Unit 24

Radioactivity and Nuclear Reactions

This unit online, with its interactives:
science-playground.com/s/12f24

In February 1896, in Paris, Henri Becquerel wanted sunshine. He meant to leave crystals of a uranium salt on a photographic plate wrapped in thick black paper, out in the sun, to see whether sunlight made them give off rays that could pass through the paper. The sky stayed gray, so he put everything away in a drawer. A few days later he developed the plate anyway, expecting nothing, and found the outlines of the crystals sharp and dark on it. No sunlight, no electricity, no heating: the uranium was sending out something invisible, all by itself, and it never stopped.

That something is **radioactivity**, and in this unit you will meet its three kinds of radiation, the law that says how fast a radioactive sample dies away, and the numbers that measure it: the half-life, the decay constant, the mean life and the activity. Then three things it does for us: the Geiger–Müller tube that counts it click by click, the carbon clock that dates a 10,000-year-old bone, and the radioisotopes that find and treat disease. Grade 11 Physics, Nuclear Physics, explained what a nucleus is made of and why some are bound more tightly than others; here you will see what happens when one isn't bound tightly enough.

24.1 Radioactivity

Radioactivity is the natural process in which an unstable nucleus breaks down, or **decays**, to become more stable, giving out energy as particles or rays. What it gives out is called **radiation**. Some elements, such as uranium, thorium and radium, do it naturally, and every element has radioactive isotopes.

Radioactivity matters to physics because it opened the inside of the atom: Rutherford found the nucleus by firing alpha particles from radium at gold foil (Grade 11, Rutherford's experiment). It matters to everyone else because of its uses, in medicine, industry, dating the past and nuclear power, and because of its dangers.

Becquerel had found it by accident, as the opening tells. He showed that the rays came from the uranium in any compound of it (he first used a **potassium uranyl sulphate**, a double sulphate of potassium and uranium), that they made air conduct electricity, and that their strength didn't depend on light, heat or chemistry. They were called **Becquerel rays**, or uranic rays.

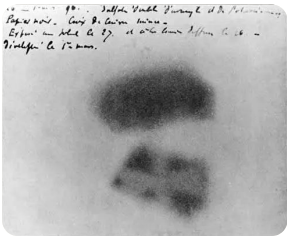


Figure 24.1 *Becquerel's photographic plate of 1896, darkened by a uranium salt kept on it in a dark drawer. The dark shape at the bottom is the shadow of a metal cross placed between the salt and the plate: the rays were partly stopped by the metal. The handwriting is his own.*
Photo: Henri Becquerel, Public domain

A STORY FROM SCIENCE

Marie Curie and the word “radioactivity”

Marie Curie (born Maria Skłodowska in Warsaw, Poland, in 1867, and working in Paris) chose Becquerel’s rays for her doctorate in 1897. Measuring them with a sensitive electrometer built by her husband Pierre, she found that thorium gives them out too, and that the ore **pitchblende** was four times more active than its uranium could explain. So it had to contain something new. Grinding, dissolving and separating tons of pitchblende in a leaking shed, Marie and Pierre found two new elements in 1898: **polonium**, named after her occupied homeland, and **radium**, a million times more active than uranium. She coined the word **radioactivity**.

In 1903 she shared the Nobel Prize in Physics with Pierre Curie and Henri Becquerel, the first woman to win a Nobel Prize; in 1911 she won the Nobel Prize in Chemistry alone, for discovering polonium and radium. She is still the only person to have won Nobel Prizes in two different sciences. During the First World War she drove X-ray vans to the front herself, and from 1922 she was a member of France’s Academy of Medicine, working on the medical uses of radium. She died in 1934 of aplastic anemia, almost certainly caused by her years of work with radiation; her notebooks are still radioactive and are kept in lead-lined boxes.



Figure 24.2 *Marie Curie (1867–1934), about 1920.*
Photo: Henri Manuel, Public domain

DID YOU KNOW?

You are radioactive

About 0.012% of all potassium is the radioactive isotope potassium-40, and a tiny fraction of the carbon in you is carbon-14. In an adult’s body, about 8,000 nuclei decay every second, mostly potassium-40 and carbon-14. A banana, rich in potassium, holds about 15 Bq of potassium-40 (15 decays a second), and eating one gives you a radiation dose of about 0.1 microsievert. A chest X-ray is a few hundred to a thousand bananas. Your body soon gets rid of the extra potassium, so bananas are perfectly safe: it is only a fun way to compare doses.

Check 1

What made Becquerel’s plate dark in the drawer?

a sunlight stored in the uranium crystals

b a chemical reaction between the uranium salt and the plate

c radiation given out by the uranium nuclei themselves, which passed through the black paper

d heat from the uranium salt

24.2 Natural and artificial radioactivity

Natural radioactivity

Natural radioactivity is the spontaneous decay of unstable nuclei of elements found in nature, giving out alpha (α) particles, beta (β) particles or gamma (γ) rays. Nothing has to start it: it goes on by itself. It is found mostly in heavy elements, such as uranium, thorium and radium, and in a few light ones, such as potassium-40 and carbon-14.

Artificial radioactivity

Artificial (or **induced**) **radioactivity** is the radioactivity of nuclei made unstable on purpose, by a nuclear reaction. Stable nuclei are bombarded with particles such as neutrons, protons or alpha particles; the new nuclei formed are unstable, and they decay, giving out radiation. The radioactive isotopes made in this way are called **artificial radioisotopes**.

It was discovered in 1934 by Irène Joliot-Curie (Marie Curie's daughter) and her husband Frédéric Joliot-Curie. They fired alpha particles from polonium at aluminium foil, and found that the foil went on giving out radiation after the alpha source was taken away. The alpha particles had made phosphorus-30, which doesn't exist in nature:

$$^{27}_{13}\mathrm{Al} + {}^{4}_{2}\mathrm{He} \rightarrow {}^{30}_{15}\mathrm{P} + {}^{1}_{0}\mathrm{n}$$

and the phosphorus-30 then decayed, with a half-life of about 2.5 minutes, giving out positrons (positive electrons) and becoming stable silicon-30. They won the 1935 Nobel Prize in Chemistry for it.

Today nuclear reactors and particle accelerators make hundreds of artificial radioisotopes: cobalt-60 for treating cancer, technetium-99m for medical scans, iodine-131 for the thyroid, americium-241 for smoke detectors. Natural and artificial radioactivity together are used in medicine, in radiocarbon dating and in nuclear power.

Check 2

Which of these is an example of **artificial** radioactivity?

a radon gas seeping out of granite rocks

b potassium-40 decaying in a banana

c cobalt-60 made by putting cobalt-59 in a nuclear reactor, where it captures a neutron

d uranium decaying in the Earth's crust

24.3 Radioactive radiations

Radioactive substances give out three kinds of radiation, named by Rutherford with the first three letters of the Greek alphabet in the order of how far they get through matter: **alpha (α)**, **beta (β)** and **gamma (γ)**.

They can be told apart by passing them through an electric or a magnetic field, in a vacuum, as in Figure 24.3:

- **Between charged plates** (an electric field), alpha particles, being heavy and positive, bend towards the negative plate; beta particles, light and negative, bend towards the positive plate, much more sharply; gamma rays go straight on, so they have no charge.
- **In a magnetic field**, alpha and beta particles curve in opposite directions, so their charges are opposite; gamma rays again go straight on: they are neutral, a kind of electromagnetic wave.

The force on a charge q moving at v across a magnetic field B is $F = qvB$, and it bends the particle into a circle of radius $r = \dfrac{mv}{qB}$ (Unit 20, an electron beam in a magnetic field). An alpha particle has twice the charge of a beta particle but about 7,300 times its mass, so in the same field its circle is far larger: it hardly bends at all. That is why Rutherford needed a very strong magnet to show, in 1903, that alpha particles are positive.

Cause of radioactivity

A nucleus is held together by the **strong nuclear force**, an attraction between all its nucleons (protons and neutrons) that acts only over about 10^{-15} m, between neighbors. Against it pushes the **electric repulsion** between its protons, which reaches across the whole nucleus. A nucleus is stable only when the two are well balanced, and that needs the right mix of neutrons and protons (Grade 11, Forces in the nucleus).

When the balance is wrong, the nucleus is **unstable**, and sooner or later it changes into a more stable one by giving out radiation:

- a nucleus that is **too big** (heavier than lead, with more than 82 protons, where the repulsion of so many protons is hard to hold) gives out an **alpha particle** and gets smaller;
- a nucleus with **too many neutrons** for its protons turns a neutron into a proton and gives out a **beta particle** (an electron);
- a nucleus left with **too much energy**, in an excited state after an alpha or beta decay, gives the extra energy out as a **gamma ray**, as an excited atom gives out light.

So the cause of radioactivity is an **unstable nucleus**: one whose repulsion is too much for its binding, or whose neutrons and protons are out of balance.

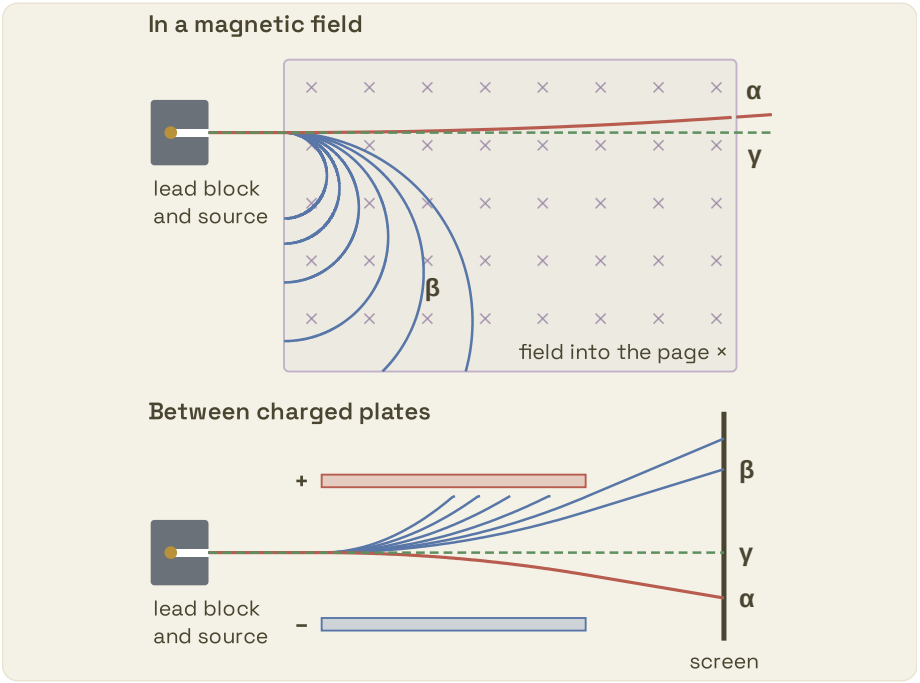


Figure 24.3 *The three radiations from a source in a lead block, in a vacuum, worked out to scale for 5 MeV alpha particles and beta particles of 0.3 to 2.3 MeV. **Top:** in a magnetic field into the page, beta particles curve strongly one way, alpha particles very slightly the other, gamma rays not at all. **Bottom:** between charged plates, alpha particles bend towards the − plate, beta particles towards the + plate, gamma rays go straight on.*

CAREFUL

Draw the directions right

Many books draw the alpha beam bending almost as much as the beta beam in a magnetic field, only the other way. In an exam, draw the **directions** right (alpha and beta opposite, gamma straight; alpha towards the negative plate between plates) and add that alpha bends far less than beta, because it is much heavier.

