Unit 16 · Equation

STUDY NOTES

Key terms

Equation: a mathematical sentence saying two expressions are equal, with = between them: $x + 5 = 15$.

Solving an equation: finding the value of the letter that makes it true.

Solution: that value.

Equality facts: the four ways to change both sides and keep them equal.

An equation is a level balance

- the = sign is the balance point
- do **the same to both sides** → it stays level (true)
- change only one side → it tips
- a level see-saw is an equation too: $x - 3$ on one end, 18 on the other → $x - 3 = 18$, $x = 21$

The equality facts

1. **addition:** add the same number to both sides
2. **subtraction:** take the same number from both sides
3. **multiplication:** multiply both sides by the same number
4. **division:** divide both sides by the same number (**not 0**)

Aim: get the letter **alone** on one side; the other side is its value.

Done to x	Undo it by
$+ 3$	$- 3$ on both sides
$- 7$	$+ 7$ on both sides
$\times 7$	$\div 7$ on both sides
$\div 2$	$\times 2$ on both sides

One step

- $x - 3 = 8$: $+ 3$ on both sides → $x = 11$
- $x + 3 = 9$: $- 3$ → $x = 6$
- $4x = 12$: $\div 4$ → $x = 3$ (4 equal parts make 12; one part 3)
- $x/5 = 4$: $\times 5$ → $x = 20$ (5 equal parts of 4)
- $7x = 56$ → $x = 8$; $y/2 = 8$ → $y = 16$
- the letter on the right: $9 = x - 4$ → $+ 4$ → $x = 13$
- write the step in brackets beside each line; tiles: a red −1 and a green +1 make 0

Two steps

- undo **+ or − first**, then **× or ÷**

$$\begin{aligned} & 2x + 3 = 7 \\ 2x = 4 \quad & [- 3 \text{ on both sides}] \\ x = 2 \quad & [\div 2 \text{ on both sides}] \end{aligned}$$

- like terms first: $2x + 3x = 15$ → $5x = 15$ → $x = 3$
- x on both sides: take the smaller x term off both, like a bag off each pan
- $7x + 5 = 9 + 5x$: $- 5x$ → $2x + 5 = 9$; $- 5$ → $2x = 4$; $\div 2$ → $x = 2$
- the answer can be a fraction: $3x = 4$ → $x = 4/3 = 1\ 1/3$

When x is taken away

- $10 - x = 4$: what must I take from 10 to leave 4?
- $10 - 4 = 6$, so $x = 6$; check $10 - 6 = 4$ ✓

Word problems

1. let the unknown be x
2. write the story as an equation
3. solve it, one step a line
4. check in the story; answer in words

- "adding x to 5 gives 9" → $5 + x = 9$
- "taking 9 from y leaves 5" → $y - 9 = 5$
- "8 times y is 32" → $8y = 32$
- "p divided by 6 is 9" → $p/6 = 9$, $p = 54$

Story	Equation	Solution
paid Rs. 500, Rs. 120 back	$x + 120 = 500$	Rs. 380
gave 5 guavas, 7 left	$x - 5 = 7$	12 guavas
4 weeks' money + Rs. 50 = Rs. 450	$4x + 50 = 450$	Rs. 100 a week
Rs. y shared by 3, Rs. 400 each	$y/3 = 400$	Rs. 1,200
Utsab x, Apsan twice, Rs. 300 in all	$x + 2x = 300$	Rs. 100, Rs. 200
ages x and $x + 6$, sum 18	$2x + 6 = 18$	6 and 12 years

Check every answer

- put it back into the **first** equation
- the wrong undo, $+ 3$ → $x = 12$: $12 + 3 = 15$, not 9; the check catches it

COMMON MISTAKES

✗ something done to one side only
✓ the same to both sides, or it is no longer true.

✗ $x + 3 = 9$ solved by adding 3: $x = 12$
✓ undo + 3 with − 3: $x = 6$.

✗ $10 - x = 4$ → $x = 14$
✓ what is taken from 10 to leave 4: $x = 6$.

✗ $2x + 3 = 7$ divided by 2 first
✓ undo + or − first, then × or ÷.