Unit 6 · Wave Motion

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. wave
2. progressive wave
3. wave number
4. principle of superposition
5. stationary wave
6. nodes
7. antinodes
8. phase change

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. λ
2. a
3. $\varphi = 2\pi x/\lambda$ behind O
4. v

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **a.** towards $+x$ at 20 m/s — A · 1

$\omega t - kx$ means the $+x$ direction; $v = \omega/k = 60/3 = 20$ m/s.

ii. **b.** 0.60 m — A · 1

$\lambda = v/f = 330/550 = 0.60$ m.

iii. **a.** $\lambda/4$ — K · 1

Nodes and antinodes alternate a quarter of a wavelength apart; two nodes are $\lambda/2$ apart.

iv. **a.** a node, since it can't move — U · 1

A fixed end can't move, so its displacement is always zero: a node. The wave is reflected there with a phase change of π.

v. **b.** 1.2 m — A · 1

Each loop is $\lambda/2$ long: $3 \times \lambda/2 = 1.8$ m, so $\lambda = 1.2$ m.

4 Short answers

a. MARKING SCHEME — K · 2

- ✓ Any one, e.g. a progressive wave carries energy forward; a stationary wave carries none past its nodes.
- ✓ Any second, e.g. in a progressive wave every particle has the same amplitude; in a stationary wave the amplitude changes from zero at the nodes to 2a at the antinodes (or: a progressive wave has no nodes or antinodes).

b. MARKING SCHEME — U · 2

- ✓ A wave carries energy but not matter.
- ✓ Each particle only vibrates about its own mean position.

c. MARKING SCHEME — A · 3

- ✓ $\lambda = 1.2$ m.
- ✓ $\pi/2$ for 0.30 m apart.
- ✓ $\pi/2$ for 1.0 ms apart.

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5 Long answer

ANSWER U · 4

The particle at O moves in SHM; counting time from when it passes its mean position going up, $y = a\sin \omega t$.

The disturbance reaches P, a distance x along, a little later, so P does what O did but lags behind it in phase by an angle φ: $y = a\sin(\omega t - \varphi)$.

A path difference of λ is a phase difference of 2π, so a path difference x is $\varphi = \frac{2\pi}{\lambda}x = kx$.

So $y = a\sin(\omega t - kx)$, with $k = 2\pi/\lambda$ the wave number.

MARKING SCHEME

- ✓ At O, y = a sin ωt.
- ✓ P lags O by a phase φ: y = a sin(ωt − φ).
- ✓ φ = 2πx/λ.
- ✓ y = a sin(ωt − kx) with k = 2π/λ.

6 Numerical problems

a. **ANSWER** A · 4

Compare with $y = a\sin(\omega t - kx)$: a = 0.004 m, $\omega = 150\pi$ rad/s, k = 2.5π rad/m.

i. $f = \frac{\omega}{2\pi} = \mathbf{75\ Hz}$; $\lambda = \frac{2\pi}{k} = \frac{2}{2.5} = \mathbf{0.80\ m}$.

ii. $v = f\lambda = 75 \times 0.80 = \mathbf{60\ m/s}$.

iii. The greatest particle speed is $a\omega = 0.004 \times 150\pi = \mathbf{1.9\ m/s}$, far less than the wave's speed.

MARKING SCHEME

- ✓ f = 75 Hz.
- ✓ λ = 0.80 m.
- ✓ v = 60 m/s.
- ✓ Greatest particle speed aω = 1.9 m/s.

b. **ANSWER** A · 4

i. $y = y_1 + y_2 = 2 \times 0.02\sin 80\pi t\cos 2\pi x = \mathbf{0.04\cos 2\pi x \sin 80\pi t}$.

ii. Nodes where cos 2πx = 0: $2\pi x = \pi/2, 3\pi/2$, so **x = 0.25 m and 0.75 m** (λ = 1 m, nodes λ/2 apart).

iii. $A = 0.04\cos(2\pi/6) = 0.04\cos 60^\circ = \mathbf{0.02\ m}$.

MARKING SCHEME

- ✓ y = 0.04 cos 2πx sin 80πt.
- ✓ Nodes where cos 2πx = 0.
- ✓ x = 0.25 m and 0.75 m.
- ✓ Amplitude 0.02 m at x = 1/6 m.

7 Think

ANSWER HA · 3

The speed stays the same: it is set by the rope (its tension and mass per length), not by how fast you shake it. The frequency doubles, and since v = fλ with v unchanged, the wavelength halves: the crests come twice as close together.

MARKING SCHEME

- ✓ The speed doesn't change: the medium fixes it.
- ✓ The frequency doubles.
- ✓ So, from v = fλ, the wavelength halves.

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8 Activity: Count the loops

(not marked)

WHAT STUDENTS SHOULD FIND

The rope settles into steady loops only at certain rates of shaking. The post and (nearly) your hand are nodes; the middle of each loop is an antinode. Each loop is half a wavelength, so the wavelength is twice a loop's length: it gets shorter as you shake faster and the loops increase.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	9	7	14	3	**33**

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