Unit 19 · Alternating Current

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

K · 3

1. rms value → **f.** the steady current that would heat a resistor at the same rate
2. reactance → **e.** the opposition of a pure inductor or capacitor to a.c., in ohms
3. impedance → **b.** the total opposition of a circuit to a.c., $\sqrt{R^2 + (X_L - X_C)^2}$
4. resonance → **c.** the state of an LCR circuit when $X_L = X_C$ and the current is greatest
5. power factor → **a.** cos φ, the ratio of the real power to $V_{\text{rms}} I_{\text{rms}}$
6. wattless current → **g.** a current 90° out of phase with the voltage, using no power on average

Each carries 0.5 mark.

2 Label the diagram

K · 2

1. φ
2. L
3. C
4. VL– VC

Each carries 0.5 mark.

3 Multiple choice

i. **b.** 120 V and 60 Hz — A · 1

$V_{\text{rms}} = V_0/\sqrt{2} = 170/1.414 = 120$ V; $\omega = 120\pi$, so f = $\omega/2\pi = 60$ Hz. (108 V is the mean over half a cycle, 0.637 V_0.)

ii. **c.** 157 Ω — A · 1

$X_L = 2\pi fL = 2\pi \times 50 \times 0.5 = 157\ \Omega$.

iii. **a.** 637 Ω — A · 1

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi \times 50 \times 5 \times 10^{-6}} = 637\ \Omega$. ($1.6 \times 10^{-3}$ Ω is 2πfC, the formula turned upside down.)

iv. **b.** 318 Hz — A · 1

$f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.025 \times 10^{-5}}} = 318$ Hz.

v. **b.** leads the voltage by 90° — K · 1

The capacitor's charge, and so its p.d., builds up after the current: the current reaches each peak a quarter of a cycle before the voltage.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ d.c.: f = 0 and X_C is infinite, so no steady current.
- ✓ a.c.: it charges and discharges each cycle, so current flows (more at higher f).

b. MARKING SCHEME — U · 2

- ✓ $\varphi = 90°$, so cos $\varphi = 0$ and the average power is zero.
- ✓ Energy is stored in the field and returned to the source each cycle.

c. MARKING SCHEME — U · 2

- ✓ The average over a cycle is zero.
- ✓ The rms value gives the same heating as that steady current (heating depends on I^2).

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5 Long answer

ANSWER U · 4

The same current I flows through all three, so I is the reference. $V_R = IR$ is in phase with I; $V_L = IX_L$ leads it by 90°; $V_C = IX_C$ lags it by 90°. V_L and V_C are opposite, so they combine to $V_L - V_C$, at right angles to V_R. The supply voltage is the vector sum: $V^2 = V_R{}^2 + (V_L - V_C)^2 = I^2[R^2 + (X_L - X_C)^2]$, so $Z = \frac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$. The voltage leads the current by φ, where $\tan\varphi = \frac{X_L - X_C}{R}$.

MARKING SCHEME

- ✓ A phasor diagram with V_R along I, V_L up and V_C down.
- ✓ $V^2 = V_R{}^2 + (V_L - V_C)^2$.
- ✓ $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- ✓ $\tan\varphi = (X_L - X_C)/R$.

6 Numerical problems

a. **ANSWER** A · 4

i. $X_C = \frac{1}{2\pi \times 50 \times 50 \times 10^{-6}} = \mathbf{63.7\ \Omega}$.

ii. $Z = \sqrt{40^2 + 63.7^2} = \mathbf{75.2\ \Omega}$.

iii. I = 230 ÷ 75.2 = **3.06 A**.

iv. Only the resistor uses power: $P = I^2R = 3.06^2 \times 40 = \mathbf{374\ W}$ (the same as $V_{rms}I_{rms}\cos\varphi$, with $\cos\varphi$ = R/Z = 0.53).

MARKING SCHEME

- ✓ $X_C = 63.7\ \Omega$.
- ✓ Z = 75.2 Ω.
- ✓ I = 3.06 A.
- ✓ P = 374 W.

b. **ANSWER** A · 3

$f_0 = \frac{1}{2\pi\sqrt{0.20 \times 5.0 \times 10^{-6}}} = \mathbf{159\ Hz}$.

At resonance Z = R: I = 10 ÷ 25 = **0.40 A**.

$Q = \frac{1}{R}\sqrt{\frac{L}{C}} = \frac{1}{25}\sqrt{\frac{0.20}{5.0 \times 10^{-6}}} = \frac{200}{25} = \mathbf{8}$.

MARKING SCHEME

- ✓ $f_0 = 159$ Hz.
- ✓ I = 0.40 A.
- ✓ Q = 8.

c. **ANSWER** A · 3

$I_{rms} = 230 \div 60 = \mathbf{3.83\ A}$; $I_0 = \sqrt{2} \times 3.83 = \mathbf{5.42\ A}$.

A resistor has $\cos\varphi = 1$: $P = 230 \times 3.83 = \mathbf{882\ W}$.

MARKING SCHEME

- ✓ $I_{rms} = 3.83$ A.
- ✓ $I_0 = 5.42$ A.
- ✓ P = 882 W.

7 Think

ANSWER HA · 3

With the capacitor, $X_C = 1/(2\pi fC)$ falls as f rises, so the current rises and the lamp gets **brighter**. With the coil, $X_L = 2\pi fL$ rises with f, so the lamp gets **dimmer**. On d.c. the capacitor stops the current once charged (the lamp goes out), while the coil, with no reactance at f = 0 and little resistance, lets the lamp light fully.

MARKING SCHEME

- ✓ Capacitor: X_C falls with f, lamp brighter.
- ✓ Coil: X_L rises with f, lamp dimmer.
- ✓ d.c.: no light through the capacitor, full light through the coil.

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8 Activity: Telling a.c. from d.c. with two LEDs (Activity 19.1)

(not marked)

WHAT STUDENTS SHOULD FIND

On the battery only one LED lights, and swapping the wires lights the other; waved, it draws a smooth streak. On a.c. both light, and waved they draw dotted streaks whose red and green dashes take turns: each LED is lit only in the half cycle when the current flows its way. The a.c. supply has no fixed polarity: its current keeps reversing, 50 times a second each way.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	6	10	14	3	**33**

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