Unit 4 · Simplification

STUDY NOTES

Key terms

Operations: adding, taking away, multiplying, dividing (+, −, ×, ÷).

Mathematical statement: a whole problem as one line of numbers and signs: $10 \times 7 - 37$.

Simplify: work it out, step by step, until one number is left; the work is **simplification**.

Order of operations: the one order everyone agrees on, so a written sum means the same to all.

The rule

1. **first × and ÷**, from **left to right** (whichever comes first)
2. **then + and −**, from **left to right**

- multiply and divide before you add and take away
- **another way for + and −:** add the first number and every number with + in front; take away all the rest
- $8 - 10 + 30$: $8 + 30 = 38$; $38 - 10 = \mathbf{28}$ (handy when going from the left would take away more than you have)
- write one step a line, the operation beside it: [multiply], [add]

One sum, two answers

$55 \div 5 + 7 \times 3$

- by the rule: $11 + 21 = \mathbf{32}$ ✓
- left to right: $11 + 7 = 18$; $18 \times 3 = 54$ ✗
- the × 3 belongs only to the 7
- "add 7, then multiply everything by 3" needs **brackets**: $(55 \div 5 + 7) \times 3$ (Grade 6)

Steps, shown

Statement	Steps	Answer
$10 \times 7 - 37$	$70 - 37$	33
$55 \div 11 + 5 \times 2$	$5 + 5 \times 2 = 5 + 10$	15
$17 - 9 \div 3 \times 2$	$17 - 3 \times 2 = 17 - 6$	11
$10 \times 9 + 84 \div 12$	$90 + 84 \div 12 = 90 + 7$	97
$48 \div 6 \times 2 + 20 - 5$	$8 \times 2 + 20 - 5 = 16 + 20 - 5 = 36 - 5$	31
$24 \div 8 \times 6$	3×6	18
$37 - 15 + 8$	$22 + 8$	30
$20 - 4 - 2$	$16 - 2$	14

Words into signs

- 2 times 5 → 2×5
- one third of 36 → $36 \div 3$
- two times one third of 9 → $9 \div 3 \times 2$
- the product of 8 and 13 → 8×13
- the sum of 121 and 83 → $121 + 83$
- 666 divided into 6 parts → $666 \div 6$
- "take 8×13 away from $121 + 83$" → $121 + 83 - 8 \times 13 = \mathbf{100}$

A missing number

- work out the known part, then undo the last step
- $6 \times 6 - \ldots = 25$: $36 - 25 = \mathbf{11}$
- $\ldots \times 4 + 3 = 23$: $23 - 3 = 20$; $20 \div 4 = \mathbf{5}$; check $5 \times 4 + 3 = 23$ ✓
- $10 \times 40 \div \ldots = 40$: $400 \div \mathbf{10} = 40$

Word problems

1. find what must come first (the story tells you)
2. write **one** mathematical statement
3. simplify it, one step a line
4. answer in words; check it against the story

- 7 packets of 10 bars, 37 sold: $10 \times 7 - 37 = \mathbf{33}$ **bars** (the 70 bars before taking 37 away)
- 10 pencils to each of 15, 4 left: $15 \times 10 + 4 = \mathbf{154}$ **pencils**
- 3 buses of 42, 9 got off: $3 \times 42 - 9 = \mathbf{117}$ **students**
- 2 heaps of $9 \div 3$ oranges given from 17: $17 - 9 \div 3 \times 2 = \mathbf{11}$

Why everyone must agree

- bill $2 \times 90 + 3 \times 50$: by the rule $180 + 150 =$ **Rs. 330**
- left to right: $180 + 3 = 183$; $183 \times 50 =$ Rs. 9,150, a very different bill
- a shop bill: "how many × price" for each thing, then add
- simple shop calculators: $1 + 2 \times 3 = \mathbf{9}$ (each step done as you press)
- scientific and phone calculators: **7**, by the rule
- spreadsheets follow the rule: =5+2*3 gives 11

COMMON MISTAKES

✗ $55 \div 5 + 7 \times 3$ worked left to right = 54
✓ × and ÷ first: 32.

✗ "DMAS" read as add before take away: $37 - 15 + 8 = 37 - 23 = 14$
✓ take away, then add, from the left: 30.

✗ $20 - 4 - 2 = 20 - 2 = 18$
✓ from the left: $16 - 2 = 14$.

✗ $24 \div 8 \times 6$ with × first: $24 \div 48$
✓ left to right: $3 \times 6 = 18$.

✗ several steps at once on one line
✓ one operation a line, named beside it.