Grade 12 Physics, Explained

Unit 4 · First Law of Thermodynamics

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. isolated system
2. internal energy
3. indicator diagram
4. isothermal
5. adiabatic
6. isochoric
7. degrees of freedom
8. reversible

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. $dV = A\,dx$
2. dx
3. piston, area A
4. $F = PA$

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **b.** its internal energy doesn't change, and the heat it takes in equals the work it does — U · 1

The internal energy of an ideal gas depends only on its temperature, so $\Delta U = 0$ and, by the first law, $Q = W$.

ii. **a.** 1.67 — K · 1

A single atom has 3 degrees of freedom, so $\gamma = 1 + 2/f = 1 + 2/3 = 1.67$.

iii. **b.** rises by 350 J — A · 1

Work done on the gas means $W = -100$ J. $\Delta U = Q - W = 250 - (-100) = +350$ J.

iv. **b.** 900 J — A · 1

$W = P\Delta V = 1.5 \times 10^5 \times (10.0 - 4.0) \times 10^{-3} = 900$ J.

v. **b.** γ times as steep — U · 1

(dP/dV) is $-P/V$ for the isotherm and $-\gamma P/V$ for the adiabat: γ times as steep, since $\gamma > 1$.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ The fast compression is (nearly) adiabatic: little heat escapes.
- ✓ The work done on the air raises its internal energy and temperature.

b. MARKING SCHEME — K · 2

- ✓ Any one, e.g. a reversible process can be run backwards with the system and surroundings both returned to their first states; an irreversible one can't.
- ✓ Any second, e.g. a reversible process is very slow, through equilibrium states, with nothing dissipated (no friction); an irreversible one passes through non-equilibrium states and dissipates energy (e.g. a balloon bursting, a car braking).

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c. MARKING SCHEME A · 3

- ✓ 1870 J at constant volume.
- ✓ 1246 J more at constant pressure (nRΔT).
- ✓ The extra heat goes into the work of expansion.

5 Long answer

ANSWER U · 4

Isothermal: PV = constant. Differentiating, P dV + V dP = 0, so $(dP/dV)_{\text{iso}} = -\text{P/V}$.
Adiabatic: PV^{γ} = constant. Differentiating, $\gamma PV^{\gamma-1}\,dV + V^{\gamma}\,dP = 0$. Dividing by V^{γ}: $\gamma(\text{P/V})\,dV + dP = 0$, so $(dP/dV)_{\text{adia}} = -\gamma\text{P/V}$.
So $(dP/dV)_{\text{adia}} = \gamma\,(dP/dV)_{\text{iso}}$. Since $\gamma > 1$, the adiabat is the steeper.

MARKING SCHEME

- ✓ From PV = constant, slope −P/V.
- ✓ Differentiating PV^{γ} = constant.
- ✓ Adiabatic slope $-\gamma$P/V.
- ✓ So the adiabat is γ times as steep ($\gamma > 1$).

6 Numerical problems

a. ANSWER A · 4

i. $T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1} = 290 \times 15^{0.4} = \mathbf{857\ K}$ (about 584 °C).

ii. $P_2 = P_1\left(\frac{V_1}{V_2}\right)^{\gamma} = 1.0 \times 10^5 \times 15^{1.4} = \mathbf{4.4 \times 10^6\ Pa}$.

iii. Work done by the gas per mole, $W = \frac{R(T_1 - T_2)}{\gamma - 1} = \frac{8.31 \times (290 - 857)}{0.4} = -1.18 \times 10^4$ J, so about **11.8 kJ** of work is done **on** each mole.

MARKING SCHEME

- ✓ $T_2 = T_1(V_1/V_2)^{\gamma-1} = 857$ K.
- ✓ $P_2 = P_1(V_1/V_2)^{\gamma} = 4.4 \times 10^6$ Pa.
- ✓ $W = R(T_1 - T_2)/(\gamma - 1)$.
- ✓ About 11.8 kJ done on each mole.

b. ANSWER A · 4

i. $W = nRT\ln\frac{V_2}{V_1} = 0.50 \times 8.31 \times 350 \times \ln\frac{2.0}{6.0} = \mathbf{-1.60 \times 10^3\ J}$ (1.60 kJ done on the gas).

ii. ΔU = 0, so Q = W: it gives out **1.60 kJ** of heat.

iii. $P_1 = \frac{nRT}{V_1} = \frac{0.50 \times 8.31 \times 350}{6.0 \times 10^{-3}} = \mathbf{2.4 \times 10^5\ Pa}$; $P_1V_1 = P_2V_2$, so $P_2 = 3P_1 = \mathbf{7.3 \times 10^5\ Pa}$.

MARKING SCHEME

- ✓ $W = nRT\ln(V_2/V_1) = -1.60$ kJ.
- ✓ ΔU = 0, so 1.60 kJ of heat is given out.
- ✓ $P_1 = 2.4 \times 10^5$ Pa.
- ✓ $P_2 = 7.3 \times 10^5$ Pa.

7 Think

ANSWER HA · 3

The change in internal energy is the same: U is a state variable, fixed by the states A and B alone. The work done is the area under the path, which differs from path to path, so it need not be the same. By the first law Q = ΔU + W, so the heat differs too.

MARKING SCHEME

- ✓ ΔU is the same: internal energy is a state variable.
- ✓ W differs: it is the area under each path.
- ✓ Q = ΔU + W differs as well.

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8 Activity: Squeeze the air, feel the change

(not marked)

WHAT STUDENTS SHOULD FIND

The pump's end gets warm: quick compressions are nearly adiabatic, and the work done on the air raises its internal energy and temperature. In the syringe the plunger is hard to push and springs back: the squeezed air has a higher pressure. Pushing in, you do work on the air (negative work by the gas); springing back, the air does work on the plunger (positive work by the gas). The syringe's temperature change is too small to feel.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	9	8	13	3	**33**

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