Unit 2 · Whole Number

STUDY NOTES

Key terms

Natural numbers: the counting numbers, $N = \{1, 2, 3, \ldots\}$; smallest 1, no end (infinity).

Whole numbers: the natural numbers and 0, $W = \{0, 1, 2, 3, \ldots\}$; smallest 0.

Simplification: working out a mathematical sentence one step a line.

Divisible: divides exactly, nothing left over: $366 = 3 \times 122$.

Factor: a number that divides another exactly; $F_{12} = \{1, 2, 3, 4, 6, 12\}$.

Multiple: the number × 1, 2, 3, …; $M_5 = \{5, 10, 15, \ldots\}$, never ends.

Prime number: exactly two factors, 1 and itself: 2, 3, 5, 7, 11, 13, …

Composite number: more than two factors: 4, 6, 8, 9, 10, 12, …

Prime factorization: a number written as a product of primes.

Square number: a number times itself, $7 \times 7 = 7^2 = 49$; **square root** of 49 = 7.

HCF: highest common factor, the largest number dividing them all.

LCM: lowest common multiple, the smallest number they all divide.

Natural and whole numbers

- 0 is whole, **not natural**; every natural number is whole
- natural + natural, natural × natural: always natural; natural − natural: not always ($8 - 8 = 0$)
- ten digits 0–9; a digit's value depends on its **place**
- smallest with n digits: 1 then zeros (1000); largest: all 9s (9999); $999 + 1 = 1000$
- **largest** from given digits: biggest to smallest (851 from 1, 5, 8)
- **smallest**: smallest to biggest (158), but **never start with 0**: 5, 0, 1 → 105, not 015
- smallest with 6, 0, 2, 9: **2069** (smallest non-zero digit, then 0, then the rest)
- 3 digits → $3 \times 2 \times 1 = 6$ numbers; 4 digits → 24
- three **different** digits: largest 987, smallest 102
- sum and difference of the largest and smallest: 5, 0, 1 → $510 - 105 = 405$

Simplification

Order

1. small brackets ( )
2. curly brackets { }
3. big brackets [ ]
4. × and ÷ from the left, then + and − from the left

- inside each bracket: × and ÷ before + and −
- a bracket with one number left is no longer needed: $\{16\} = 16$
- a number before a bracket means **multiply**: $8\{17 - 15\} = 8 \times 2 = 16$
- brackets change the order: "add 5 and 3, then × 4" $= (5 + 3) \times 4 = 32$; $5 + 3 \times 4 = 17$
- 12 chocolates, keep 4, rest shared by 2: $(12 - 4) \div 2 = 4$; without brackets $12 - 4 \div 2 = 10$ ✗
- $[20 \times \{40 - 6 \times (7 - 2)\}] + 16 = [20 \times \{40 - 30\}] + 16 = 200 + 16 =$ **216**
- one "=" line for each step
- words into a sentence: "the sum of 182 and 8 divided by the difference of 75 and 65" $= (182 + 8) \div (75 - 65) = 19$; "17 from 3 times the sum of 7 and 4, divided by 8" $= \{3 \times (7 + 4) - 17\} \div 8 = 2$

Divisibility tests

Test for	Divisible when
2	last digit 0, 2, 4, 6 or 8 (even)
3	sum of the digits divisible by 3
5	last digit 0 or 5
7	cut off last digit, **double** it, subtract from the rest: answer divisible by 7 (repeat if big)
11	cut off last digit, subtract it from the rest, repeat: answer divisible by 11

- 12345: $1 + 2 + 3 + 4 + 5 = 15$ → divisible by 3
- 252: $25 - 2 \times 2 = 21$ → divisible by 7
- 2431: $243 - 1 = 242$; $24 - 2 = 22$ → divisible by 11
- 9: the sum of the digits divisible by 9

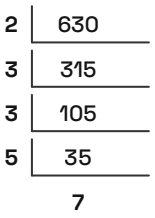
- why 2 and 5: ten is divisible by 2 and 5, so only the ones decide
- why 3: $2343 = (2 \times 999 + 3 \times 99 + 4 \times 9) + (2 + 3 + 4 + 3)$; the 9s, 99s, 999s divide by 3
- both 2 and 5 $\rightarrow$ last digit **0**
- other test for 11: alternate digits from the right, difference of the two sums: 2431: $(1 + 4) - (3 + 2) = 0$ ✓

Factors and multiples

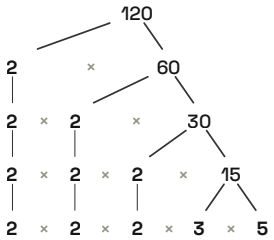
- $5 \times 6 = 30$: 5 and 6 are **factors** of 30; 30 is a **multiple** of 5 and of 6
- every number has factors 1 and itself (1 has only 1)
- find factors in **pairs**: $18 = 1 \times 18 = 2 \times 9 = 3 \times 6$ $\rightarrow F_{18} = \{1, 2, 3, 6, 9, 18\}$
- stop when the number tried is already in a pair
- a square number has an odd number of factors: $F_{16} = \{1, 2, 4, 8, 16\}$ (4 counted once)
- factors: few; multiples: never end
- common factors: list both sets, keep the members in both: $F_{12}, F_{18} \rightarrow \{1, 2, 3, 6\}$
- a bell every 45 min in 3 h: $180 \div 45 = 4$ rings after the start
- rectangles of squares: sides are the factors; a prime makes only one strip (1×7)