Properties of alpha particles

1. Alpha particles are **helium nuclei**, ^4_2He: two protons and two neutrons, a helium atom that has lost its two electrons. So an alpha particle has **twice the positive charge** of a proton ($+2e = +3.2 \times 10^{-19}$ C) and about **four times its mass** (6.64×10^{-27} kg).
2. Their speeds are about 1.4×10^7 to 2.05×10^7 m/s (5–7% of the speed of light), depending on the substance that gives them out. All the alpha particles from one kind of nucleus come out with the same energy, between about 4 and 9 MeV (4.2 to 6.8 MeV for most of the usual sources).
3. Although they carry a lot of energy, they are the **least penetrating** of the three. Being heavy and doubly charged, they lose their energy quickly: a sheet of paper, aluminium about 0.02 mm thick, or the dead outer layer of your skin stops them.
4. Their **range** in air at normal pressure is 3 to 8 cm, depending on their energy.
5. They have a very great **ionizing power**: each produces about a hundred thousand ion pairs (tens of thousands in every centimeter of air) before it stops.
6. They make some substances, such as zinc sulphide and barium platinocyanide, glow (**fluorescence**).
7. They are deflected by electric and magnetic fields (only slightly, being heavy).
8. They affect a photographic plate.

9. They are **scattered** when they pass through thin metal foils (Rutherford's experiment).
10. They can cause **nuclear reactions**, and so make artificial radioactivity, as the Joliot-Curies found.

Properties of beta particles

1. Beta particles are **fast-moving electrons**, coming from the nucleus (not from the electron shells): charge $-e = -1.6 \times 10^{-19}$ C, mass 9.11×10^{-31} kg.
2. They come out with **every speed** from almost zero up to about 99% of the speed of light. Each decay shares its energy between the electron and an antineutrino in a different way, so the electrons have every energy from zero up to a maximum, which for most beta emitters is a few MeV or less. With so little mass, even the fastest carry modest energies.
3. They penetrate more than alpha particles: a few millimeters of aluminium stops them, and their range in air is up to several meters.
4. They ionize gases, but their ionizing power is much smaller than that of alpha particles: very roughly a hundred ion pairs per centimeter of air.
5. They make zinc sulphide and barium platinocyanide glow.
6. They are deflected by electric and magnetic fields, much more than alpha particles, and the other way.
7. They affect a photographic plate.
8. They can cause nuclear reactions, and so produce artificial radioactivity.

Properties of gamma rays

1. Gamma rays are **electromagnetic waves** of very short wavelength (about 10^{-11} m and shorter), like X-rays but usually of higher energy. They travel at the speed of light.
2. They are **highly penetrating**: they pass through several centimeters of iron or lead. Lead and thick concrete only reduce them; each 1.2 cm of lead halves the gamma rays of cobalt-60.
3. Their ionizing power is small.
4. They make some substances, such as willemite (a zinc mineral), fluoresce.
5. They affect a photographic plate.
6. They are **not deflected** by electric or magnetic fields: they have no charge.
7. They knock electrons out of the surfaces they fall on (the photoelectric effect).
8. They heat the surfaces that absorb them.
9. They can cause some nuclear reactions (a gamma ray can break a deuterium nucleus into a proton and a neutron).
10. High-energy (hard) gamma rays are used in hospitals for radiotherapy, to kill cancer cells.

Property	Alpha (α)	Beta (β)	Gamma (γ)
What it is	helium nucleus, $^4_2\mathrm{He}$	fast electron from the nucleus	electromagnetic wave (photons)
Charge	$+2e$	$-e$	0
Mass	about 4 u	about $\frac{1}{1823}$ u	0 (no rest mass)
Speed	5–7% of c	up to about 99% of c	c
Ionizing power	greatest	moderate	least
Penetrating power	least: stopped by paper	moderate: stopped by a few mm of aluminium	greatest: reduced by cm of lead
Range in air	3–8 cm	up to several meters	very long
In electric and magnetic fields	deflected (a little)	deflected (a lot, the other way)	not deflected

Notice that the order of ionizing power is the exact reverse of penetrating power. That is no accident: each ion a radiation makes costs it some energy. An alpha particle, making tens of thousands of ions per centimeter, spends all its energy in a few centimeters of air; a gamma ray, making very few, keeps going.

Check 3

An unknown source is placed in front of a Geiger–Müller tube. The count rate drops a lot when a sheet of paper is put in front, drops a little more with 5 mm of aluminium, and drops only slightly further with 1 cm of lead. What does the source give out?

a alpha only
b beta and gamma only
c alpha, beta and gamma
d gamma only

24.4 Laws of radioactive decay

Radioactive decay is **spontaneous**: it happens by itself. The decay of a nucleus is called **radioactive disintegration**. From their experiments, Rutherford and Soddy gave these laws (1902–1903):

a. **Radioactive decay is random and spontaneous**, and it is **not affected by external conditions** such as temperature, pressure, chemical combination or electric and magnetic fields. Heating uranium to 1000 °C or cooling it in liquid nitrogen leaves its rate of decay exactly the same: the energies inside a nucleus are millions of times larger than anything heat or chemistry can supply.

b. **In one decay a nucleus gives out either an alpha particle or a beta particle, never both at once**, and one atom gives out only one such particle at a time. When it does, it becomes an atom of a **different element**, by these rules (the **displacement laws** of Soddy and Fajans):

i. Alpha decay. The nucleus loses two protons and two neutrons. Its mass number A goes down by 4 and its atomic number Z by 2, so it moves two places back in the periodic table:

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A-4}_{Z-2}\mathrm{Y} + {}^{4}_{2}\mathrm{He} + Q $$

Q is the **Q value**, the energy released, which mostly becomes the alpha particle's kinetic energy. For example, uranium-238 decays by giving out an alpha particle and becomes thorium-234:

$$ {}^{238}_{92}\mathrm{U} \rightarrow {}^{234}_{90}\mathrm{Th} + {}^{4}_{2}\mathrm{He} + Q $$

ii. Beta decay. Inside the nucleus a **neutron turns into a proton**, and an electron (the beta particle, ${}^{0}_{-1}\mathrm{e}$) is created and thrown out, with an antineutrino $\bar{\nu}$ (a neutral particle with almost no mass). The nucleus has one proton more and one neutron fewer: Z goes **up by 1**, and A stays the **same**. It moves one place forward in the periodic table:

$$ {}^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\mathrm{e} + \bar{\nu} + Q $$

For example, carbon-14 gives out a beta particle and becomes nitrogen-14:

$$ {}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} + \bar{\nu} + Q $$

iii. Gamma emission. A gamma ray carries away energy only: A and Z don't change. It usually follows an alpha or a beta decay at once, as the new nucleus settles from an excited state.

CAREFUL

Where does the electron come from?

A beta particle is **not** one of the atom's orbiting electrons, and the nucleus does not contain electrons waiting to come out. The electron is **created** at the moment of decay, when a neutron becomes a proton: ${}^{1}_{0}\mathrm{n} \rightarrow {}^{1}_{1}\mathrm{p} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$. Many exam answers leave out the antineutrino; that is accepted at this level, but it is there in every beta decay, and it is why beta particles come out with a spread of energies.

Check 4

Radium-226 (${}^{226}_{88}\mathrm{Ra}$) gives out an alpha particle. The new nucleus is:

a ${}^{226}_{86}\mathrm{Rn}$
b ${}^{222}_{86}\mathrm{Rn}$
c ${}^{222}_{87}\mathrm{Fr}$
d ${}^{226}_{89}\mathrm{Ac}$

c. **The rate of disintegration at any time is directly proportional to the number of radioactive atoms present at that time.** This is the **radioactive decay law**, or law of radioactive disintegration, and it gives radioactivity its numbers.

The decay equation

Why should the rate be proportional to the number present? Because each nucleus has the same chance of decaying in the next second, whatever has happened before and however many neighbors it has. A thousand nuclei give a thousand chances; twice as many give twice as many decays.

Let a sample start with N_0 radioactive atoms at $t = 0$, and let N be the number not yet decayed at time t. In the short time dt after that, dN of them decay. Because N is going down, dN is negative, and the **rate of disintegration** is $-\dfrac{dN}{dt}$, a positive number.

1. By the decay law, the rate is proportional to the number present:
$$-\frac{dN}{dt} \propto N$$
2. Put in a constant of proportionality λ, the **decay constant** (or disintegration constant) of the substance:
$$\frac{dN}{dt} = -\lambda N \qquad (24.1)$$
The minus sign says that the number of parent atoms goes **down** as time goes on.
3. Separate the variables, N on one side and t on the other:
$$\frac{dN}{N} = -\lambda\, dt$$
4. Integrate both sides, from N_0 (at $t = 0$) to N (at time t):
$$\int_{N_0}^{N} \frac{dN}{N} = -\lambda \int_0^t dt \qquad (24.2)$$
5. The integral of $\dfrac{1}{N}$ is $\ln N$:
$$\big[\ln N\big]_{N_0}^{N} = \big[-\lambda t\big]_0^t \quad \Rightarrow \quad \ln N - \ln N_0 = -\lambda t \quad \Rightarrow \quad \ln \frac{N}{N_0} = -\lambda t$$
6. Take the exponential of both sides:
$$\boxed{N = N_0\, e^{-\lambda t}} \qquad (24.3)$$

This is the **decay equation**: it gives the number of atoms not yet decayed at any time t. It is an **exponential** decrease, the same law that describes a cooling cup of tea or a capacitor discharging. Since $e^{-\lambda t}$ is zero only when $t = \infty$, a radioactive sample never quite disappears in theory: its graph of N against t gets ever closer to the time axis without touching it (it is **asymptotic** to it), Figure 24.4.

