Unit 2 · Periodic Motion

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. periodic motion
2. simple harmonic motion
3. amplitude
4. frequency
5. hertz
6. phase
7. damping
8. resonance

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. l
2. T
3. mg sin θ
4. mg cos θ

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **b.** a straight line through the origin with a negative slope — K · 1

$a = -\omega^2 y$: a straight line through the origin with slope $-\omega^2$. The acceleration always points against the displacement.

ii. **c.** 2 s — K · 1

It beats seconds: each swing one way takes 1 s, so a complete oscillation takes 2 s.

iii. **b.** 0.63 s — A · 1

$T = 2\pi\sqrt{(m/k)} = 2\pi\sqrt{(0.5/50)} = 2\pi \times 0.1 = 0.63$ s.

iv. **c.** four times as much — U · 1

$E = \frac{1}{2}m\omega^2 r^2$, so $E \propto r^2$: twice the amplitude, four times the energy.

v. **c.** returns to its rest position in the shortest time without overshooting — U · 1

Critical damping brings a body back to its mean position in the shortest possible time with no overshoot. Heavy damping creeps back slowly; light damping keeps oscillating.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ The restoring force grows in the same proportion as the mass.
- ✓ So the acceleration (g/l)x, and the period, don't change.

b. MARKING SCHEME — K · 2

- ✓ Any one, e.g. a damped oscillation loses energy and its amplitude dies away; in a forced oscillation an outside periodic force keeps supplying energy, so it keeps going.
- ✓ Any second, e.g. a damped oscillation is at (about) its natural frequency; a forced oscillation is at the driving frequency, whatever the natural frequency.

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c. MARKING SCHEME A · 3

✓ Amplitude 0.05 m.

✓ f = 2 Hz.

✓ Greatest speed $r\omega = 0.63$ m/s.

5 Long answer

ANSWER U · 4

When the string makes an angle θ with the vertical, the weight mg has a part mg cos θ along the string (balanced by the tension) and a part mg sin θ along the arc, towards the mean position. This is the restoring force: $F = -mg \sin \theta$.

For small angles, $\sin \theta \approx \theta$ (in radians), and $\theta = x/l$, where x is the distance along the arc. So $F = -(mg/l)x$, and $a = F/m = -(g/l)x$.

The acceleration is proportional to the displacement and points towards the mean position: SHM, with $\omega^2 = g/l$.

So $T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{l}{g}}$.

MARKING SCHEME

✓ The restoring force is $-mg \sin \theta$ (the part of the weight along the arc).

✓ For small θ, $\sin \theta \approx \theta = x/l$, so $a = -(g/l)x$.

✓ $a \propto -x$, so the motion is SHM with $\omega = \sqrt{(g/l)}$.

✓ $T = 2\pi\sqrt{(l/g)}$.

6 Numerical problems

a. ANSWER A · 4

i. $\omega = 2\pi \times 1.5 = 9.42$ rad/s, so $k = m\omega^2 = 0.2 \times 9.42^2 = \mathbf{17.8\ N/m}$.

ii. $E = \frac{1}{2}m\omega^2r^2 = \frac{1}{2} \times 0.2 \times 9.42^2 \times 0.08^2 = \mathbf{0.057\ J}$.

iii. $KE = \frac{1}{2}m\omega^2(r^2 - y^2) = \frac{1}{2} \times 0.2 \times 9.42^2 \times (0.08^2 - 0.04^2) = \mathbf{0.043\ J}$ (three quarters of E). $v = \omega\sqrt{(r^2 - y^2)} = 9.42 \times \sqrt{0.0048} = \mathbf{0.65\ m/s}$.

MARKING SCHEME

✓ $\omega = 9.42$ rad/s and k = 17.8 N/m.

✓ E = 0.057 J.

✓ KE = 0.043 J.

✓ v = 0.65 m/s.

b. ANSWER A · 4

i. $T = 35.9 \div 20 = \mathbf{1.795\ s}$.

ii. $g = \frac{4\pi^2 l}{T^2} = \frac{4\pi^2 \times 0.80}{1.795^2} = \mathbf{9.80\ m/s^2}$.

iii. For T = 2 s: $l = \frac{gT^2}{4\pi^2} = \frac{9.80 \times 4}{4\pi^2} = 0.993$ m, so it must be made longer by $0.993 - 0.80 = \mathbf{0.19\ m}$.

MARKING SCHEME

✓ T = 1.795 s.

✓ $g = 4\pi^2 l/T^2 = 9.80$ m/s^2.

✓ Length for T = 2 s, 0.993 m.

✓ Longer by 0.19 m.

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7 Think

ANSWER HA · 3

The **spring clock**. Its period, $2\pi\sqrt{(m/k)}$, has no g in it: gravity only moves the point it oscillates about. The pendulum's period, $2\pi\sqrt{(l/g)}$, grows as g falls: on the Moon it swings $\sqrt{6} \approx 2.45$ times as slowly, so the pendulum clock runs very slow.

MARKING SCHEME

- ✓ The spring clock keeps time.
- ✓ $T = 2\pi\sqrt{(m/k)}$ doesn't depend on g.
- ✓ $T = 2\pi\sqrt{(l/g)}$ is longer where g is smaller ($\sqrt{6}$ times on the Moon), so the pendulum clock runs slow.

8 Activity: How the period depends on length

(not marked)

WHAT STUDENTS SHOULD FIND

The period grows with the length, but not in proportion: T^2/l comes out about the same each time, close to $4\pi^2/9.8 \approx 4.0$ s^2/m, so $T^2 \propto l$ and $g \approx 4\pi^2 \div 4.0 \approx 9.8$ m/s^2. Timing 20 swings, not one, makes the error in starting and stopping the watch small. The length must be measured to the middle of the stone (the effective length), and the swings kept small.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Sand Lissajous Pendulum** (exhibit): The pendulum swings in two directions at once, each a simple harmonic motion; their frequency ratio and phase difference set the curve drawn in the sand.
- **The Clay Owl** (exhibit): Count the owl's swings in 10 seconds, pushed gently, then harder: for small swings the period depends on the string's length, not the amplitude.
- **Coupled Pendulums** (exhibit): Start one pendulum: through the shared support it nudges the other once a swing, at that pendulum's natural frequency, so the other's swing builds: resonance.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	10	8	12	3	**33**

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