Unit 4 · First Law of Thermodynamics

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 31

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

The gas laws need the **absolute** temperature. $T_1 = 47 + 273 = 320$ K, so $T_2 = 320 \times (1/3)^{0.4} =$ **206 K**, about **−67 °C**.

MARKING SCHEME

- ✓ The mistake: the temperature used in °C, not kelvin.
- ✓ $T_1 = 320$ K, so $T_2 = 320 \times (1/3)^{0.4}$.
- ✓ $T_2 = 206$ K (about −67 °C).

2 Read the cycle

ANSWER A · 4

i. A → B: isochoric (volume constant, pressure rises). B → C: isobaric expansion. The cycle is traced clockwise.

ii. A → B: **0**. B → C: $P\Delta V = 3.0 \times 10^5 \times 3.0 \times 10^{-3} =$ **+900 J**. C → A: compression, minus the trapezium under the line: $-\frac{1}{2}(3.0 + 1.0) \times 10^5 \times 3.0 \times 10^{-3} =$ **−600 J**.

iii. Net $= 0 + 900 - 600 =$ **+300 J**. Loop: a triangle, $\frac{1}{2} \times 3.0 \times 10^{-3} \times 2.0 \times 10^5 = 300$ J, positive since clockwise.

MARKING SCHEME

- ✓ A → B isochoric, B → C isobaric.
- ✓ $W_{AB} = 0$ and $W_{BC} = +900$ J.
- ✓ $W_{CA} = -600$ J.
- ✓ Net 300 J, equal to the triangle's area.

3 Predict, then explain

ANSWER HA · 3

i. The **adiabatic** one: it also cools, so its pressure falls by a factor $(1/2)^{\gamma}$, more than the isothermal one's ½.

ii. The **isothermal** one. From the same starting point the adiabat is steeper and lies below the isotherm all the way to 2V, so the area under it, the work, is smaller. (In the isothermal case heat flows in to keep the temperature up; in the adiabatic case the work comes only from the internal energy.)

MARKING SCHEME

- ✓ The adiabatic sample ends at the lower pressure.
- ✓ The adiabat is steeper and lies below the isotherm.
- ✓ So the isothermal expansion has the larger area under it: more work.

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4 Show that

ANSWER A · 3

For a monatomic gas $C_V = \frac{3}{2}R$ and $C_P = C_V + R = \frac{5}{2}R$.

Heat taken in: $Q = nC_P\Delta T = \frac{5}{2}nR\Delta T$. Work: $W = P\Delta V = nR\Delta T$. So $\frac{W}{Q} = \frac{nR\Delta T}{\frac{5}{2}nR\Delta T} = \frac{2}{5}$.

The rest, $\Delta U = nC_V\Delta T = \frac{3}{2}nR\Delta T$, is $\frac{3}{5}$ of Q.

MARKING SCHEME

- ✓ $C_V = (3/2)R$ and $C_P = (5/2)R$.
- ✓ $W = nR\Delta T$, so $W/Q = 2/5$.
- ✓ $\Delta U/Q = 3/5$.

5 A real setting

ANSWER A · 4

i. Isochoric: $P_2 = P_1T_2/T_1 = 3.0 \times 10^5 \times 330/300 = \mathbf{3.3 \times 10^5\ Pa}$.

ii. $n = PV/RT = 3.0 \times 10^5 \times 0.012 \div (8.31 \times 300) = \mathbf{1.44\ mol}$.

iii. $\Delta U = nC_V\Delta T = 1.44 \times 2.5 \times 8.31 \times 30 = \mathbf{900\ J}$ (all of the heat taken in, since no work is done).

MARKING SCHEME

- ✓ $P_2 = 3.3 \times 10^5$ Pa.
- ✓ $n = 1.44$ mol.
- ✓ $\Delta U = nC_V\Delta T$.
- ✓ $\Delta U = 900$ J.

6 What if?

ANSWER A · 3

$T_2 = T_1 \times 2^{\gamma-1}$. Monatomic: $300 \times 2^{2/3} = \mathbf{476\ K}$. Diatomic: $300 \times 2^{0.4} = \mathbf{396\ K}$.
The **monatomic** gas ends hotter: with fewer degrees of freedom, the work done on it is shared among fewer ways of storing energy, so more of it goes into the motion that sets the temperature.

MARKING SCHEME

- ✓ $T_2 = T_1 \times 2^{\gamma-1}$.
- ✓ 476 K and 396 K.
- ✓ The monatomic gas ends hotter (fewer degrees of freedom).

7 Work backwards

ANSWER A · 2

$W = nRT\ln(V_2/V_1)$, so $\ln(V_2/V_1) = 2000 \div (1 \times 8.31 \times 400) = 0.602$, and $V_2/V_1 = e^{0.602} \approx \mathbf{1.8}$.

MARKING SCHEME

- ✓ $\ln(V_2/V_1) = W/(nRT) = 0.602$.
- ✓ $V_2/V_1 \approx 1.8$.

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8 Correct the misconceptions

ANSWER U · 3

1. Heat is a process variable, not a state variable: a gas has internal energy, not "heat"; how much heat is exchanged depends on the path.
2. In an adiabatic process the temperature changes: work is done at the expense of the internal energy (expansion cools, compression heats). It is the isothermal process that keeps T constant.
3. C_P is greater than C_V (by R): at constant pressure the gas also does work as it expands, so it needs more heat for the same rise in temperature.

MARKING SCHEME

- ✓ Heat is a process (path) variable, not a state variable.
- ✓ Adiabatic: no heat, but the temperature changes.
- ✓ $C_P > C_V$, because the expanding gas also does work.

9 Think

ANSWER HA · 3

The box is insulated, so Q = 0. The gas pushes against nothing (a vacuum), and the box doesn't move, so W = 0. Then $\Delta U = Q - W = \mathbf{0}$. The internal energy of an ideal gas depends only on its temperature, so its **temperature doesn't change**. The process is **irreversible**: the gas never goes back into its half by itself, and it passes through states that are not in equilibrium.

MARKING SCHEME

- ✓ Q = 0 and W = 0, so $\Delta U = 0$.
- ✓ For an ideal gas, T stays the same.
- ✓ It is irreversible.

10 Make your own

ANSWER HA · 3

For example: A (1.0 L, 1.0×10^5 Pa), B (1.0 L, 2.0×10^5 Pa), C (6.0 L, 2.0×10^5 Pa), D (6.0 L, 1.0×10^5 Pa), traced A → B → C → D → A, clockwise. Net work = area = $(2.0 - 1.0) \times 10^5 \times (6.0 - 1.0) \times 10^{-3}$ = 500 J. (Any rectangle with $\Delta P \times \Delta V = 500$ J, traced clockwise.)

MARKING SCHEME

- ✓ Four corners forming a rectangle on the P–V diagram.
- ✓ Its area $\Delta P \times \Delta V = 500$ J.
- ✓ Traced clockwise, so the work is done by the gas.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	6	16	9	**31**

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