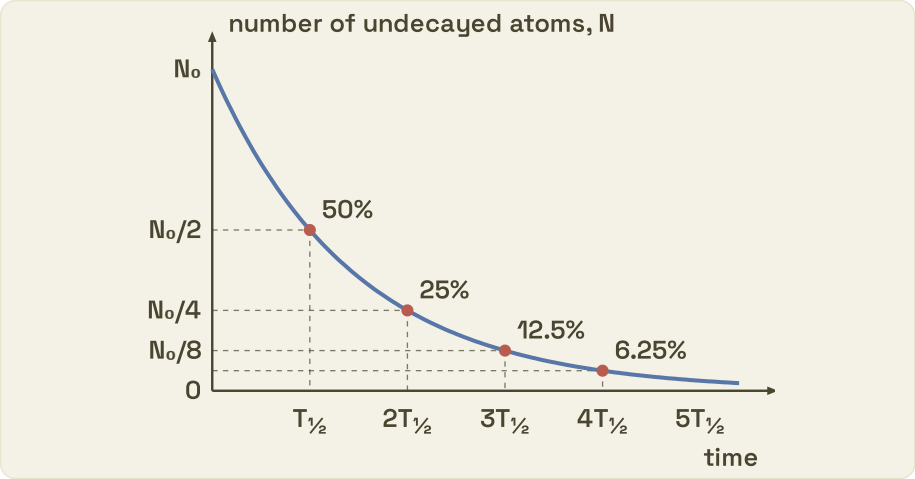


Figure 24.4 *The number of undecayed atoms N against time, drawn from $N = N_0 e^{-\lambda t}$. In each half-life $T_{1/2}$ the number falls to half of what it was: 50% left after one half-life, 25% after two, 12.5% after three, 6.25% after four.*

From the equation and its graph:

1. Radioactive decay is **exponential**: fast at first, while there are many atoms to decay, then slower and slower.
2. The **larger the decay constant** λ, the faster the number of atoms falls.
3. Whatever the substance, a sample would take infinite time to decay completely, so in that sense all radioactive substances look alike. They are not: some are far more active than others. To compare them we use the **half-life** or the **mean life**, below.

Check 5

A sample of a radioactive substance is heated from 20 °C to 500 °C. Its rate of decay:

a increases, because the atoms move faster
b decreases, because the nuclei expand
c stays the same
d becomes zero

24.5 Half-life

The **half-life** $T_{1/2}$ of a radioactive substance is the time in which **half the atoms** of a sample decay; or, the time in which its number of undecayed atoms (or its activity) falls to half.

To find it, put $t = T_{1/2}$ and $N = \dfrac{N_0}{2}$ into the decay equation (24.3):

1. $$\frac{N_0}{2} = N_0\, e^{-\lambda T_{1/2}}$$
2. Cancel N_0 and turn both sides over:
$$e^{-\lambda T_{1/2}} = \frac{1}{2} \quad \Rightarrow \quad e^{\lambda T_{1/2}} = 2$$
3. Take natural logarithms:
$$\lambda T_{1/2} = \ln 2 = 0.693$$
4. So
$$\boxed{T_{1/2} = \frac{0.693}{\lambda}} \qquad (24.4)$$

The half-life is **inversely proportional to the decay constant**, and it is a property of the nucleus alone: nothing done to the sample changes it. Half-lives range from far less than a second to far longer than the age of the universe:

Isotope	Decays by	Half-life
polonium-214	α	0.16 milliseconds
radon-222	α	3.8 days
iodine-131	β	8.0 days
cobalt-60	β, γ	5.3 years
radium-226	α	about 1600 years
carbon-14	β	5730 years
uranium-238	α	4.47 billion years

Take radium, with a half-life of about 1600 years (some older books give 1620). Start with N_0 atoms. In 1600 years, half of them, $\frac{N_0}{2}$, will each have given out an alpha particle and become radon, and $\frac{N_0}{2}$ radium atoms are left. In the next 1600 years half of **those** decay, leaving $\frac{N_0}{4}$; then $\frac{N_0}{8}$, and so on. Each half-life takes away half of **what is left**, not half of what you started with.

Number of atoms left after n half-lives

Time	Atoms left
0	N_0
$T_{1/2}$	$\frac{N_0}{2} = N_0\left(\frac{1}{2}\right)^1$
$2T_{1/2}$	$\frac{N_0}{4} = N_0\left(\frac{1}{2}\right)^2$
$3T_{1/2}$	$\frac{N_0}{8} = N_0\left(\frac{1}{2}\right)^3$
$nT_{1/2}$	$N_0\left(\frac{1}{2}\right)^n$

So after a time t, which is $n = \dfrac{t}{T_{1/2}}$ half-lives,

$$\boxed{N = N_0\left(\frac{1}{2}\right)^n, \qquad n = \frac{t}{T_{1/2}}} \qquad (24.5)$$

This works for any time, not only whole numbers of half-lives (n can be 2.5), and it is the same equation as (24.3): $\left(\frac{1}{2}\right)^{t/T_{1/2}} = e^{-t\ln 2/T_{1/2}} = e^{-\lambda t}$. Use whichever is easier: (24.5) when the time is a whole number of half-lives.

Check 6

A sample has a half-life of 3 hours. What fraction is left undecayed after 12 hours?

a $\frac{1}{4}$ b $\frac{1}{8}$
c $\frac{1}{16}$ d none of it

24.6 Radioactive decay constant

From equation (24.1), $\dfrac{dN}{dt} = -\lambda N$, so

$$\lambda = -\frac{dN/dt}{N} \qquad (24.6)$$

The **decay constant** of a radioactive substance is the ratio of its instantaneous rate of disintegration to the number of atoms present at that time: the **fraction of the atoms that decay per unit time**. Its unit is s^{-1} (or per day, per year). If $\lambda = 0.01$ per second, then about 1% of the atoms present decay in each second; or, for one nucleus, λ is the **probability per second** that it decays.

There is a second way to see it. Put $t = \dfrac{1}{\lambda}$ in the decay equation:

$$N = N_0\, e^{-\lambda \times 1/\lambda} = N_0\, e^{-1} = \frac{N_0}{2.718} = 0.368\, N_0$$

So **the decay constant is the reciprocal of the time in which the number of atoms falls to $\frac{1}{e}$, about 37%, of its starting value.** That time, $\frac{1}{\lambda}$, is the mean life, next.

CAREFUL

λ is not a time

Some books say the decay constant "is the time during which the number of atoms falls to 37%". It is the **reciprocal** of that time: λ is in s^{-1}, and the time is $\frac{1}{\lambda}$, in seconds.

24.7 Average life or mean life

Some nuclei of a sample decay in the first second, some last a thousand years; each nucleus's life is a matter of chance. The **mean life** (or average life) τ of a radioactive substance is the average time a nucleus of it exists before it decays:

$$\text{Mean life} = \frac{\text{total life of all the atoms}}{\text{total number of atoms present at the start}}$$

The mean life is 1/λ

1. Of the N_0 nuclei present at the start, the number that decay between t and $t + dt$ is the rate times the time, $\lambda N\,dt = \lambda N_0 e^{-\lambda t}\,dt$.
2. Each of them lived for a time t, so together they lived $t \times \lambda N_0 e^{-\lambda t}\,dt$.
3. Add up the lives of all the nuclei, from $t = 0$ to $t = \infty$, and divide by N_0:
$$\tau = \frac{1}{N_0}\int_0^\infty t\,\lambda N_0 e^{-\lambda t}\,dt = \lambda\int_0^\infty t\,e^{-\lambda t}\,dt$$
4. The integral, by parts, is $\dfrac{1}{\lambda^2}$ (the term $\left[-\frac{t}{\lambda}e^{-\lambda t}\right]_0^\infty$ is zero at both limits, leaving $\frac{1}{\lambda}\int_0^\infty e^{-\lambda t}dt = \frac{1}{\lambda^2}$). So
$$\boxed{\tau = \frac{1}{\lambda}} \qquad (24.7)$$

The **mean life is the reciprocal of the decay constant**. Using $\lambda = \dfrac{0.693}{T_{1/2}}$ from (24.4):

$$\tau = \frac{T_{1/2}}{0.693} = 1.443\,T_{1/2} \qquad \text{or} \qquad T_{1/2} = 0.693\,\tau \qquad (24.8)$$

The mean life is **1.443 times the half-life**; the half-life is **0.693 times the mean life**.

Check 7

A substance has a half-life of 6.93 hours. Its mean life is:

a 4.80 hours
b 6.93 hours
c 10.0 hours
d 13.9 hours

24.8 Activity of a radioactive substance

The **activity** R (or A) of a radioactive substance is its rate of disintegration: the number of nuclei that decay per second.

$$R = -\frac{dN}{dt}$$

The minus sign makes the activity a positive number while N is falling. By the decay law (24.1), $-\dfrac{dN}{dt} = \lambda N$, so

$$R = \lambda N$$

The activity is the decay constant times the number of radioactive atoms present. At $t = 0$, $R_0 = \lambda N_0$; at time t, $R = \lambda N$. Dividing, and using (24.3):

$$\frac{R}{R_0} = \frac{N}{N_0} = e^{-\lambda t} \quad \Rightarrow \quad \boxed{R = R_0\,e^{-\lambda t}} \qquad (24.9)$$

The activity falls **exponentially at the same rate** as the number of atoms, with the same half-life. So we can also write

$$\frac{R}{R_0} = \left(\frac{1}{2}\right)^n, \qquad n = \frac{t}{T_{1/2}}$$

This is why a half-life can be measured with a Geiger–Müller counter: the **count rate** (counts per second, a fixed fraction of the activity, after the background count is taken away) halves in each half-life, just as the number of atoms does. Every graph in this unit's exercise is a count rate against time.

