Unit 3 · Fluid Mechanics

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. Pascal's law
2. upthrust
3. metacenter
4. surface tension
5. angle of contact
6. viscosity
7. terminal velocity
8. Reynolds number

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. F
2. U
3. W
4. terminal velocity

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **b.** Pa s (N s/m^2) — K · 1

$\eta = F \div (A \times dv/dx)$, so its unit is N $\div$ (m^2 $\times$ s^{-1}) = N s/m^2 = Pa s. The poise is the CGS unit (0.1 Pa s).

ii. **b.** a convex meniscus and falls in a capillary — U · 1

For mercury on glass cohesion is stronger than adhesion: $\theta \approx 140°$, more than 90°, so the meniscus is convex and cos θ is negative, a capillary fall.

iii. **c.** 75 000 N — A · 1

$F_2 = F_1 \times A_2/A_1 = 150 \times 0.20 \div (4 \times 10^{-4}) = 150 \times 500 = 75\,000$ N.

iv. **a.** laminar, since the Reynolds number is about 72 — A · 1

$R_e = \rho v D/\eta = 900 \times 0.4 \times 0.010 \div 0.05 = 72$, far below 2000: laminar.

v. **b.** the fast air above it has a lower pressure than the still air inside — U · 1

By Bernoulli's principle, where the air moves faster its pressure is lower. The still air inside is at the ordinary pressure, so the net push on the roof is upwards.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ Soap lowers the surface tension of water.
- ✓ So the water spreads into the fibers and under the grease instead of forming drops.

b. MARKING SCHEME — U · 2

- ✓ Liquid: faster molecules break away from their neighbors' attraction more easily, so η falls.
- ✓ Gas: faster molecules carry more momentum between layers, so η rises.

c. MARKING SCHEME — A · 3

- ✓ $h = 2T \cos\theta/(r\rho g)$.
- ✓ h = 3.7 cm.
- ✓ 7.3 cm in the narrower tube.

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5 Long answer

ANSWER U · 4

Let its top be at depth h_1 and its bottom at $h_2 = h_1 + l$. The pressure on the top is $P_0 + h_1\sigma g$, pushing down with a force $(P_0 + h_1\sigma g)A$; on the bottom it is $P_0 + h_2\sigma g$, pushing up with $(P_0 + h_2\sigma g)A$. The sideways pushes on the curved side cancel.

Net upward force: $U = (h_2 - h_1)\sigma gA = lA\sigma g = V\sigma g$, where $V = lA$ is the volume of the cylinder.

$V\sigma g$ is the weight of a volume V of the liquid, the liquid the cylinder pushes aside: the upthrust equals the weight of the liquid displaced (Archimedes' principle).

MARKING SCHEME

- ✓ Downward force on the top $(P_0 + h_1\sigma g)A$ and upward force on the bottom $(P_0 + h_2\sigma g)A$.
- ✓ The sideways forces cancel.
- ✓ $U = (h_2 - h_1)\sigma gA = V\sigma g$.
- ✓ $V\sigma g$ is the weight of the liquid displaced.

6 Numerical problems

a. **ANSWER** A · 4

i. $v = \dfrac{2r^2(\rho - \sigma)g}{9\eta} = \dfrac{2 \times (1.5 \times 10^{-3})^2 \times (7800 - 950) \times 9.8}{9 \times 0.95} = \mathbf{0.035\ m/s}$ (3.5 cm/s).

ii. At terminal velocity $F = W - U = \dfrac{4}{3}\pi r^3(\rho - \sigma)g = \dfrac{4}{3}\pi \times (1.5 \times 10^{-3})^3 \times 6850 \times 9.8 = \mathbf{9.5 \times 10^{-4}}$ **N**. (Check: $6\pi\eta rv = 6\pi \times 0.95 \times 1.5 \times 10^{-3} \times 0.0353 = 9.5 \times 10^{-4}$ N.)

MARKING SCHEME

- ✓ $v = 2r^2(\rho - \sigma)g/(9\eta)$.
- ✓ $v = 0.035$ m/s.
- ✓ At terminal velocity $F = W - U$.
- ✓ $F = 9.5 \times 10^{-4}$ N.

b. **ANSWER** A · 4

i. $A \propto d^2$, so the area is a quarter: $v_2 = 4 \times 1.2 = \mathbf{4.8\ m/s}$.

ii. $P_2 = P_1 - \dfrac{1}{2}\rho(v_2^2 - v_1^2) = 1.8 \times 10^5 - 500 \times (23.04 - 1.44) = 1.8 \times 10^5 - 10\,800 = \mathbf{1.69 \times 10^5\ Pa}$.

iii. $Av = \pi \times 0.040^2 \times 1.2 = \mathbf{6.0 \times 10^{-3}\ m^3/s}$ (6 liters a second).

MARKING SCHEME

- ✓ $v_2 = 4.8$ m/s from continuity.
- ✓ Bernoulli for a horizontal pipe.
- ✓ $P_2 = 1.69 \times 10^5$ Pa.
- ✓ Rate of flow 6.0×10^{-3} m^3/s.

7 Think

ANSWER HA · 3

The water speeds up as it falls, under gravity. In steady flow the same volume passes every cross-section each second, Av = constant. So where v is larger, lower down, the area A of the stream must be smaller: the stream narrows.

MARKING SCHEME

- ✓ The water speeds up as it falls.
- ✓ Av is the same at every cross-section (continuity).
- ✓ So a larger v lower down means a smaller area.

8 Activity: Watch a ball reach terminal velocity

(not marked)

WHAT STUDENTS SHOULD FIND

The ball speeds up at first, but within a few centimeters its speed stops rising: the later stretches take the same time. It has reached its terminal velocity, where the viscous drag and the upthrust together balance its weight ($F + U = W$). A smaller ball, or a thicker liquid, gives a lower terminal velocity.

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See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **The Rock That Floats** (exhibit): Pumice floats because its trapped gas bubbles make its average density less than water's: it sinks only until the upthrust equals its weight.
- **Galileo Thermometer** (exhibit): Each sealed bulb floats or sinks by Archimedes' principle: as the warming liquid's density falls, so does each bulb's upthrust, and they sink one by one.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	7	10	13	3	**33**