  - 1 paisa = 4 daam
- Tarai: 1 bigha = 20 kattha, 1 kattha = 20 dhur

COMMON MISTAKES

✗ 1/2 + 1/3 = 2/5
✓ never add the denominators: 3/6 + 2/6 = 5/6.

✗ 1/2 = (1 + 1)/(2 + 1) = 2/3
✓ only × or ÷ both keeps the amount.

✗ 1/8 > 1/4 because 8 > 4
✓ bigger denominator, smaller parts: 1/8 < 1/4.

✗ 2/10 equivalent to 2/5
✓ only the bottom changed; 6/15 is.

✗ 2 1/2 × 1 1/3 = 2 1/6 (wholes and parts apart)
✓ improper first: 3 1/3.

✗ 2/3 ÷ 1/6 = 3/2 × 1/6 = 1/4
✓ turn over the divisor: 2/3 × 6 = 4.

✗ reciprocal of 2 1/3 = 3 1/2
✓ 7/3 → 3/7.

✗ 3/4 of 20 = 5
✓ 5 is one quarter; 3/4 is 15.

✗ 9/12 given as the final answer
✓ lowest terms: 3/4.