24.9 Units of radioactivity

- The **becquerel** (Bq) is the SI unit of activity: **1 Bq = 1 disintegration per second.** It is named after Henri Becquerel.
- The **curie** (Ci) is the older unit: **1 Ci = 3.7 × 10¹⁰ disintegrations per second.** It was chosen to be about the activity of 1 g of radium-226, and named after Marie and Pierre Curie. A curie is a large activity, so smaller units are used:
  - 1 millicurie (mCi) = 10^{-3} Ci = 3.7×10^7 disintegrations per second;
  - 1 microcurie (μCi) = 10^{-6} Ci = 3.7×10^4 disintegrations per second.
- Because the curie is tied to radium, the **rutherford** (Rd) was proposed in 1946: **1 Rd = 10⁶ disintegrations per second**, with
  - 1 millirutherford (mRd) = 10^3 disintegrations per second;

- 1 microrutherford (μRd) = 1 disintegration per second.

The rutherford is hardly used now; the becquerel replaced both, but the curie is still common in hospitals and industry.

DID YOU KNOW?

How big is a becquerel?

A becquerel is tiny. A household smoke detector holds about 37,000 Bq (1 μCi) of americium-241. Your body has about 8,000 Bq of its own. The cobalt-60 source of a radiotherapy machine is about 3×10^{14} Bq, several thousand curies.

Worked examples

WORKED EXAMPLE

24.1 Why "rays" and "particles"?

Why is gamma called a "ray" when alpha and beta are called "particles"?

Answer

A gamma ray is electromagnetic radiation, a stream of photons, which have **no rest mass** and no charge, so it is called a ray, like a ray of light. Alpha and beta radiations are made of particles that **have rest mass** (and charge): helium nuclei and electrons. So they are called particles.

WORKED EXAMPLE

24.2 What changes when a beta particle is emitted?

What change takes place in the nucleus when a β-particle is emitted?

Answer

A neutron in the nucleus changes into a proton, and the electron created is thrown out (with an antineutrino). The daughter nucleus has one more proton and one fewer neutron: its **atomic number goes up by one**, and its **mass number stays the same**:

$$^{A}_{Z}\mathrm{X} \rightarrow {}^{A}_{Z+1}\mathrm{Y} + {}^{0}_{-1}\mathrm{e}$$

For example, radium-225 becomes actinium-225:

$$^{225}_{88}\mathrm{Ra} \rightarrow {}^{225}_{89}\mathrm{Ac} + {}^{0}_{-1}\mathrm{e}$$

WORKED EXAMPLE

24.3 All three together?

Can a single nucleus emit an α-particle, a β-particle and a γ-ray together?

Answer

No. In one decay a nucleus gives out **either** an alpha particle **or** a beta particle, never both at once, and only one at a time. Its daughter is often left excited, and then gives out a gamma ray, usually straight after; but that is a separate step. A sample of many nuclei can give out all three kinds at the same moment, from different nuclei.

WORKED EXAMPLE

24.4 Why alpha particles?

How do radioactive nuclei come to emit α-particles?

Answer

In a very large nucleus (more than about 82 protons), the electric repulsion between all the protons, which acts across the whole nucleus, becomes too strong for the strong nuclear force, which acts only between neighboring nucleons. The nucleus becomes more stable by throwing out a group of two protons and two neutrons, bound tightly together as a helium nucleus: an alpha particle. An alpha particle is so tightly bound that this releases energy, which it carries away.

WORKED EXAMPLE

24.5 Opposite deflections

Why are α-particles deflected in the opposite direction to β-particles in an electric field?

Answer

The force on a charge in an electric field is $F = qE$. Alpha particles are positive and beta particles negative, so the forces on them are in **opposite directions**, and so are their deflections: alpha towards the negative plate, beta towards the positive plate.

WORKED EXAMPLE

24.6 Two alphas and two betas

A uranium-238 nucleus, ${}^{238}_{92}\mathrm{U}$, emits two α-particles and two β-particles. What are the atomic number and the mass number of the daughter nucleus?

Answer

Two alpha particles take away a mass number of $2 \times 4 = 8$ and an atomic number of $2 \times 2 = 4$. Two beta particles add 2 to the atomic number and nothing to the mass number.

- Mass number: $238 - 8 = \mathbf{230}$
- Atomic number: $92 - 4 + 2 = \mathbf{90}$

$$ {}^{238}_{92}\mathrm{U} \rightarrow {}^{230}_{90}\mathrm{Th} + 2\,{}^{4}_{2}\mathrm{He} + 2\,{}^{0}_{-1}\mathrm{e} $$

Element 90 is **thorium**: the daughter is thorium-230. This is a real step of uranium's decay chain: U-238 → (α) Th-234 → (β) Pa-234 → (β) U-234 → (α) Th-230.

WORKED EXAMPLE

24.7 Disintegrations per second

At a certain instant a piece of radioactive material contains 10^{12} atoms. The half-life of the material is 15 days. Calculate the number of disintegrations per second.

Answer

Half-life: $T_{1/2} = 15 \times 24 \times 60 \times 60 = 1.296 \times 10^{6}$ s.

Decay constant: $\lambda = \dfrac{0.6931}{T_{1/2}} = \dfrac{0.6931}{1.296 \times 10^{6}} = 5.348 \times 10^{-7}\ \mathrm{s^{-1}}$.

Activity: $R = \lambda N = 5.348 \times 10^{-7} \times 10^{12} = \mathbf{5.35 \times 10^{5}\ disintegrations\ per\ second}$.

WORKED EXAMPLE

24.8 The mass of a tiny source

Calculate the mass of a sample of lead-214 that has an activity of 3.7×10^{4} decays per second (1 μCi). Its half-life is 26.8 minutes. (Avogadro number $= 6.02 \times 10^{23}$ per mole.)

Answer

Half-life $T_{1/2} = 26.8 \times 60 = 1608$ s.

Decay constant: $\lambda = \dfrac{0.693}{1608} = 4.31 \times 10^{-4}\ \mathrm{s^{-1}}$.

From $R = \lambda N$, the number of atoms is

$$ N = \frac{R}{\lambda} = \frac{3.7 \times 10^{4}}{4.31 \times 10^{-4}} = 8.58 \times 10^{7} $$

One mole of lead-214, 214 g, has 6.02×10^{23} atoms, so one atom has a mass of $\dfrac{214}{6.02 \times 10^{23}} = 3.555 \times 10^{-22}$ g, and

$$ m = 8.58 \times 10^{7} \times 3.555 \times 10^{-22} = 3.05 \times 10^{-14}\ \mathrm{g} $$

The mass is about $\mathbf{3.1 \times 10^{-14}\ g}$: an activity big enough to count easily, from a sample far too small to weigh.

WORKED EXAMPLE

24.9 How many alphas and betas?

After a series of alpha and beta decays, plutonium-239 ($^{239}_{94}\text{Pu}$) becomes lead-207 ($^{207}_{82}\text{Pb}$). How many alpha and beta particles are emitted in the complete decay chain? Give the reason for your answer.

Answer

Let m alpha particles and n beta particles be emitted:

$$^{239}_{94}\text{Pu} \rightarrow {}^{207}_{82}\text{Pb} + m\,{}^{4}_{2}\text{He} + n\,{}^{0}_{-1}\text{e}$$

In every nuclear reaction the **mass number** and the **charge (atomic number)** are conserved: they add up to the same on both sides.

- Mass numbers: $239 = 207 + 4m + 0$, so $4m = 32$ and $m = 8$.
- Atomic numbers: $94 = 82 + 2m - n = 82 + 16 - n$, so $n = 4$.

8 alpha particles and 4 beta particles. Only alpha decay changes the mass number, so it is found first; then the betas make up the atomic number. (Plutonium-239 really does end as lead-207, through uranium-235.)

WORKED EXAMPLE

24.10 One percent of radium

Calculate the time needed for 1% of a sample of radium to disintegrate. Take the half-life of radium to be 1500 years.

Answer

If 1% decays, 99% is left: $\dfrac{N}{N_0} = \dfrac{99}{100}$.

From $\dfrac{N}{N_0} = e^{-\lambda t}$:

$$e^{\lambda t} = \frac{100}{99} \quad \Rightarrow \quad \lambda t = \ln\frac{100}{99}$$

With $\lambda = \dfrac{0.693}{T_{1/2}}$:

$$t = \frac{T_{1/2}}{0.693}\ln\frac{100}{99} = \frac{1500}{0.693} \times 0.01005 = 21.75 \text{ years}$$

The time needed is **about 21.8 years**. (Using the measured half-life of radium-226, 1600 years, it would be 23.2 years.)

WORKED EXAMPLE

24.11 A half-life from the activity

A radioactive source has decayed to $\frac{1}{128}$ of its initial activity after 50 days. What is its half-life?

Answer

By the decay equation. $\dfrac{R}{R_0} = e^{-\lambda t} = \dfrac{1}{128}$, so $e^{\lambda t} = 128$ and $\lambda t = \ln 128$.

$$\lambda = \frac{\ln 128}{50} = \frac{4.852}{50} = 0.0970 \text{ per day}, \qquad T_{1/2} = \frac{0.693}{0.0970} = 7.14 \text{ days}$$

By counting half-lives (quicker). $\dfrac{1}{128} = \left(\dfrac{1}{2}\right)^7$, so 50 days is 7 half-lives:

$$T_{1/2} = \frac{50}{7} = 7.14 \text{ days}$$

The half-life is **7.14 days**.

WORKED EXAMPLE

24.12 Atoms decayed and energy released

The initial number of atoms of a radioactive element is 6.0×10^{20}, and its half-life is 10 hours. Calculate (a) the number of atoms that decay in 30 hours, and (b) the energy released, if each decay releases 4.0×10^{-15} J.

Answer

a. 30 hours is $n = 30/10 = 3$ half-lives, so the number left is

$$N = N_0 \left(\frac{1}{2}\right)^3 = \frac{6.0 \times 10^{20}}{8} = 0.75 \times 10^{20}$$

(The decay equation gives the same: $N = 6.0 \times 10^{20} \times e^{-0.0693 \times 30} = 6.0 \times 10^{20} \times e^{-2.079} = 0.75 \times 10^{20}$.)
Number decayed: $N_0 - N = 6.0 \times 10^{20} - 0.75 \times 10^{20} = \mathbf{5.25 \times 10^{20}}$.
b. Energy released: $E = 5.25 \times 10^{20} \times 4.0 \times 10^{-15} = \mathbf{2.1 \times 10^{6}\ J}$.

Test yourself

YOUR TURN

24.1 A decay chain

A radioactive nucleus A undergoes a series of decays: $A \rightarrow (\alpha) A_1 \rightarrow (\beta^-) A_2 \rightarrow (\alpha) A_3 \rightarrow (\gamma) A_4$. The mass number and atomic number of A are 180 and 72. What are they for A_4?

