Unit 6 · Wave Motion

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 29

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

The speed is right, but the direction is wrong. With $(\omega t + kx)$, a point further along +x leads, not lags: the wave travels along the **−x** direction at 12 m/s. Only $(\omega t - kx)$ goes towards +x.

MARKING SCHEME

✓ v = 12 m/s is right.
✓ The mistake: the sign of kx.
✓ (ωt + kx) travels towards −x.

2 Show that

ANSWER A · 3

Particle velocity $\frac{dy}{dt} = a\omega\cos(\omega t - kx)$, greatest $a\omega$. Wave speed $v = \omega/k$.

$\frac{a\omega}{\omega/k} = ak = \frac{2\pi a}{\lambda}$.

If a is much smaller than λ, the particles move much more slowly than the wave itself.

MARKING SCHEME

✓ Greatest particle speed aω.
✓ Ratio aω ÷ (ω/k) = ak = 2πa/λ.
✓ Small a/λ: the particles are much slower than the wave.

3 Predict, then explain

ANSWER HA · 3

i. **Flat**: by superposition +2 cm and −2 cm add to zero everywhere.
ii. **No.** The rope is flat but moving: its parts have their greatest speed there, so all the energy is kinetic for that instant. A moment later the two pulses come out again, each unchanged.

MARKING SCHEME

✓ The rope is flat at that instant.
✓ The energy is still there, as kinetic energy of the moving rope.
✓ The pulses pass through and go on unchanged.

4 Read the data

ANSWER A · 3

i. Nodes are λ/2 apart: (1.11 − 0.15)/3 = 0.32 m, so λ = **0.64 m**.
ii. v = fλ = 125 × 0.64 = **80 m/s**.
iii. Halfway between the nodes, λ/4 from each: **0.31, 0.63 and 0.95 m**.

MARKING SCHEME

✓ λ/2 = 0.32 m, so λ = 0.64 m.
✓ v = 80 m/s.
✓ Antinodes at 0.31, 0.63 and 0.95 m.

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5 A real setting

ANSWER A · 3

One loop: the string is $\lambda/2$, so $\lambda = 2 \times 0.90 =$ **1.8 m**, and $f = v/\lambda = 270/1.8 =$ **150 Hz**.
Three loops: $3 \times \lambda/2 = 0.90$ m, $\lambda = 0.60$ m, so $f = 270/0.60 =$ **450 Hz**, three times as high.

MARKING SCHEME

- ✓ $\lambda = 1.8$ m for one loop.
- ✓ $f = 150$ Hz.
- ✓ $f = 450$ Hz for three loops.

6 What if?

ANSWER HA · 3

Where the two arrive in opposite phase, they no longer cancel: the amplitude is $5 - 3 =$ **2 mm**, not zero, so there are no true nodes. Where they arrive in phase it is $5 + 3 =$ **8 mm**. The 3 mm parts of the two waves still make a stationary wave, but 2 mm of the larger wave is left over as a progressive wave, so **some energy is carried along**, in the larger wave's direction.

MARKING SCHEME

- ✓ At the old nodes, 2 mm: no true nodes.
- ✓ At the antinodes, 8 mm.
- ✓ Some energy is carried, in the larger wave's direction.

7 Measure it

ANSWER A · 3

$f = 20/16 =$ **1.25 Hz**. $v = f\lambda = 1.25 \times 0.40 =$ **0.50 m/s**. In one bob (one period) a crest travels one wavelength, **0.40 m**, while the leaf itself stays in place.

MARKING SCHEME

- ✓ $f = 1.25$ Hz.
- ✓ $v = 0.50$ m/s.
- ✓ One wavelength, 0.40 m, in one period.

8 Work backwards

ANSWER A · 2

$\omega = 2\pi \times 40 = 80\pi$ rad/s; $\lambda = 32/40 = 0.80$ m, so $k = 2\pi/0.80 = 2.5\pi$ rad/m. Along −x:
$y =$ **$0.005\sin(80\pi t + 2.5\pi x)$**.

MARKING SCHEME

- ✓ $\omega = 80\pi$ rad/s and $k = 2.5\pi$ rad/m.
- ✓ $y = 0.005 \sin(80\pi t + 2.5\pi x)$, with + for −x.

9 Correct the misconceptions

ANSWER U · 3

1. The amplitude changes along a loop, from zero at the nodes to 2a at the antinode in the middle ($2a \cos kx$).
2. A stationary wave carries no energy across its nodes: the energy of each loop stays in it, changing between kinetic and potential.
3. A free end is an antinode: it is reflected without a change of phase and moves furthest. A fixed end is a node.

MARKING SCHEME

- ✓ The amplitude varies from 0 at a node to 2a at an antinode.
- ✓ No energy flows past the nodes.
- ✓ A free end is an antinode (a fixed end a node).

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10 Make your own

HA · 3

ANSWER

For example, with f = 100 Hz: nodes $\lambda/2 = 0.25$ m apart, so $\lambda = 0.50$ m and $k = 2\pi/0.50 = 4\pi$ rad/m; antinode amplitude 2a = 6 mm, so a = 3 mm.

$y_1 = 0.003\sin(200\pi t - 4\pi x)$, $y_2 = 0.003\sin(200\pi t + 4\pi x)$. (Any ω, with the same a and k.)

MARKING SCHEME

- ✓ $\lambda = 0.50$ m, so $k = 4\pi$ rad/m.
- ✓ a = 3 mm for each wave.
- ✓ Two such waves with the same ω, traveling in opposite directions.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	6	14	9	**29**

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