Unit 8 · Waves in Pipes and Strings

ANSWER KEY · FOR TEACHERS

Full marks 32.5

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. fundamental frequency
2. harmonics
3. overtones
4. closed organ pipe
5. node
6. end correction
7. linear mass density
8. sonometer

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. $l = \lambda_1/4$ $f_1 = v/4l$
2. $l = 3\lambda_2/4$ $3f_1$
3. $l = 5\lambda_3/4$ $5f_1$

K · 1.5

Each carries 0.5 mark.

3 Multiple choice

i. **b.** 250 Hz — A · 1

For an open pipe $f_1 = \frac{v}{2l} = \frac{340}{2 \times 0.68} = 250$ Hz.

ii. **b.** only the odd harmonics — U · 1

Its column holds an odd number of quarter-wavelengths, $l = (2n - 1)\lambda/4$, so only f_1, $3f_1$, $5f_1$, … are possible.

iii. **b.** 200 Hz — A · 1

$f_1 = \frac{v}{2l} = \frac{240}{1.2} = 200$ Hz.

iv. **b.** third harmonic — K · 1

An open pipe produces every harmonic; the first overtone is the second harmonic, the second overtone the third.

v. **a.** a slightly lower note — U · 1

The end correction, about 0.6r, is larger for a wider pipe, so the effective length l + e is larger and the frequency lower.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ An open pipe has all harmonics; a closed pipe only the odd ones.
- ✓ More harmonics make the quality richer.

b. MARKING SCHEME — K · 2

- ✓ Law of length: the frequency is inversely proportional to the vibrating length ($f \propto 1/l$), μ and T constant.
- ✓ Law of tension: the frequency is proportional to the square root of the tension ($f \propto \sqrt{T}$), l and μ constant.

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c. MARKING SCHEME A · 3

- ✓ e = 0.9 cm.
- ✓ f_1 = 324 Hz.
- ✓ 336 Hz without the end correction.

5 Long answer

ANSWER U · 4

The closed end is always a node and the open end an antinode.

First mode: one quarter-wavelength, $l = \lambda_1/4$, so $f_1 = \frac{v}{4l}$.

Second mode: three quarter-wavelengths, $l = 3\lambda_2/4$, so $f_2 = \frac{3v}{4l} = 3f_1$.

Third mode: five quarter-wavelengths, $l = 5\lambda_3/4$, so $f_3 = \frac{5v}{4l} = 5f_1$.

The column always holds an odd number of quarter-wavelengths, so the frequencies are f_1, $3f_1$, $5f_1$, ...: only odd harmonics.

MARKING SCHEME

- ✓ Node at the closed end, antinode at the open end.
- ✓ $f_1 = v/4l$.
- ✓ $f_2 = 3f_1$ and $f_3 = 5f_1$.
- ✓ Always an odd number of quarter-wavelengths: odd harmonics only.

6 Numerical problems

a. ANSWER A · 4

i. $f = \frac{1}{2l}\sqrt{\frac{T}{\mu}} = \frac{1}{1.6}\sqrt{\frac{49}{2.5 \times 10^{-3}}} = \frac{140}{1.6} = \mathbf{87.5\ Hz}$.

ii. $f \propto 1/l$: twice the frequency needs half the length, **0.40 m**.

iii. $f \propto \sqrt{T}$: twice the frequency needs four times the tension, **196 N**.

MARKING SCHEME

- ✓ $f = (1/2l)\sqrt{(T/\mu)}$.
- ✓ f = 87.5 Hz.
- ✓ l = 0.40 m.
- ✓ T = 196 N.

b. ANSWER A · 4

i. Between the two resonances the column grows by half a wavelength: $\lambda/2 = 52.4 - 17.0 = 35.4$ cm, so $\lambda = \mathbf{0.708\ m}$.

ii. $v = f\lambda = 480 \times 0.708 = \mathbf{340\ m/s}$.

iii. At the first resonance $l_1 + e = \lambda/4 = 17.7$ cm, so $e = 17.7 - 17.0 = \mathbf{0.7\ cm}$.

iv. $d = e/0.3 = \mathbf{2.3\ cm}$.

MARKING SCHEME

- ✓ $\lambda = 2(l_2 - l_1) = 0.708$ m.
- ✓ v = 340 m/s.
- ✓ $e = \lambda/4 - l_1 = 0.7$ cm.
- ✓ d = 2.3 cm.

7 Think

ANSWER HA · 3

$f \propto \sqrt{T}$, so for f to rise 1.10 times, T must rise $1.10^2 = 1.21$ times: by **21%**. The frequency depends on the square root of the tension, so a given rise in frequency needs about twice as large a rise, in percent, in the tension.

MARKING SCHEME

- ✓ $f \propto \sqrt{T}$.
- ✓ T must rise 1.21 times, by 21%.
- ✓ The square root makes the tension's change larger.

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8 Activity: A bottle organ

(not marked)

WHAT STUDENTS SHOULD FIND

Each bottle is a closed pipe: the water is the closed end, a node, and the mouth an antinode. The bottle with the shortest air column (the most water) gives the highest note, since $f_1 = v/4(l + e)$: the shorter the column, the higher the frequency.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	8.5	8	13	3	**32.5**

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