Answer

- α: ${}^{180}_{72}\mathrm{A} \rightarrow {}^{176}_{70}\mathrm{A}_1$ $(A - 4, Z - 2)$
- β^-: ${}^{176}_{70}\mathrm{A}_1 \rightarrow {}^{176}_{71}\mathrm{A}_2$ $(Z + 1)$
- α: ${}^{176}_{71}\mathrm{A}_2 \rightarrow {}^{172}_{69}\mathrm{A}_3$
- γ: no change, ${}^{172}_{69}\mathrm{A}_4$

A_4 has **mass number 172 and atomic number 69**.

YOUR TURN

24.2 Penetrating and ionizing

The penetrating power of a β-particle is greater than that of an α-particle, but its ionizing power is less. Why?

Answer

An alpha particle is about 7,300 times heavier, carries twice the charge and moves more slowly, so it pulls strongly on every electron it passes and makes many ions per centimeter. Each ion costs it energy, so it uses up its energy in a few centimeters of air. A beta particle, light, singly charged and fast, makes far fewer ions per centimeter, so it loses its energy more slowly and travels farther. More ionization means less penetration.

YOUR TURN

24.3 Artificial radioisotopes

Explain the term “artificial radioisotopes”.

Answer

Artificial radioisotopes are radioactive isotopes that are not found in nature (or not in useful amounts) but are made by nuclear reactions: stable nuclei are bombarded with neutrons (in a reactor), protons or alpha particles (in an accelerator), and the new nuclei formed are unstable and decay. Examples: phosphorus-30 (the first, made by the Joliot-Curies in 1934), cobalt-60, iodine-131, technetium-99m.

YOUR TURN

24.4 Decay constant

A radioactive substance has a half-life of 30 days. What is its decay constant?

Answer

$\lambda = \dfrac{0.693}{T_{1/2}} = \dfrac{0.693}{30} = \mathbf{0.0231\ per\ day}$, or in s^{-1}: $\dfrac{0.693}{30 \times 86400} = \mathbf{2.67 \times 10^{-7}\ s^{-1}}$.

YOUR TURN

24.5 In the store

4 g of a radioactive material of half-life 10 years is kept in a store for 15 years. How much of it has disintegrated?

Answer

$n = 15/10 = 1.5$ half-lives. Left: $4 \times \left(\frac{1}{2}\right)^{1.5} = 4 \times 0.3536 = 1.41$ g.
Disintegrated: $4 - 1.41 = \mathbf{2.6\ g}$.

YOUR TURN

24.6 The half-life of radium

The atomic weight of radium is 226. One gram of radium is found to emit 3.67×10^{10} alpha particles per second. Calculate the half-life of radium in years.

Answer

Atoms in 1 g: $N = \dfrac{6.02 \times 10^{23}}{226} = 2.66 \times 10^{21}$.
Each decay gives one alpha particle, so the activity is $R = 3.67 \times 10^{10}\ \text{s}^{-1}$, and

$$\lambda = \frac{R}{N} = \frac{3.67 \times 10^{10}}{2.66 \times 10^{21}} = 1.38 \times 10^{-11}\ \text{s}^{-1}$$

$$T_{1/2} = \frac{0.693}{1.38 \times 10^{-11}} = 5.03 \times 10^{10}\ \text{s} = \frac{5.03 \times 10^{10}}{3.15 \times 10^{7}}\ \text{years} \approx 1.6 \times 10^{3}\ \text{years}$$

The half-life of radium is **about 1600 years**.
(About 1595 years, close to the measured value.)

YOUR TURN

24.7 Decayed in 8 days

The half-life of a radioactive substance is 2 days. What fraction of the original number of atoms has decayed in 8 days?

Answer

8 days is 4 half-lives, so the fraction left is $\left(\frac{1}{2}\right)^4 = \frac{1}{16}$, and the fraction **decayed** is $1 - \frac{1}{16} = \frac{15}{16}$.

YOUR TURN

24.8 Technetium in a patient

Technetium nuclei used in medicine (mass number 99) have a half-life of 6 hours. A solution containing 10^{-12} g of it is injected into the bladder of a patient. Find its activity at the beginning and after one hour.

Answer

Atoms: $N_0 = \dfrac{10^{-12}}{99} \times 6.02 \times 10^{23} = 6.08 \times 10^{9}$.
$\lambda = \dfrac{0.693}{6 \times 3600} = 3.21 \times 10^{-5}\ \text{s}^{-1}$.
At the beginning: $R_0 = \lambda N_0 = 3.21 \times 10^{-5} \times 6.08 \times 10^{9} = \mathbf{1.95 \times 10^{5}\ s^{-1}}$.
After one hour, $n = \frac{1}{6}$ of a half-life: $R = R_0 \left(\frac{1}{2}\right)^{1/6} = 1.95 \times 10^{5} \times 0.891 = \mathbf{1.74 \times 10^{5}\ s^{-1}}$.
(The medical isotope is technetium-99m, an excited state of technetium-99 that gives out a gamma ray.)

YOUR TURN

24.9 Down to 10^8 atoms

At a certain instant a piece of radioactive material contains 10^{12} atoms. Its half-life is 30 days. How long will it be before only 10^8 atoms remain?

Answer

$\frac{N}{N_0} = \frac{10^8}{10^{12}} = 10^{-4} = e^{-\lambda t}$, so $\lambda t = \ln 10^4 = 9.210$.

$$t = \frac{9.210}{\lambda} = \frac{9.210 \times 30}{0.693} = \mathbf{398.7\ days}$$

YOUR TURN

24.10 The activity of a gram of radium

Radium-226 decays by alpha emission with a half-life of 1620 years. What is the activity of 1 g of radium-226? (Avogadro number = 6.022×10^{23} per mole.)

Answer

$N = \frac{6.022 \times 10^{23}}{226} = 2.665 \times 10^{21}$ atoms.

$\lambda = \frac{0.693}{1620} = 4.28 \times 10^{-4}$ per year.

$R = \lambda N = 4.28 \times 10^{-4} \times 2.665 \times 10^{21} = \mathbf{1.14 \times 10^{18}\ decays\ per\ year}$, which is $\frac{1.14 \times 10^{18}}{3.156 \times 10^7} = 3.6 \times 10^{10}$ per second: about 1 curie, as the curie was meant to be.

YOUR TURN

24.11 After 50 seconds

A sample of an isotope with $\lambda = 0.10\ \text{s}^{-1}$ contains 5.0×10^9 undecayed nuclei at the start of an experiment. Find (a) the number of undecayed nuclei after 50 s, and (b) its activity after 50 s.

Answer

a. $N = N_0 e^{-\lambda t} = 5.0 \times 10^9 \times e^{-0.10 \times 50} = 5.0 \times 10^9 \times e^{-5} = 5.0 \times 10^9 \times 6.74 \times 10^{-3} = \mathbf{3.4 \times 10^7}$.

b. $R = \lambda N = 0.10 \times 3.4 \times 10^7 = \mathbf{3.4 \times 10^6\ Bq}$.

24.10 Detection of radiation: the GM tube

Radiation can't be seen, heard or felt, so we need instruments to detect it: photographic film, cloud chambers, scintillation counters, semiconductor detectors. The simplest and commonest is the **Geiger–Müller counter** (GM counter), made by Hans Geiger and Walther Müller in 1928. Each particle of radiation that enters it gives one click.



Figure 24.5 *A portable GM counter. The probe on the cable holds the GM tube; the box holds the high-voltage supply, the counter and a loudspeaker that clicks for each count.*
Photo: Boffy b, CC BY-SA 3.0

Construction

The GM tube is shown in Figure 24.6.

- **C** is a metal **cylinder** (or a glass tube coated inside with metal), the cathode. It is filled with a gas at low pressure, about 10 cm of mercury: usually argon, with a little halogen or alcohol vapor.
- **W** is a fine **tungsten wire** stretched along the middle of the cylinder, the anode.
- The cylinder and the wire are insulated from each other by **ebonite plugs** E.
- The wire is connected to the **positive** terminal of a high-voltage supply (about a kilovolt) through a very high **resistance R** (of the order of megohms); the cylinder is connected to the **negative** terminal.
- A thin **window** W′ of mica at one end lets the radiation in; even alpha particles can get through it.
- The voltage across R is passed to an **amplifier** and a **counter**.

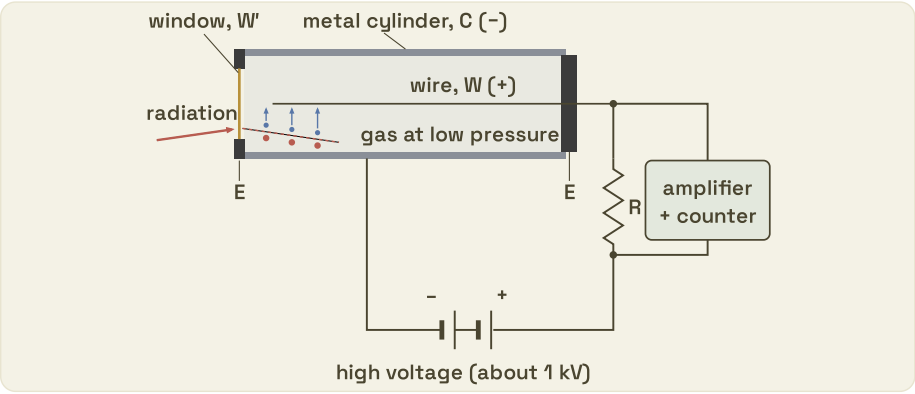


Figure 24.6 *A Geiger–Müller counter. Radiation enters through the thin mica window W′ and ionizes the gas along its track: the electrons freed (blue) drift towards the wire, the positive ions (red) towards the cylinder. The wire W, the anode, is about a kilovolt positive of the cylinder C through a large resistance R; each avalanche of electrons reaching the wire sends a pulse of current through R, and the pulse of voltage across R is amplified and counted.*

Working

1. With the power on, the wire is positive and the cylinder negative, so there is a strong electric field between them, strongest close to the thin wire.
2. An ionizing particle (an alpha or beta particle, or an electron knocked out by a gamma ray) enters through the window and **ionizes** a few gas atoms, making electrons and positive ions.
3. The electrons are **accelerated** towards the wire. Near the wire the field is so strong that each gains enough energy between collisions to ionize more atoms; the new electrons do the same, and so on. A single electron becomes an **avalanche** of millions of electrons, which spreads along the whole wire.
4. The avalanche reaches the wire and flows through the resistance R, giving a short **pulse** of voltage across R.
5. The pulse is **amplified** and **counted**: each one adds 1 to the counter, and often a loudspeaker clicks.

