Grade 12 Physics, Explained

Unit 1 · Rotational Dynamics and Oscillatory Motion

ANSWER KEY · FOR TEACHERS

Full marks 33

Give the marks for the points in the marking scheme; an answer in other words that says the same is right.

1 Key words

1. rigid body
2. moment of inertia
3. kg m^2
4. radius of gyration
5. torque
6. angular momentum
7. conservation of angular momentum
8. pure rolling

K · 4

Each carries 0.5 mark.

2 Label the diagram

1. v
2. P is at rest
3. 2v
4. $\sqrt{2}$ v

K · 2

Each carries 0.5 mark.

3 Multiple choice

i. **b.** kg m^2/s — K · 1

$L = I\omega$: kg m^2 × rad/s = kg m^2/s (the same as J s). kg m^2 is the unit of moment of inertia, N m of torque.

ii. **c.** four times as large — U · 1

$\frac{1}{3}Ml^2 \div \frac{1}{12}Ml^2 = 4$: about the end, half of the rod is much farther from the axis.

iii. **b.** 1.6 J — A · 1

For a ring $K^2/R^2 = 1$, so KE = ½$Mv^2(1 + 1) = Mv^2 = 0.4 \times 2^2 = 1.6$ J: half moving, half turning.

iv. **c.** 4 rad/s^2 — A · 1

$\tau = I\alpha$, so $\alpha = 6 \div 1.5 = 4$ rad/s^2.

v. **b.** the Sun's pull gives no torque about the Sun, so mvr stays constant — U · 1

The pull points straight at the Sun, so its torque about the Sun is zero and the angular momentum mvr is conserved: a smaller r needs a larger v.

4 Short answers

a. MARKING SCHEME — U · 2

- ✓ The ring has the larger moment of inertia ($K^2/R^2 = 1$ against ½).
- ✓ More of its energy goes into turning, so its speed down the slope is less.

b. MARKING SCHEME — U · 2

- ✓ K = $\sqrt{(I/M)}$, and both I and M double.
- ✓ So K depends on the spread of the mass about the axis, not the amount.

c. MARKING SCHEME — A · 3

- ✓ I = Σmr^2.
- ✓ I = 0.15 kg m^2.
- ✓ K = 0.27 m.

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5 Long answer

ANSWER U · 4

Take the end O as the origin, with x along the rod. The mass per unit length is M/l, so a small piece of length dx at a distance x from O has mass $dm = \frac{M}{l}dx$ and adds $dI = \frac{M}{l}x^2\,dx$.

All the pieces lie between x = 0 and x = l, so I = $\int$ (M/l)x^2 dx from 0 to l = (M/l) × $l^3/3 = \frac{Ml^2}{3}$.

From $I = MK^2$: $K = \frac{l}{\sqrt{3}} \approx 0.58\,l$.

MARKING SCHEME

- ✓ The mass of a piece, dm = (M/l) dx, and dI = dm x^2.
- ✓ The integral from 0 to l.
- ✓ I = $Ml^2/3$.
- ✓ K = $l/\sqrt{3}$.

6 Numerical problems

a. **ANSWER** A · 4

i. $v = \sqrt{\frac{2gh}{1 + K^2/R^2}} = \sqrt{\frac{2 \times 9.8 \times 1.4}{1.4}} = \sqrt{19.6} = \mathbf{4.4\ m/s}$.

ii. Rotational ÷ total = $\frac{2/5}{1 + 2/5} = \mathbf{2/7}$ (about 29%).

iii. $a = \frac{g\sin\theta}{1 + K^2/R^2} = \frac{9.8 \times 0.5}{1.4} = \mathbf{3.5\ m/s^2}$.

MARKING SCHEME

- ✓ v = $\sqrt{(2gh/(1 + K^2/R^2))}$.
- ✓ v = 4.4 m/s.
- ✓ Fraction rotational 2/7.
- ✓ a = 3.5 m/s^2.

b. **ANSWER** A · 4

i. $I = \frac{1}{2}MR^2 = \frac{1}{2} \times 12 \times 0.3^2 = \mathbf{0.54\ kg\ m^2}$.

ii. $\omega = 2\pi \times 2 = 12.57$ rad/s, so $\alpha = 12.57 \div 4 = 3.14$ rad/s^2, and $\tau = I\alpha = 0.54 \times 3.14 = \mathbf{1.70\ N\ m}$.

iii. $\theta = ½\alpha t^2 = ½ \times 3.14 \times 4^2 = 25.1$ rad (4 turns), so $W = \tau\theta = 1.70 \times 25.1 = \mathbf{42.6\ J}$. Check: $½I\omega^2 = ½ \times 0.54 \times 12.57^2 = 42.6$ J.

MARKING SCHEME

- ✓ I = 0.54 kg m^2.
- ✓ $\omega = 12.57$ rad/s and $\alpha = 3.14$ rad/s^2.
- ✓ $\tau = 1.70$ N m.
- ✓ W = $\tau\theta$ = 42.6 J, equal to $½I\omega^2$.

7 Think

ANSWER HA · 3

Roll them from rest down the same slope, side by side. The solid one wins. The hollow one has all its mass near the rim, so its moment of inertia is larger (MR^2 against $\frac{1}{2}MR^2$); more of the energy Mgh goes into turning and less into moving forward, so it is slower. Mass and radius can't tell them apart, since the speed $\sqrt{2gh/(1 + K^2/R^2)}$ depends only on the shape.

MARKING SCHEME

- ✓ Roll both down the same slope from rest: the solid one arrives first.
- ✓ The hollow one has the larger moment of inertia (more mass far from the axis).
- ✓ So more of its energy goes into turning and it moves forward more slowly.

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8 Activity: Spin faster

(not marked)

WHAT STUDENTS SHOULD FIND

You turn faster with the books pulled in. Pulling them in moves mass nearer the axis, so the moment of inertia falls; with no outside torque (only a little friction), $I\omega$ stays nearly constant, so ω rises, as for a skater pulling in her arms. Friction slows you a little each turn, so start counting at once.

See it at Science Playground

Exhibits and plants at the center, Bhaktapur, that go with this unit. Book a class visit at science-playground.com/booking.

- **Uphill Cone - Defying Gravity?** (exhibit): As the double cone seems to climb the spreading rails, its center of gravity drops, and the lost potential energy becomes both rotational and translational kinetic energy.
- **Magnetic Levitation** (exhibit): Ask the guide to spin the floating globe: with almost no frictional torque on it, its angular deceleration is tiny, so it keeps turning a long time.
- **Spin Art Top** (exhibit): A fast top stays upright: gravity's torque only makes its large angular momentum precess slowly; as friction slows it, it wobbles more and draws loops.

Marks by level	Knowledge	Understanding	Application	Higher ability	Total
Marks	7	10	13	3	**33**

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