Unit 23 · Quantization of Energy

ANSWER KEY · FOR TEACHERS

Full marks 34

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. stationary orbit
2. ionization energy
3. wave number
4. Balmer
5. de Broglie
6. uncertainty principle
7. hard
8. Bragg's law

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. focusing cup, M
2. filament, C
3. target, T
4. cooling fins

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **c.** 8.5 Å — A · 1

$r_n = 0.53\,n^2$ Å $= 0.53 \times 16 = 8.5$ Å.

ii. **c.** 3.4 eV — A · 1

$E_2 = -13.6/4 = -3.4$ eV, so 3.4 eV takes the electron to $E_\infty = 0$.

iii. **b.** 1.23 Å — A · 1

$$\lambda = \frac{12.27}{\sqrt{V}}\ \text{Å} = \frac{12.27}{10} = 1.23\ \text{Å}.$$

iv. **b.** 0.41 Å — A · 1

$\lambda_{\min} = \dfrac{hc}{eV} = \dfrac{6.63 \times 10^{-34} \times 3.0 \times 10^{8}}{1.6 \times 10^{-19} \times 30\,000} = 4.1 \times 10^{-11}$ m $= 0.41$ Å; it depends only on the voltage, not the target.

v. **a.** gives more X-rays of the same wavelengths — U · 1

A hotter filament gives out more electrons, so more X-ray photons (greater intensity); their wavelengths are set by the voltage.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ Zero is the free electron at rest far away.
- ✓ A bound electron needs energy supplied to free it, so its energy is less than zero.

b. MARKING SCHEME — U · 2

- ✓ A whole number of de Broglie wavelengths fits round the orbit: $2\pi r = n\lambda$.
- ✓ With $\lambda = h/mv$ this gives $mvr = nh/2\pi$, Bohr's condition.

c. MARKING SCHEME — U · 2

- ✓ X-ray wavelengths (about 1 Å) are far smaller than a ruled grating's spacing.
- ✓ A crystal's planes of atoms are spaced about 1 Å apart, like a grating for X-rays.

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5 Long answer

ANSWER U · 4

The electric attraction gives the centripetal force: $\frac{1}{4\pi\varepsilon_0}\frac{e^2}{r_n^2} = \frac{mv_n^2}{r_n}$. Bohr's condition is $mv_n r_n = \frac{nh}{2\pi}$, so $v_n = \frac{nh}{2\pi m r_n}$. Putting it in: $\frac{e^2}{4\pi\varepsilon_0} = mr_n\frac{n^2h^2}{4\pi^2m^2r_n^2}$, so $r_n = \frac{\varepsilon_0 n^2h^2}{\pi me^2} \propto n^2$.

An electron in a stationary orbit does not radiate. When it jumps from a higher level E_2 to a lower E_1, it gives out one photon of frequency $f = (E_2 - E_1)/h$. The levels have fixed energies, so only certain frequencies appear: a line spectrum. Jumps to n = 1, 2, 3 … give the Lyman, Balmer, Paschen … series.

MARKING SCHEME

- ✓ Electric force = centripetal force, and $mvr = nh/2\pi$.
- ✓ $r_n = \varepsilon_0 n^2h^2/\pi me^2$, so $r_n \propto n^2$.
- ✓ A jump gives one photon with $hf = E_2 - E_1$.
- ✓ Fixed levels give fixed lines; the series by the lower level.

6 Numerical problems

a. **ANSWER** A · 4

i. $E_5 = -13.6/25 = \mathbf{-0.544\ eV}$; $E_2 = -13.6/4 = \mathbf{-3.4\ eV}$.

ii. $E = -0.544 - (-3.4) = \mathbf{2.86\ eV}$.

iii. $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = 1.097 \times 10^7 \times 0.21$, so $\lambda = \mathbf{4.34 \times 10^{-7}\ m}$ (434 nm).

iv. It ends on n = 2: the **Balmer** series, **visible** (violet-blue).

MARKING SCHEME

- ✓ −0.544 eV and −3.4 eV.
- ✓ 2.86 eV.
- ✓ λ = 434 nm.
- ✓ Balmer, visible.

b. **ANSWER** A · 3

i. $n = \frac{I}{e} = \frac{5.0 \times 10^{-3}}{1.6 \times 10^{-19}} = \mathbf{3.1 \times 10^{16}\ a\ second}$.

ii. $P = VI = 40\,000 \times 5.0 \times 10^{-3} = \mathbf{200\ W}$.

iii. 99 % becomes heat: **198 J each second** (198 W).

MARKING SCHEME

- ✓ n = 3.1×10^{16} a second.
- ✓ P = 200 W.
- ✓ Heat 198 W.

c. **ANSWER** A · 3

$\sin\theta = \frac{n\lambda}{2d}$. n = 1: $\sin\theta = 1.5 \div 4.0 = 0.375$, $\theta = \mathbf{22}°$. n = 2: $\sin\theta = 0.75$, $\theta = \mathbf{49}°$. n = 3: $\sin\theta = 1.125$, more than 1, so **no** third order.

MARKING SCHEME

- ✓ First order 22°.
- ✓ Second order 49°.
- ✓ No third order ($\sin\theta$ would be over 1).

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7 Think

ANSWER HA · 3

$\lambda = \dfrac{h}{mv} = \dfrac{6.63 \times 10^{-34}}{0.15 \times 30} = \mathbf{1.5 \times 10^{-34}\ m}$. Diffraction is seen only when the wavelength is about the size of the gap or object the wave meets. Nothing is anywhere near 10^{-34} m across (a nucleus is about 10^{-15} m), so the ball's wave nature can never show. An electron's wavelength is about an ångström, the spacing of atoms in a crystal, so crystals diffract electrons.

MARKING SCHEME

- ✓ $\lambda = 1.5 \times 10^{-34}$ m.
- ✓ Diffraction needs a gap or obstacle about the wavelength's size.
- ✓ Nothing is that small for a ball; atoms' spacing matches an electron's λ.

8 Activity: Models of the hydrogen atom (Activity 23.1)

(not marked)

WHAT STUDENTS SHOULD FIND

656 nm is absorbed only when the electron is already in n = 2 (the 2 → 3 jump), and 486 nm likewise (2 → 4). 500 nm is never absorbed: no two levels of hydrogen differ by exactly that photon's energy, and an atom takes in only photons whose energy fits a jump.

See it at Science Playground

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- **Plasma Ball** (exhibit): The glowing gas gives an emission spectrum: excited atoms drop to lower levels, giving out only their own wavelengths, each gas its own color.
- **The Spectroscope** (exhibit): Look at different light sources through it: a hot filament or sunlight gives a continuous band; a fluorescent tube's glowing gas adds bright lines of its own.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	6	11	14	3	**34**

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