So **each ionizing particle that enters gives one count**, whatever its energy: the tube counts particles but can't tell their energies apart.

> DID YOU KNOW?
> **Why the tube needs a moment's rest**
>
> After each avalanche, the slow positive ions take about a ten-thousandth of a second to drift out to the cylinder, and while they surround the wire the field is too weak for another avalanche. A particle arriving in this **dead time** is missed, so a GM counter undercounts very active sources. The halogen or alcohol in the gas **quenches** the discharge: it soaks up the energy that would otherwise start a second, false avalanche when the ions reach the cylinder.

Plateau of the GM counter

If the source is kept fixed and the **count rate** (counts per minute) is measured at different applied voltages, the graph is as in Figure 24.7.

- Below a certain voltage, the **threshold voltage** V_{TH}, the field is too weak for an avalanche big enough to count, so the count rate is **zero**.

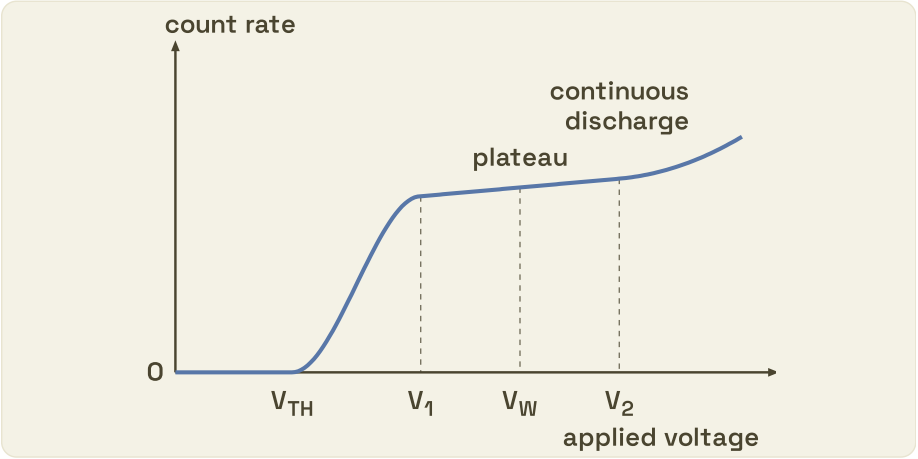


Figure 24.7 *Count rate against the voltage across a GM tube, with a fixed source. Below the threshold VTH, nothing is counted; the count rate rises quickly to the **plateau** from $V1$ to $V2$, where it hardly changes, then shoots up as the tube breaks into continuous discharge. The tube is used at the working voltage VW, the middle of the plateau.*

- Above V_{TH}, more and more of the ionizing particles start avalanches big enough to count, and the count rate rises quickly.
- Then **every** particle that enters gives a counted pulse. Raising the voltage further can't give more counts than there are particles, so the curve is almost **flat**: this flat part, from V_1 to V_2, is the **plateau**.
- Beyond the plateau the field is so strong that the tube breaks into **continuous discharge**, with no particles needed: the count rate shoots up, and the tube can be damaged. This region is always avoided.

Working voltage. On the plateau the count rate hardly depends on the voltage, so it gives the true count of particles. The best **working voltage** is taken in the middle of the plateau, where small changes in the supply don't matter:

$$V_W = \frac{V_1 + V_2}{2}$$

Check 8

Why is a GM tube worked on the plateau of its characteristic?

a the count rate is largest there

b there, every ionizing particle gives one count, so small changes in the voltage don't change the count rate

c the tube can tell alpha, beta and gamma apart there

d there is no dead time on the plateau

24.11 Radiocarbon dating

Finding the age of an archaeological object (a bone, a piece of charcoal, a cloth) or a geological one (a rock) is called **dating**. It is done by measuring how much of a radioactive isotope is left in it. For things that were once alive, the isotope is **carbon-14**, with a half-life of **5730 years**.

How carbon-14 gets into living things

1. **Cosmic rays** from space hit the upper atmosphere and knock neutrons out of nuclei. These neutrons hit nitrogen-14, the commonest nucleus in the air, and make carbon-14:

$$^{14}_{7}\mathrm{N} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{14}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H}$$

2. The carbon-14 joins oxygen as carbon dioxide and mixes through the air. Plants take it in during photosynthesis, along with ordinary carbon-12; animals eat the plants. So **every living thing** contains carbon-14, in the same small proportion as the air: about 1.2 carbon atoms in every million million (10^{12}) are carbon-14. While it lives, it keeps taking in new carbon, so the proportion stays the same.
3. Carbon-14 is radioactive and decays back to nitrogen:

$$^{14}_{6}\mathrm{C} \to {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}$$

In living carbon this gives an activity of about 0.23 Bq for every gram of carbon (about 14 decays a minute).

4. When the plant or animal **dies**, it stops taking in carbon. Its carbon-12 stays, but its carbon-14 decays away with a half-life of 5730 years, and nothing replaces it. So the less carbon-14 is left, compared with the carbon-12, the longer ago it died.

The age

Let N_0 be the number of carbon-14 atoms the sample had when it died, and N the number now. Because the carbon-12 doesn't change, N_0 is found from the sample's carbon-12 and the living ratio; or, more simply, we compare the activity R of the sample with the activity R_0 of the same mass of carbon from a living thing today.

1. By the decay law, $N = N_0 e^{-\lambda t}$, so $\dfrac{N_0}{N} = e^{\lambda t}$.
2. Activity is proportional to the number of atoms ($R = \lambda N$), so $\dfrac{R_0}{R} = \dfrac{\lambda N_0}{\lambda N} = \dfrac{N_0}{N}$.
3. Take logarithms: $\lambda t = \ln \dfrac{R_0}{R}$, and put in $\lambda = \dfrac{0.693}{T_{1/2}}$:

$$\boxed{t = \frac{T_{1/2}}{0.693} \ln \frac{R_0}{R} = \frac{T_{1/2}}{0.693} \ln \frac{N_0}{N}} \qquad (24.10)$$

where $T_{1/2} = 5730$ years is carbon-14's half-life, R_0 the activity of living carbon and R that of the same mass of carbon from the sample. This gives the time since the organism died: its age.

CAREFUL

Carbon-14 becomes nitrogen, not carbon-12

Some books start the derivation with $N_0 = N_{12} + N_{14}$, saying that a carbon-14 atom "decays into carbon-12", and end with $t = \frac{T_{1/2}}{0.693} \ln\left(\frac{N_{12}}{N_{14}} + 1\right)$. That is wrong: carbon-14 decays into **nitrogen-14**, and the carbon-12 in a sample was there all along. Use the activity form (24.10), with R_0 the activity of an equal mass of **living** carbon; it gives the right age, and it is the form the exam expects.

WORKED EXAMPLE

Charcoal from a hearth

Charcoal from an old hearth has a carbon-14 activity of 0.0575 Bq per gram of carbon. Living wood has 0.23 Bq per gram. How old is the charcoal? (Half-life of carbon-14 = 5730 years.)

Answer

$$\frac{R_0}{R} = \frac{0.23}{0.0575} = 4 = 2^2$$

The activity has fallen to a quarter, so two half-lives have passed: $t = 2 \times 5730 = \mathbf{11{,}460\ years}$.

By the formula: $t = \dfrac{5730}{0.693} \ln 4 = 8268 \times 1.386 = 11{,}460$ years.

After about ten half-lives (57,000 years) only a thousandth of the carbon-14 is left, too little to measure reliably, so radiocarbon dating works back to about **50,000 years**. Older things, and rocks, are dated with longer-lived isotopes: uranium-238 decaying to lead-206 (half-life 4.47 billion years) dates the oldest rocks, and gives the age of the Earth, 4.54 billion years.

A STORY FROM SCIENCE

Willard Libby's clock

The American chemist Willard Libby worked out radiocarbon dating in 1946–1949. To test it, he dated wood from the tombs of Egyptian pharaohs, whose ages were known from history, and his carbon ages came out right. He won the Nobel Prize in Chemistry in 1960. The method has since been refined: the carbon-14 in the air has not been perfectly constant (the Sun's activity and the Earth's magnetism change how many cosmic rays get in, and nuclear bomb tests of the 1950s and 1960s nearly doubled it for a while), so ages are corrected using tree rings, which can be counted year by year for over 12,000 years. In Nepal, radiocarbon dating has dated human remains from the cliff caves of Upper Mustang to as early as about 3,000 years ago.

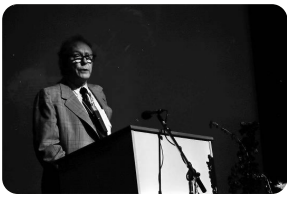


Figure 24.8 *Willard Libby (1908–1980), who invented radiocarbon dating, lecturing in 1974.*
Photo: Willy Pragher, CC BY 3.0 de

Check 9
Why can't radiocarbon dating find the age of a stone tool?

a stone has too much carbon-14
b stone was never alive, so it never took in carbon-14 from the air in a known proportion
c stone doesn't let radiation out
d carbon-14 decays too slowly in stone

24.12 Medical uses of radiation

Radiation has vital uses in medicine. Radioisotopes of elements such as carbon, chlorine, sodium, iodine and technetium are made, mostly in nuclear reactors, and used in two ways: to find out what is wrong (**diagnosis**) and to treat it (**therapy**).

i. Diagnosis

A small amount of a radioisotope, a **tracer**, is given to the patient by mouth or injection. It goes where that element goes in the body, and its gamma rays, which pass out of the body, are picked up by a detector or a **gamma camera** to show where it is and how fast it moves.

a. **Blood circulation**: a little sodium-24 in the blood shows where the circulation is blocked or slow.
b. **Brain tumors**: tracers collect in the tumor, which shows up as a bright spot. Today this is usually done by **PET scans** with fluorine-18, which gives out positrons (Grade 11, Annihilation and creation).
c. **The heart's pumping**: tracers in the blood (sodium-24 in early studies, technetium-99m today) show how much blood the heart pumps with each beat.
d. **Kidneys, liver and lungs**: how well they work, in health and disease, from how fast a tracer passes through them.
e. **Bone**: a bone scan with technetium-99m shows defects in bone metabolism, fractures and cancers that have spread to bone.
f. **Anemia**: iron-59 shows how the body takes up iron to make red blood cells.