Primes and prime factorization

- **1 is neither** prime nor composite (one factor)
- **2 is the only even prime**
- every way of breaking a number up ends with the **same primes**: $24 = 2 \times 2 \times 2 \times 3$
- **division method**: divide by the smallest prime that goes in; repeat until the answer is prime
- **factor tree**: split into two factors; split each composite again until all branches are prime
- $486 = 2 \times 3 \times 3 \times 3 \times 3 \times 3$
- $630 = 2 \times 3 \times 3 \times 5 \times 7$
- $120 = 2 \times 2 \times 2 \times 3 \times 5$
- $588 = 2 \times 2 \times 3 \times 7 \times 7$



Division method: $630 = 2 \times 3 \times 3 \times 5 \times 7$.



Factor tree: $120 = 2 \times 2 \times 2 \times 3 \times 5$.

- **check**: multiply the primes back; 1 is never written
- the primes never run out (Euclid)

Squares and square roots

- squares up to 100: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100
- 7^2 read "7 squared": two 7s multiplied
- squaring and the square root undo each other: $6^2 = 36$; square root of 36 = 6
- 81 students in a square: $9 \times 9 = 81$ $\rightarrow$ 9 in a row
- 9 in a row, 3 left over: $9 \times 9 + 3 = 84$ students
- 8 in a row, 5 more needed: $8 \times 8 - 5 = 59$ students
- square of an odd number is odd: $3^2 = 9$, $5^2 = 25$

HCF

- **listing**: $F_{27} = \{1, 3, 9, 27\}$, $F_{36} = \{1, 2, 3, 4, 6, 9, 12, 18, 36\}$; common $\{1, 3, 9\}$ $\rightarrow$ HCF **9**
- **prime factors**: multiply the primes common to **all** the numbers, as often as in both
- $24 = 2 \times 2 \times 2 \times 3$, $60 = 2 \times 2 \times 3 \times 5$ $\rightarrow$ HCF $= 2 \times 2 \times 3 =$ **12**
- 12, 15, 18: only one 3 in all three $\rightarrow$ HCF **3**
- no prime in common (8 and 15) $\rightarrow$ HCF **1**

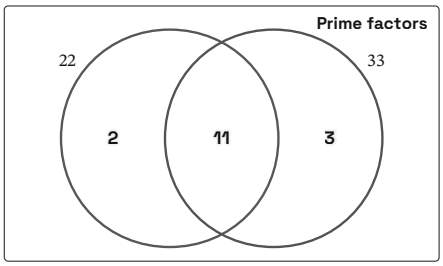
LCM

- **listing**: $M_{12} = \{12, 24, 36, \ldots\}$, $M_{18} = \{18, 36, \ldots\}$ $\rightarrow$ LCM **36**
- **prime factors**: common primes $\times$ all the primes left over

- $24 = 2 \times 2 \times 2 \times 3$, $36 = 2 \times 2 \times 3 \times 3$: common 12, left over $2 \times 3 \rightarrow$ LCM $= 12 \times 6 =$ **72**
- $22 = 2 \times 11$, $33 = 3 \times 11$: HCF 11, LCM $2 \times 11 \times 3 = 66$

HCF × LCM = PRODUCT OF THE TWO NUMBERS

- $11 \times 66 = 726 = 22 \times 33$
- $12 \times 72 = 864 = 24 \times 36$
- LCM = product ÷ HCF: 24 and 40, HCF 8 → LCM $= 960 \div 8 = 120$



22 and 33's prime factors: HCF = 11 (the middle), LCM = $2 \times 11 \times 3 = 66$.

HCF	LCM
at most the smallest number	at least the largest number
cutting or sharing into the **largest equal parts**	things that **repeat and meet again**
ropes 12 m and 18 m → pieces of **6 m**	buses every 10 and 15 min → together every **30 min** (7:30)
ribbons 12 cm, 18 cm → 6 cm	lights every 4 s and 6 s → 12 s

- rods 14 cm and 35 cm: shortest length both measure = LCM = 70 cm
- adding 1/4 + 1/6 needs the LCM of 4 and 6, 12; simplifying 18/24 divides by the HCF, 6

COMMON MISTAKES

✗ smallest natural number is 0
✓ 1; 0 is whole only.

✗ smallest number from 5, 0, 1 is 015
✓ can't start with 0: 105.

✗ $12 - 4 \div 2$ for "keep 4, share the rest by 2"
✓ brackets: $(12 - 4) \div 2 = 4$.

✗ $30 - (10 - 4) = 16$
✓ bracket first: $30 - 6 = 24$.

✗ test for 7: subtract the last digit once
✓ double it; 81: $8 - 1 = 7$, but 81 isn't divisible by 7.

✗ all primes are odd
✓ 2 is an even prime.

✗ 1 called prime
✓ one factor only: neither prime nor composite.

✗ $45 = 5 \times 9$
✓ 9 isn't prime: $45 = 3 \times 3 \times 5$.

✗ HCF of 12, 15, 18 = 6
✓ 15 has no 2: HCF 3.

✗ HCF bigger than a number, LCM smaller
✓ something went wrong.