Unit 3 · Integers

STUDY NOTES

Key terms

Negative numbers: numbers less than 0: -1 ("minus one", "negative one"), -2, -3, ...

Positive numbers: numbers greater than 0; $+3$ and 3 are the same.

Integers: negative whole numbers, 0 and positive whole numbers: $Z = \{..., -3, -2, -1, 0, 1, 2, 3, ...\}$.

Origin: the point 0 on the number line.

Ascending order: smallest to biggest (left to right); **descending**: biggest to smallest.

Why we need them

- whole numbers: $+$ and $\times$ always work; $-$ only when the first is not smaller
- $2 - 3$: 2 steps right from 0, then 3 steps left $\rightarrow$ one step past 0 $\rightarrow$ **-1**
- $4 - 6 = -2$; $3 - 6 = -3$
- on the number line: adding $\rightarrow$ right; subtracting $\rightarrow$ left

The integers

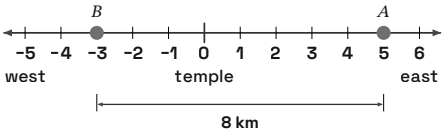
- **positive integers** $Z^+ = \{+1, +2, +3, ...\}$: right of the origin
- **zero**: neither positive nor negative, the border
- **negative integers** $Z^- = \{-1, -2, -3, ...\}$: left of the origin
- dots at **both** ends: no biggest, no smallest integer
- 3 and -3: same distance from 0, opposite sides
- -4: 4 steps **left** of 0
- 4 units **left** of a number: subtract 4: $3 \rightarrow -1$, $-2 \rightarrow -6$
- integers **between** two: the ends left out: between -2 and 3 $\rightarrow -1, 0, 1, 2$
- not integers: parts such as 1/2 (between 0 and 1); 0 **is** an integer
- Z from German *Zahlen*, "numbers"

Comparing

- the number further **right** is bigger
- every positive $> 0 >$ every negative
- two negatives: smaller number after the minus is bigger: $-2 > -6$
- $-7 < 2$
- $-3 > -9$
- $-8 < -1$
- colder night = smaller temperature: -6 °C colder than -2 °C
- owing Rs 6 is worse than owing Rs 5: $-6 < -5$
- ascending: $-4, 7, 0, -10, 3 \rightarrow$ **$-10, -4, 0, 3, 7$**
- coldest to warmest (a made-up night, °C)
  - Mustang -8
  - Jumla -5
  - Kathmandu 2
  - Pokhara 6

Distance between two points

- opposite sides of 0: **add** the distances from 0: from -3 to $5 = 3 + 5 = 8$ steps
- same side: subtract: $+5$ and $+3 \rightarrow 5 - 3 = 2$
- temple at 0, east $\rightarrow$ right: Pratiksha $+5$ km, Abinash -3 km $\rightarrow$ **8 km** apart, not 2



East +, west −: A at +5, B at −3, 8 km apart.

Where it is used

- temperatures below freezing: Jumla, average January night about -4.8 °C
- heights from sea level, 0 m: Everest 8,849 m; Dead Sea surface about -440 m
- time zones: Nepal UTC +5:45; New York in winter UTC -5
- lift buttons: B1, B2 or -1, -2 for floors under the ground
- money received +, money owed −; Brahmagupta's negatives were debts

COMMON MISTAKES

✗ $2 - 3 = 1$
✓ that is $3 - 2$; $2 - 3 = -1$.

✗ $-6 > -5$ because $6 > 5$
✓ -6 is further left: $-6 < -5$.

✗ 0 called positive (or negative)
✓ neither: the border.

✗ 0 not an integer
✓ 0 is an integer, the origin.

✗ homes 5 km east and 3 km west are 2 km apart
✓ opposite sides: $3 + 5 = 8$ km.

✗ $-3, -1, 0, 2$ called descending
✓ smallest first: ascending.