The most used medical isotope is **technetium-99m**: its 6-hour half-life is long enough for a scan and short enough to leave the body soon, and it gives out only a gamma ray, easy to detect and gentle on the patient.

ii. Therapy

Here the radiation is used to **destroy** cells that are harmful, such as cancer cells, which are more easily killed by radiation than healthy cells because they divide more often.

a. **The thyroid**: the thyroid gland takes up iodine, so radioactive **iodine-131** given as a drink goes straight there, and its beta particles destroy an overactive thyroid or thyroid cancer from the inside.
b. **Cancer**: gamma rays from **cobalt-60**, in a beam aimed at the tumor from several directions, kill the cancer cells while sparing the tissue around. (Today most beams come from machines called linear accelerators, which make high-energy X-rays instead of using a radioactive source.) Small sealed sources, such as **gold-198** seeds or iridium-192, can be placed inside or next to a tumor (**brachytherapy**).
c. **Skin and blood diseases**: phosphorus-32, a beta emitter, has been used on skin conditions and to treat some blood disorders.

Gamma rays are also used to **sterilize** medical equipment (syringes, gloves, bandages) sealed in their packets, killing every germ without heating them.

DID YOU KNOW?

Radiotherapy in Nepal

Nepal's cancer hospitals treat patients with radiotherapy, among them B.P. Koirala Memorial Cancer Hospital in Bharatpur and Bhaktapur Cancer Hospital, and the larger hospitals in Kathmandu have nuclear medicine departments that use radioisotopes such as technetium-99m and iodine-131 for scans and thyroid treatment.

WORKED EXAMPLE

24.13 The GM tube

What is the function of a GM tube?

Answer

A GM tube **detects and counts ionizing radiation**: alpha and beta particles, and gamma rays (through the electrons they knock out). Each particle that enters it gives one pulse, which is counted.

WORKED EXAMPLE

24.14 Carbon dating

What is carbon dating used for?

Answer

Carbon dating finds the **age of things that were once alive**, such as bones, wood, charcoal, cloth and seeds from archaeological sites, up to about 50,000 years old, from how much of their carbon-14 is left.

WORKED EXAMPLE

24.15 The thyroid

Which radioactive isotope is commonly used for thyroid problems?

Answer

Iodine-131, used both to diagnose and to treat thyroid disorders: the thyroid collects iodine, so the isotope goes straight to it.

YOUR TURN

24.12 One medical use

Name one medical use of radioisotopes.

Answer

Any one: iodine-131 to treat an overactive thyroid or thyroid cancer; cobalt-60 gamma rays to kill cancer cells; technetium-99m for bone and heart scans; fluorine-18 for PET scans; gamma rays to sterilize syringes and bandages.

YOUR TURN

24.13 Inside the GM tube

What happens inside a GM tube when radiation enters?

Answer

The radiation ionizes a few gas atoms. The electrons freed are accelerated towards the positive central wire and, in the strong field near it, ionize more atoms, which makes an avalanche of electrons. The avalanche reaches the wire and flows through the resistance R, giving a voltage pulse, which is amplified and counted: one count for each particle.

YOUR TURN

24.14 Carbon-14 and dating

How does carbon-14 help to date ancient objects?

Answer

Living things keep the same proportion of carbon-14 as the air, because they keep taking in carbon. When one dies, it stops taking it in, and its carbon-14 decays with a half-life of 5730 years. Comparing the sample's carbon-14 activity R with that of living carbon R_0 gives the time since death: $t = \frac{5730}{0.693} \ln \frac{R_0}{R}$ years.

Health hazards of radiation

The radiation that heals can also harm, for the same reason: it **ionizes**. When it passes through a living cell it knocks electrons off atoms and breaks molecules, above all the DNA that carries the cell's instructions.

- **A large dose in a short time** kills many cells at once. The body can't replace them fast enough, and the person suffers **radiation sickness**: vomiting, burns, hair loss, damage to the bone marrow that makes blood, and death after very large doses. These effects come only above a threshold dose, and get worse as the dose grows.
- **Smaller doses** may damage the DNA of a cell without killing it. If the cell can't repair it, the damage can lead, years later, to **cancer** (leukemia, thyroid cancer and others), and damage in the sex cells can cause **mutations** passed on to children. These effects have no safe threshold: the larger the dose, the higher the **chance**.

The harm done is measured as the **dose** in sieverts (Sv). Everyone receives about **2.4 mSv a year** on average from natural background radiation (radon gas from the ground, rocks, cosmic rays, the potassium in our food); a chest X-ray gives 0.02 to 0.1 mSv, a CT scan 2 to 10 mSv. A dose of 4 to 5 Sv in a short time would kill about half the people who received it.

Which radiation is most dangerous depends on where the source is.

- **Outside the body**, alpha particles are harmless: the dead outer layer of skin stops them. Gamma rays and beta particles, which reach the organs, are the danger.
- **Inside the body** (breathed in, swallowed, or in a wound), alpha emitters are the **most** dangerous: all their energy is dumped in a tiny volume of living tissue. Radon gas, which seeps from the ground into closed rooms, and its decay products in the lungs are the second biggest cause of lung cancer after smoking.

Protection has three rules, **time, distance and shielding**:

- spend as **little time** near a source as possible;
- keep as **far away** as possible (handle sources with long tongs; gamma radiation from a small source falls as the square of the distance);
- put **shielding** in between (lead aprons, lead or concrete walls; sources kept in lead containers);
- and in addition, never eat, drink or smoke where radioactive materials are used, and wear a **film badge** or dosimeter to record the dose received.

The danger of radioactive contamination was seen at **Chernobyl** (Ukraine, 1986), where a reactor exploded and spread radioactive cesium-137 and iodine-131 over much of Europe, and at **Fukushima** (Japan, 2011). Thirty-year cesium-137 keeps the land around Chernobyl contaminated for a century or more.

CAREFUL

Irradiated is not radioactive

Food, syringes or a patient given gamma rays do **not** become radioactive: gamma rays pass through and leave nothing behind. Something becomes radioactive only if radioactive material gets **on** or **into** it (**contamination**), or if it is bombarded with neutrons. That is why gamma rays can sterilize medical equipment, and why a patient after an X-ray or a course of radiotherapy by a beam from outside is safe to hug. (A patient given a radioisotope to swallow, such as iodine-131, is different: for a few days the isotope is inside them, and they are asked to keep apart from others.)

Check 10

A source that gives out only alpha particles is most dangerous:

a when held in the hand
b when kept a meter away
c when swallowed or breathed in
d when kept behind a sheet of paper

IN SHORT

Key ideas of Unit 24

- **Radioactivity** is the spontaneous decay of unstable nuclei, giving out α, β or γ radiation (Becquerel, 1896). **Natural**: in nature (uranium, radium, potassium-40); **artificial**: in nuclei made unstable by nuclear reactions (Joliot-Curies, 1934).
- **α**: helium nucleus, +2e, about 4 u, stopped by paper, strongly ionizing. **β**: fast electron from the nucleus, −e, stopped by a few mm of aluminium. **γ**: electromagnetic wave, no charge, reduced by cm of lead, weakly ionizing. In fields, α and β bend opposite ways (α much less), γ not at all.
- Cause: an unstable nucleus (too big, or neutrons and protons out of balance, or too much energy).
- **Laws**: decay is random and unaffected by external conditions; α decay: A − 4, Z − 2; β decay: A same, Z + 1; γ: no change. The rate of decay is proportional to the number present: $\frac{dN}{dt} = -\lambda N$, so $N = N_0 e^{-\lambda t}$.
- **Half-life** $T_{1/2} = \frac{0.693}{\lambda}$; $N = N_0 \left(\frac{1}{2}\right)^n$, $n = \frac{t}{T_{1/2}}$.
- **Decay constant** $\lambda = -\frac{dN/dt}{N}$: the fraction decaying per second; $\frac{1}{\lambda}$ is the time to fall to 37%.
- **Mean life** $\tau = \frac{1}{\lambda} = 1.443\, T_{1/2}$.
- **Activity** $R = -\frac{dN}{dt} = \lambda N$, $R = R_0 e^{-\lambda t}$. 1 Bq = 1 decay/s; 1 Ci = 3.7×10^{10} Bq; 1 Rd = 10^6 Bq.
- **GM tube**: an avalanche of ions near the positive wire gives one pulse per particle; worked at $V_W = \frac{V_1 + V_2}{2}$, the middle of the plateau.
- **Carbon dating**: $t = \frac{T_{1/2}}{0.693} \ln \frac{R_0}{R}$, with $T_{1/2} = 5730$ years; up to about 50,000 years.
- **Medicine**: tracers for diagnosis (technetium-99m, sodium-24, iodine-131), therapy (cobalt-60, iodine-131, gold-198, phosphorus-32), sterilizing.
- **Hazards**: ionization damages DNA, causing radiation sickness at large doses and a chance of cancer at small ones; protect by time, distance and shielding. Alpha is most dangerous inside the body.

Exercise

Choose the correct option

Check 11

1. An activity of 1.85×10^{12} disintegrations per second is equal to:

a 40 curie
b 50 curie
c 30 curie
d 60 curie

Check 12

2. The half-life of a radioactive substance is 2 months. Of a mass M, the amount left after 1 year is:

a $\frac{M}{64}$
b $\frac{M}{32}$
c $\frac{M}{16}$
d $\frac{M}{8}$

Check 13

3. A radioactive salt of half-life 2 days has a mass of 10 g. How much of it will be left after 10 days?

a 2.54 g
b 5 g
c 6.24 g
d 0.31 g

Check 14

4. If 20% of a radioactive mass decays in 10 days, what percentage of it remains undecayed after 20 days?

a 60%
b 75%
c 50%
d 64%

Check 15

5. Which of these is equal to one rutherford?

a 3.37×10^4 disintegrations per second
b 3.3×10^4 disintegrations per second
c 10^6 disintegrations per second
d 3.7×10^4 disintegrations per second

Check 16

6. What determines the half-life of a radioactive element?

a its atomic number alone
b its mass number alone
c the temperature
d none of the above

Check 17

7. A radioactive element has a half-life of 2.5 hours. In 10 hours, 1 g of it is reduced to:

a $\frac{1}{4}$ g
b $\frac{1}{8}$ g
c $\frac{1}{16}$ g
d $\frac{1}{32}$ g

Check 18

8. A source with a half-life of 130 days contains 10^{20} radioactive atoms at first. What is its activity after 260 days?

a $1.54 \times 10^{12}\ s^{-1}$
b $2.54 \times 10^{12}\ s^{-1}$
c $3.54 \times 10^{12}\ s^{-1}$
d $4.45 \times 10^{12}\ s^{-1}$

Check 19

9. The activity of radium decreases by about 1% every 25 years. Find its half-life.

a 1224 years
b 1424 years
c 1624 years
d 1724 years

Check 20

10. A radioactive source decays to 1% of its initial activity in 100 days. What is its half-life?

a 15 days
b 20 days
c 25 days
d 30 days

Check 21

11. The half-life of a radioactive substance is 4.8 min. Find the time needed for its activity to fall to 5%.

a 10.8 min
b 15.8 min
c 20.8 min
d 25.8 min

Answer these questions

12.
a. What is radioactivity? Write down the laws of radioactive decay.
b. Define the radioactive decay constant. Find the relation between half-life and decay constant.
c. The half-life of a radioactive element is 30 days. Calculate its decay constant, and the time needed for 75% of its atoms to disintegrate.

