Unit 1 · Rotational Dynamics and Oscillatory Motion

HARDER PRACTICE QUESTIONS · ANSWER KEY

Full marks 32

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Spot the error

ANSWER U · 3

She has left out the ring's turning. A rolling ring has kinetic energy $\frac{1}{2}Mv^2(1 + K^2/R^2)$, and for a ring $K^2/R^2 = 1$, so $Mgh = Mv^2$ and $v = \sqrt{(gh)} = \sqrt{(9.8 \times 0.8)} = \mathbf{2.8\ m/s}$. Her answer is the speed of a body sliding down without friction, which doesn't turn at all.

MARKING SCHEME

- ✓ The mistake: the rotational kinetic energy is left out.
- ✓ $Mgh = \frac{1}{2}Mv^2(1 + K^2/R^2) = Mv^2$ for a ring.
- ✓ $v = \sqrt{(gh)} = 2.8$ m/s.

2 A real setting

ANSWER A · 4

i. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 50 \times 0.4^2 = \mathbf{4.0\ kg\ m^2}$. $\omega = 2\pi \times 2400 \div 60 = 251.3$ rad/s, so $KE = \frac{1}{2} \times 4.0 \times 251.3^2 = \mathbf{1.26 \times 10^5\ J}$ (126 kJ).

ii. $t = 126\,000 \div 300 \approx \mathbf{420\ s}$, about 7 minutes.

MARKING SCHEME

- ✓ $I = 4.0$ kg m^2.
- ✓ $\omega = 251.3$ rad/s.
- ✓ $KE = 1.26 \times 10^5$ J.
- ✓ $t \approx 420$ s.

3 Predict, then explain

ANSWER A · 4

i. **No**: angular momentum is conserved (no outside torque about the axle), but the discs rub against each other until they turn together, and that friction turns some kinetic energy into heat.

ii. $I_1\omega_1 = (I_1 + I_2)\omega$: $0.6 \times 10 = 0.8\omega$, so $\omega = \mathbf{7.5\ rad/s}$.

iii. Before: $\frac{1}{2} \times 0.6 \times 10^2 = \mathbf{30\ J}$. After: $\frac{1}{2} \times 0.8 \times 7.5^2 = \mathbf{22.5\ J}$. Lost: 7.5 J, **a quarter**.

MARKING SCHEME

- ✓ Prediction: kinetic energy is lost (friction between the discs), angular momentum kept.
- ✓ $\omega = 7.5$ rad/s.
- ✓ 30 J before, 22.5 J after.
- ✓ A quarter (7.5 J) lost.

4 Show that

ANSWER A · 3

i. $K^2/R^2 = \frac{2}{3}$, so $KE = \frac{1}{2}Mv^2(1 + \frac{2}{3})$. Rotational ÷ total $= \frac{2/3}{1 + 2/3} = \frac{2/3}{5/3} = \frac{2}{5}$.

ii. $a = \frac{g\sin\theta}{1 + K^2/R^2} = \frac{g\sin\theta}{5/3} = \frac{3}{5}g\sin\theta$.

MARKING SCHEME

- ✓ $K^2/R^2 = 2/3$.
- ✓ Rotational share $(2/3) \div (5/3) = 2/5$.
- ✓ $a = g\sin\theta \div (5/3) = (3/5)\,g\sin\theta$.

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5 Check the readings

ANSWER HA · 4

i. ω rises by 3.0 rad/s each second, so the reading at **4 s** should be **12.0 rad/s**, not 13.5.

ii. $\alpha = 3.0$ rad/s^2, so $\tau = I\alpha = 0.8 \times 3.0 = \mathbf{2.4\ N\ m}$.

iii. $\theta = \frac{1}{2}\alpha t^2 = \frac{1}{2} \times 3.0 \times 5^2 = \mathbf{37.5\ rad}$ (about 6 turns).

MARKING SCHEME

- ✓ The 4 s reading; it should be 12.0 rad/s.
- ✓ $\alpha = 3.0$ rad/s^2.
- ✓ $\tau = 2.4$ N m.
- ✓ $\theta = 37.5$ rad.

6 Estimate it

ANSWER A · 3

$\omega = 2\pi \times 5 \approx 31.4$ rad/s, so KE $= \frac{1}{2} \times 1 \times 31.4^2 \approx$ **490 J**, about 500 J.

Climbing 1 m: mgh $= 50 \times 9.8 \times 1 = 490$ J: **about the same**.

MARKING SCHEME

- ✓ $\omega \approx 31.4$ rad/s.
- ✓ KE ≈ 490 J.
- ✓ About the same as mgh = 490 J.

7 Correct the misconceptions

ANSWER U · 3

1. The radius of gyration depends on how the mass is spread, not how much: any thin ring has K = R.
2. The point touching the road is at rest at that instant (its forward v and its backward Rω cancel); the top moves at 2v.
3. A push along the spanner ($\theta = 0°$) gives no torque; the torque rF sin θ is greatest when the push is at right angles to the spanner.

MARKING SCHEME

- ✓ K doesn't depend on the mass: K = R for any thin ring.
- ✓ The lowest point of the wheel is at rest.
- ✓ The torque is zero along the spanner and greatest at 90° to it.

8 What if?

ANSWER A · 3

$v = \sqrt{\dfrac{2gh}{1 + K^2/R^2}}$, with $K^2/R^2 = \frac{1}{2}$ for any disc.

i. **v**, the same: the mass doesn't come in.

ii. **v**, the same: the radius doesn't come in either, only the shape.

iii. $\sqrt{2}$ **v** (about 1.41v): $v \propto \sqrt{h}$.

MARKING SCHEME

- ✓ Mass doubled: still v.
- ✓ Radius halved: still v (only the shape, K^2/R^2, matters).
- ✓ Height doubled: $\sqrt{2}$ v.

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9 Think

ANSWER HA · 3

The pole puts mass far from the rope, so it makes her moment of inertia about the rope much larger. A small push or lean gives a small torque, and $\alpha = \tau/I$ is then very small: she starts to topple only slowly, and has time to move and correct it before she has turned far.

MARKING SCHEME

- ✓ The pole's mass is far from the rope, so I about the rope is large.
- ✓ For a given torque, $\alpha = \tau/I$ is small.
- ✓ So she tips slowly and has time to correct herself.

10 Make your own

ANSWER HA · 2

For example, one ball on each side, both 0.1 m from the middle: $I = 2 \times 1 \times 0.1^2 = 0.02$ kg m^2. Then both at 0.2 m: $I = 2 \times 1 \times 0.2^2 = 0.08$ kg m^2, four times as much. (Any arrangement with twice the distance works, since $I \propto r^2$.)

MARKING SCHEME

- ✓ Two arrangements, the second with the balls twice as far out.
- ✓ Shown: $I \propto r^2$, so four times (e.g. 0.02 and 0.08 kg m^2).

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Uphill Cone - Defying Gravity?** (exhibit): As the double cone seems to climb the spreading rails, its center of gravity drops, and the lost potential energy becomes both rotational and translational kinetic energy.
- **Magnetic Levitation** (exhibit): Ask the guide to spin the floating globe: with almost no frictional torque on it, its angular deceleration is tiny, so it keeps turning a long time.
- **Spin Art Top** (exhibit): A fast top stays upright: gravity's torque only makes its large angular momentum precess slowly; as friction slows it, it wobbles more and draws loops.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	0	6	17	9	**32**

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