Unit 12 · Diffraction

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. diffraction
2. Fresnel diffraction
3. Fraunhofer diffraction
4. diffraction grating
5. grating element
6. resolving power
7. Rayleigh's criterion
8. numerical aperture

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. d
2. plane wave
3. lens
4. screen

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **b.** 2.0×10^{-3} rad — A · 1

$\sin\theta = \lambda/d = 580 \times 10^{-9} \div 0.29 \times 10^{-3} = 2.0 \times 10^{-3}$, and for so small an angle $\theta \approx \sin\theta$.

ii. **a.** 9.3° — A · 1

$d = 10^{-2}/3000 = 3.33 \times 10^{-6}$ m; $\sin\theta = \lambda/d = 0.162$, $\theta = 9.3°$.

iii. **b.** twice as wide and much brighter — K · 1

The central maximum runs from the first minimum on one side to the first on the other, $2\lambda D/d$, twice the others, and it holds most of the light.

iv. **b.** 3 — A · 1

$d = 10^{-2}/4500 = 2.22 \times 10^{-6}$ m, and $\sin\theta = n\lambda/d \leq 1$ gives $n \leq d/\lambda = 3.58$, so $n = 3$.

v. **a.** blue light, since its wavelength is shorter — U · 1

The smallest separation resolved, $\lambda/(2\mu \sin\theta)$, is smaller for a shorter wavelength.

4 Short answers

a. MARKING SCHEME — K · 2

- ✓ Any one, e.g. in Fresnel diffraction the source and screen are at finite distances from the opening; in Fraunhofer, effectively at infinity (with lenses).
- ✓ Any second, e.g. Fresnel: the wavefronts are spherical or cylindrical; Fraunhofer: plane. (Or: Fraunhofer's pattern is simpler to analyze.)

b. MARKING SCHEME — U · 2

- ✓ Any one, e.g. Young's bright fringes are all of the same width; in diffraction the central maximum is twice as wide as the others.
- ✓ Any second, e.g. Young's bright fringes are (nearly) equally bright; diffraction's secondary maxima get rapidly dimmer away from the center.

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c. MARKING SCHEME A · 3

- ✓ $d\theta = 1.22\lambda/D$.
- ✓ 6.7×10^{-6} rad.
- ✓ About 2.6 km apart.

5 Long answer

ANSWER U · 4

Light leaving at an angle θ has a path difference d sin θ between the two edges of the slit.
When d sin $\theta = \lambda$, divide the slit into two halves: each point in the upper half has a partner in the lower half d/2 below it, and their path difference is $\lambda/2$.
Each pair arrives in antiphase and cancels, so the whole slit gives darkness: the first minimum, sin $\theta = \lambda/d$.
When d sin $\theta = 2\lambda$, divide the slit into four quarters: neighboring quarters cancel in pairs in the same way, so there is another minimum at sin $\theta = 2\lambda/d$ (and in general at $n\lambda/d$).

MARKING SCHEME

- ✓ Path difference between the edges d sin θ.
- ✓ Two halves whose matching points differ by $\lambda/2$.
- ✓ They cancel in pairs: sin $\theta = \lambda/d$.
- ✓ Four quarters for 2λ: another minimum.

6 Numerical problems

a. ANSWER A · 4

i. $\sin\theta_1 = \lambda/d = 590 \times 10^{-9} \div 0.12 \times 10^{-3} = \mathbf{4.9 \times 10^{-3}}$ **rad**.
ii. Width $= 2\lambda D/d = 2 \times 590 \times 10^{-9} \times 1.5 \div 0.12 \times 10^{-3} =$ **14.8 mm**.
iii. Third minimum: $x = 3\lambda D/d =$ **22.1 mm** from the center.
iv. About halfway between the first and second minima, at $1.5\lambda D/d \approx$ **11 mm**.

MARKING SCHEME

- ✓ $\theta_1 = 4.9 \times 10^{-3}$ rad.
- ✓ Central maximum 14.8 mm wide.
- ✓ Third minimum 22.1 mm from the center.
- ✓ First secondary maximum about 11 mm.

b. ANSWER A · 4

i. $(a + b) = 10^{-2}/3500 = \mathbf{2.86 \times 10^{-6}}$ **m**.
ii. $\sin\theta = \lambda/(a + b)$: red 0.230, **13.3**°; blue 0.170, **9.8**°.
iii. $\sin\theta = 2 \times 656 \times 10^{-9} \div 2.86 \times 10^{-6} = 0.459$, **27.3**°.
iv. $n \leq (a + b)/\lambda = 2.86/0.656 = 4.36$, so **4**.

MARKING SCHEME

- ✓ $(a + b) = 2.86 \times 10^{-6}$ m.
- ✓ First order: 13.3° red, 9.8° blue.
- ✓ Second order red 27.3°.
- ✓ Highest order 4.

7 Think

ANSWER HA · 3

The smallest separation it can resolve is $\frac{\lambda}{2\mu\sin\theta} = \frac{500 \times 10^{-9}}{2 \times 1.4} \approx 2 \times 10^{-7}$ m, about two thousand atoms across. Diffraction blurs each point into a spot that size, and more magnification only enlarges the blur. To see finer detail needs a shorter wavelength, not stronger lenses.

MARKING SCHEME

- ✓ Limit about 2×10^{-7} m.
- ✓ That is far larger than an atom: diffraction blurs it.
- ✓ More magnification can't help; a shorter wavelength is needed.

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8 Activity: Look through cloth

(not marked)

WHAT STUDENTS SHOULD FIND

A cross or grid of spots of light, often edged with color, around the lamp. The threads act as a grating in two directions, diffracting the light. Finer cloth, with threads closer together, spreads the spots farther apart ($\sin\theta = n\lambda/d$). The pattern turns with the cloth, since it follows the threads' directions.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	9	7	14	3	**33**

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