13.
a. State the law of radioactive disintegration. What do you mean by the half-life and the mean life of a radioactive substance? Prove that the half-life of a radioactive substance is 0.693 times its mean life.
b. What is the half-life of a radioactive substance whose activity falls to $\frac{1}{10}$ of its original value in 7.5 years?

14. a. Does the neutron–proton ratio of a nucleus increase or decrease in α-decay?

b. The half-life of a radioactive substance is 30 days. How long does it take for $\frac{3}{4}$ of its original mass to disintegrate?

15. a. What percentage of a sample of a radioactive substance is left undecayed after five half-lives?

b. A stream of neutrons has a kinetic energy of 0.0327 eV. If the half-life of the neutron is 700 s, what fraction of the neutrons will decay before they travel a distance of 10 m? (Mass of a neutron $= 1.675 \times 10^{-27}$ kg.)

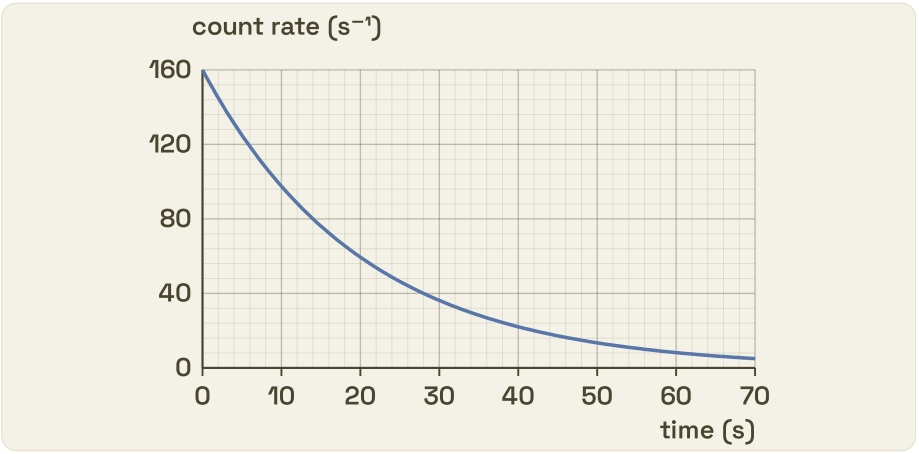


Figure 24.9 *For question 16: the count rate of a sample containing indium-116 against time.*

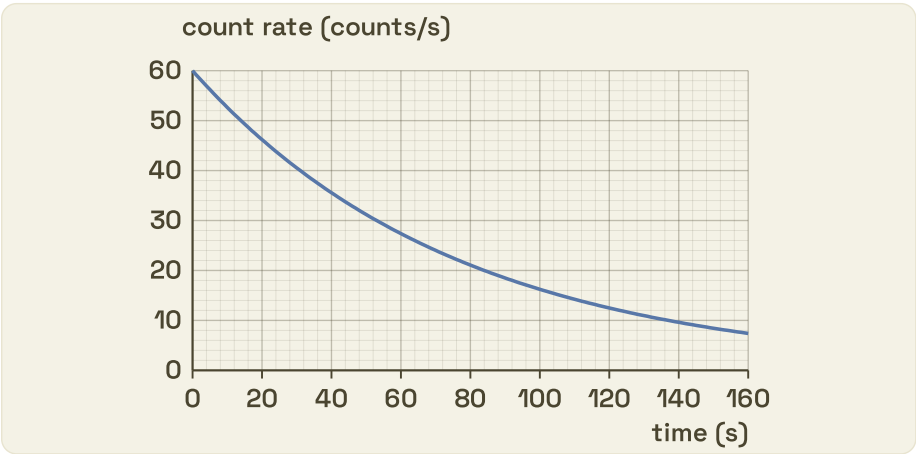


Figure 24.10 *For question 17: the count rate from a radioactive source against time.*

17. The graph shows how the count rate from a radioactive source changes with time.
    a. Describe how the count rate changes with time.
    b. i. Find from the graph how many seconds it takes for the count rate to fall from 60 counts/s to 15 counts/s.
    ii. How many half-lives is this?
    c. Calculate the half-life of the substance.
    d. How long does the count rate take to fall from 10 counts/s to 5 counts/s?

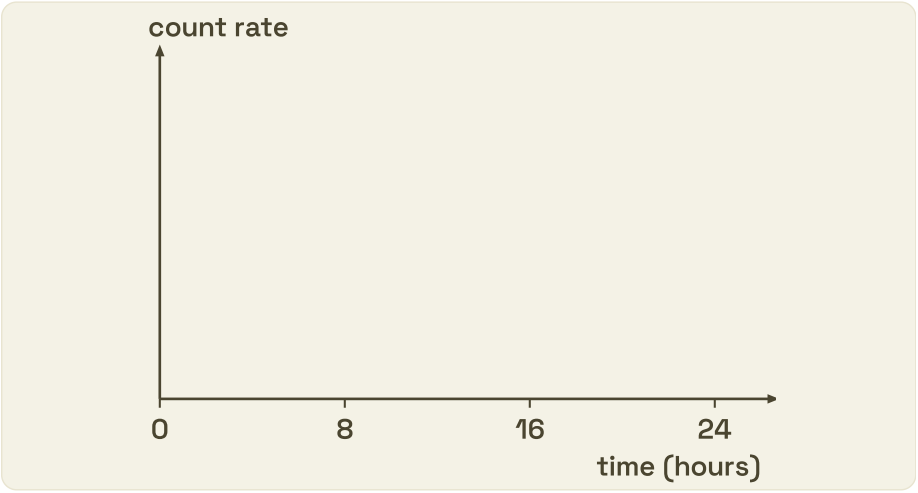


Figure 24.11 *For question 18 b i: axes on which to sketch the count rate of astatine-210 over 24 hours.*

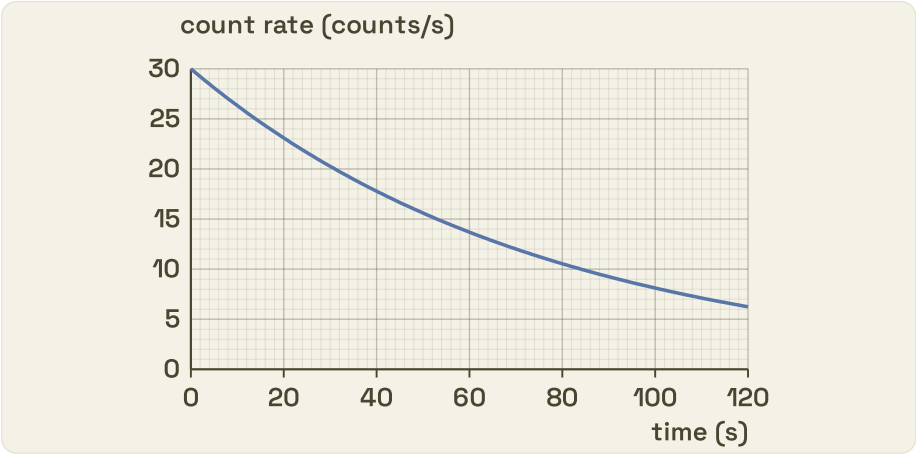


Figure 24.13 *For question 19: the count rate from a sample of radioactive material against time.*

19.

a. Complete this definition of radioactive half-life: the half-life of a radioactive substance is the time taken for …

b. The graph shows how the count rate from a sample of radioactive material changes with time.

i. Use the graph to find the half-life of the material.

ii. Use the graph to find how long it takes for the count rate to fall from 30 counts/s to 10 counts/s.

iii. Use your answers to (i) and (ii) to find how long it takes for the count rate to fall from 30 counts/s to 5 counts/s.

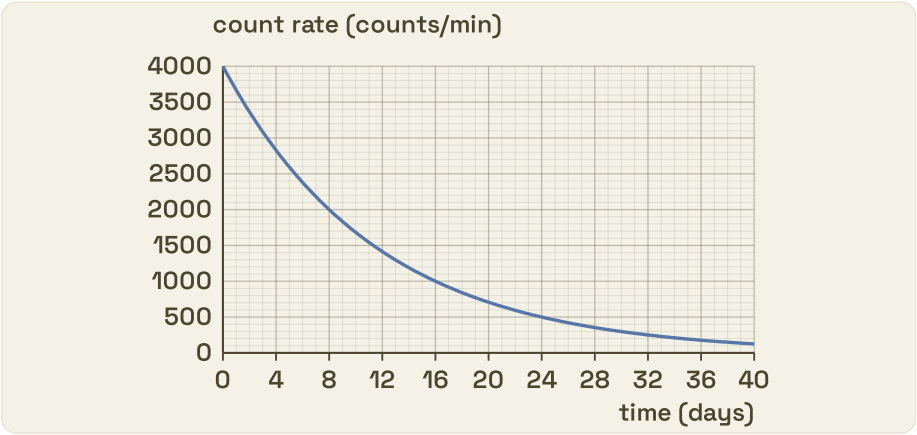


Figure 24.14 *For question 20 b: the count rate from a sample of a radioactive substance against time.*

20.

a. A radioactive nucleus decays by giving out a β-particle. State what a β-particle is, and give its charge.

b. The graph shows how the count rate from a sample of a radioactive substance varies with time. Use the graph to find the half-life. Show your working on the graph.

c. After an accident, the soil around a nuclear power station is contaminated by cesium-137, which is radioactive. A sample of the soil has a count rate of 180 counts/min from its cesium-137. Cesium-137 has a half-life of 30 years and decays by β-emission.

i. Calculate the count rate from the cesium-137 in the sample after 60 years.

ii. Suggest why people do not want to live near the power station, even after it has closed.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.