Unit 2 · Periodic Motion

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

The greatest speed is $r\omega$, where ω is the **angular** frequency, $\omega = 2\pi f$, not f. So $\omega = 2\pi \times 4 = 25.1$ rad/s and $v_{max} = r\omega = 0.03 \times 25.1 = \mathbf{0.75\ m/s}$.

MARKING SCHEME

- ✓ The mistake: f used in place of ω.
- ✓ $\omega = 2\pi f = 25.1$ rad/s.
- ✓ Greatest speed 0.75 m/s.

2 Read the data

ANSWER HA · 5

i. T^2/l is about 4.03 for every reading except $\mathbf{l = 0.60\ m}$, where it is 4.67. On the line, T^2 there should be about $\mathbf{2.42\ s^2}$.

ii. Slope $= (3.63 - 1.21) \div (0.90 - 0.30) = 2.42 \div 0.60 \approx \mathbf{4.03\ s^2/m}$.

iii. Slope $= 4\pi^2/g$, so $g = 4\pi^2 \div 4.03 \approx \mathbf{9.8\ m/s^2}$.

iv. The slope uses all the good readings at once, so a single bad reading shows up as a point off the line and can be left out, and random errors in each reading partly average out.

MARKING SCHEME

- ✓ The 0.60 m reading is off the line.
- ✓ It should be about 2.42 s^2.
- ✓ Slope $\approx$ 4.03 s^2/m.
- ✓ $g = 4\pi^2/\text{slope} \approx 9.8\ m/s^2$.
- ✓ The slope uses all the readings, so a bad one shows up and errors average out.

3 A real setting

ANSWER A · 4

i. The extra weight $240 \times 9.8 = 2352$ N compresses the springs by 0.040 m, so $k = 2352 \div 0.040 = \mathbf{58\ 800\ N/m}$.

ii. $T = 2\pi\sqrt{\dfrac{m}{k}} = 2\pi\sqrt{\dfrac{1440}{58\,800}} = \mathbf{0.98\ s}$, about one bounce a second.

MARKING SCHEME

- ✓ Extra weight 2352 N.
- ✓ k = 58 800 N/m.
- ✓ $T = 2\pi\sqrt{(m/k)}$.
- ✓ T = 0.98 s.

4 Show that

ANSWER A · 3

$v = \dfrac{dy}{dt} = r\omega\cos\omega t = r\omega\sin\left(\omega t + \dfrac{\pi}{2}\right)$. So the velocity is a sine curve with the phase $\omega t + \pi/2$: it leads the displacement by $\pi/2$, a quarter of a cycle.

The velocity is greatest a quarter of a cycle before the displacement is greatest, that is, when the displacement is zero: the greatest speed is at the mean position.

MARKING SCHEME

- ✓ $v = r\omega\cos\omega t$.
- ✓ $\cos\omega t = \sin(\omega t + \pi/2)$, so v leads y by $\pi/2$.
- ✓ So the greatest speed comes where y = 0, at the mean position.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

5 Predict, then explain

ANSWER A · 3

i. **In series**: the pair is softer than one spring.

ii. In series $\frac{1}{k_s} = \frac{1}{k} + \frac{1}{k}$, so $k_s = k/2$; in parallel $k_p = 2k$. With $T = 2\pi\sqrt{m/k}$:

$$\frac{T_s}{T_p} = \sqrt{\frac{k_p}{k_s}} = \sqrt{\frac{2k}{k/2}} = \sqrt{4} = \mathbf{2}.$$

MARKING SCHEME

- ✓ Prediction: longer in series.
- ✓ k/2 in series and 2k in parallel.
- ✓ The periods are in the ratio 2 : 1 (series : parallel).

6 Which explanation fits?

ANSWER U · 2

Utsab's. Standing still, the soldiers put the same weight on the bridge, and it is safe, so their weight alone isn't the danger. Marching in step, they push the bridge periodically; if their steps match the bridge's natural frequency it resonates and its swaying grows large. Out of step, the pushes don't add up.

MARKING SCHEME

- ✓ Resonance: the same weight standing still is safe.
- ✓ In step, periodic pushes at the bridge's natural frequency build up a large amplitude.

7 Estimate it

ANSWER A · 3

Adult: $T = 2\pi\sqrt{(0.9/9.8)} = 1.90$ s, so one swing takes about **0.95 s**. Child: $T = 2\pi\sqrt{(0.5/9.8)} = 1.42$ s, one swing about **0.71 s**.

The child's steps are quicker, by about $\sqrt{(0.9/0.5)} \approx$ **1.3 times**. (A real leg is not a simple pendulum, so these are only rough.)

MARKING SCHEME

- ✓ Adult's swing about 0.95 s.
- ✓ Child's swing about 0.71 s.
- ✓ The child steps about 1.3 times as often.

8 Correct the misconceptions

ANSWER U · 3

1. The acceleration, $-\omega^2 y$, is zero at the mean position and greatest at the extremes; it is the speed that is greatest at the mean position.
2. The period, $2\pi\sqrt{(l/g)}$, doesn't depend on the mass of the bob.
3. A hanging mass oscillates about its equilibrium position, where $kx = mg$, below the natural length.

MARKING SCHEME

- ✓ Acceleration is zero at the mean position, greatest at the extremes.
- ✓ The bob's mass doesn't change the period.
- ✓ It oscillates about the equilibrium position ($kx = mg$), not the natural length.

9 What if?

ANSWER A · 2

$T = 2\pi\sqrt{(I/\kappa)}$. The ring's moment of inertia is twice the disc's, so the period becomes $\sqrt{2}$ **times as long** (about 1.41 times).

MARKING SCHEME

- ✓ $T \propto \sqrt{I}$, with I doubled.
- ✓ The period is $\sqrt{2}$ times as long.

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking

10 Design it

ANSWER HA · 3

1. Static: hang masses m and measure the extension x for each; $kx = mg$, so k is mg/x (or the slope of a graph of mg against x).
2. Dynamic: hang one mass m, pull it down a little and time 20 oscillations to find T; then $k = 4\pi^2 m/T^2$ (repeat for a few masses; T^2 against m is a straight line of slope $4\pi^2/k$).
   Keep the same spring (and small, equal amplitudes). Both methods measure the same Hooke's-law constant, $F = -kx$, so they should agree within the errors of measurement.

MARKING SCHEME

- ✓ Static method: $k = mg/x$ from the extension.
- ✓ Dynamic method: $k = 4\pi^2 m/T^2$ from timing oscillations.
- ✓ The same spring, small amplitudes; both measure the same k in $F = -kx$.

11 Make your own

ANSWER HA · 2

$T = 2\pi\sqrt{(l/g)}$ with $T = 1$ s: $l = gT^2/4\pi^2 = 9.8 \div 39.5 =$ **0.248 m**, about 25 cm (from the support to the middle of the bob).

MARKING SCHEME

- ✓ $l = gT^2/4\pi^2$.
- ✓ $l \approx 0.25$ m, measured to the middle of the bob.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Sand Lissajous Pendulum** (exhibit): The pendulum swings in two directions at once, each a simple harmonic motion; their frequency ratio and phase difference set the curve drawn in the sand.
- **The Clay Owl** (exhibit): Count the owl's swings in 10 seconds, pushed gently, then harder: for small swings the period depends on the string's length, not the amplitude.
- **Coupled Pendulums** (exhibit): Start one pendulum: through the shared support it nudges the other once a swing, at that pendulum's natural frequency, so the other's swing builds: resonance.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	8	15	10	**33**

Science Playground, Katunje, Bhaktapur
Book a class visit: https://science-playground.com/booking