Unit 21

Photons

This unit online, with its interactives:
science-playground.com/s/12f21

Stand in the sun on a winter morning in Kathmandu and your skin warms, but it doesn't burn. Stay out at Everest Base Camp for the same time and it can burn badly, though the air there is freezing. Leave a blue shirt in strong sunlight for a season and its color fades; leave it under a bright red lamp for as long and it hardly fades. Light is not just "more" or "less". Something about its **color**, its frequency, decides what it can do to matter.

This unit is about that something. In 1900 **Max Planck** suggested, almost in despair, that light's energy is given out in small packets. In 1905 a 26-year-old clerk in the Swiss patent office, **Albert Einstein**, took the idea seriously: light *travels* as packets, now called **photons**, and one photon gives all its energy to one electron. With that one idea he explained a puzzle no wave theory could, the **photoelectric effect**, in which light knocks electrons out of a metal. It is the effect at work in the tubes that count single flashes of light in physics laboratories, and in the photocells of old film projectors and automatic doors.

A STORY FROM SCIENCE

Albert Einstein and the packets of light

Albert Einstein (1879–1955) was born in Ulm, in Germany. He disliked his Munich school's drill and left it at 15, finished school in Switzerland, then studied physics in Zurich. No university would employ him, so in 1902 he took a job checking patent applications in Bern.

In 1905, in his spare time, he wrote four papers that changed physics: one on the **photoelectric effect**, one on **Brownian motion** (which, once measured, helped prove that atoms are real), and two on **special relativity**, the second of which contains $E = mc^2$. The photoelectric paper was the one he himself called "very revolutionary". Most physicists did not believe it for years: light was a wave, as interference and diffraction had shown beyond doubt. Even Robert Millikan, who spent ten years testing Einstein's equation and found it exactly right (Section 21.5), first set out to prove it wrong.

Einstein won the **Nobel Prize in Physics for 1921** "for his services to theoretical physics, and especially for his discovery of the law of the photoelectric effect", not for relativity. In his later years he searched, without success, for one theory uniting gravity and electromagnetism. The full story.

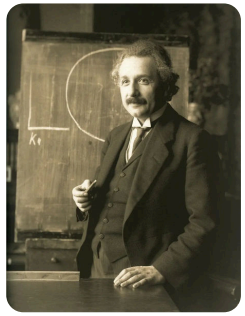


Figure 21.1 *Albert Einstein during a lecture in Vienna in 1921, the year of his Nobel Prize.*
Photo: Ferdinand Schmutzer / Adam Cuerden, Public domain

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

21.1 Quantum theory of radiation

At the end of the nineteenth century, physics seemed nearly finished, but a few results refused to fit. The most stubborn was the light given out by a hot body, **blackbody radiation**: the classical theory predicted that a hot body should pour out endless energy as ultraviolet light, which it plainly doesn't.

In 1900 the German physicist **Max Planck** found a formula that fitted the measurements exactly. To explain it, he had to assume something no one had dared to: the vibrating charges in a hot body can give out or take in energy only in definite, separate amounts, not any amount at all. Each smallest amount, a **quantum** (plural *quanta*), has energy

$$E = hf$$

where f is the frequency of the radiation and h is a new constant of nature, **Planck's constant**,

$$h = 6.63 \times 10^{-34}\ \text{J s}$$

Energy is given out or taken in as $hf, 2hf, 3hf, \ldots$, never as $1.5hf$. Just as charge comes only in whole numbers of e (Millikan's drops, Unit 20), the energy of radiation of one frequency comes only in whole numbers of hf. This idea is the **quantization of energy**, and Planck's proposal is the **quantum theory of radiation**. Planck received the Nobel Prize in Physics for 1918 (the full story).

The quantum idea went on to explain, one after another, things the classical theories could not:

- why the specific heat capacities of solids fall at low temperatures;
- the **photoelectric effect**, in which light ejects electrons from a metal (this unit);
- the **Compton effect**, the scattering of X-rays by electrons;
- the separate lines of the **hydrogen spectrum** (Unit 23);
- the origin and structure of **X-ray spectra**.

Together these successes led, in the 1920s, to **quantum mechanics**, the physics of atoms and everything built from them.



Figure 21.2 *Max Planck (1858–1947) in 1933. He began the quantum theory in 1900, at the age of 42.*
Photo: Unknown author, Public domain

The photon

Planck thought only the *giving out and taking in* of light came in packets. Einstein went further: light **travels** as packets. A packet of electromagnetic radiation is called a **photon**. A photon of light of frequency f and wavelength λ has energy

$$E = hf = \frac{hc}{\lambda}$$

since $c = f\lambda$. A photon has **no rest mass**: there is no such thing as a photon at rest. It exists only moving at the speed of light.

It still carries **momentum**. Relativity gives the momentum of anything moving at the speed of light as its energy divided by c:

$$p = \frac{E}{c} = \frac{hf}{c} = \frac{h}{\lambda}$$

A memory aid many books give: put $E = hf$ into $E = mc^2$ to get an "equivalent mass" $m = hf/c^2$, and then $p = mc = hf/c$. It leads to the right momentum, but don't take the "mass" literally: a photon has no mass, rest or otherwise; it has energy and momentum.

17. Electrons are emitted from a metal surface when it is lit by suitable electromagnetic radiation.

a. Name the effect.

b. The graph above shows how the maximum kinetic energy E_k of the emitted electrons varies with the frequency f. Use it to find (i) the threshold frequency of the radiation, and (ii) a value for the Planck constant.

c. On the graph, draw a line to show how E_k varies with f for a second metal with a lower work function.

d. The kinetic energy of the electrons is described as the maximum. Suggest why the emitted electrons have a range of kinetic energies for any one frequency.

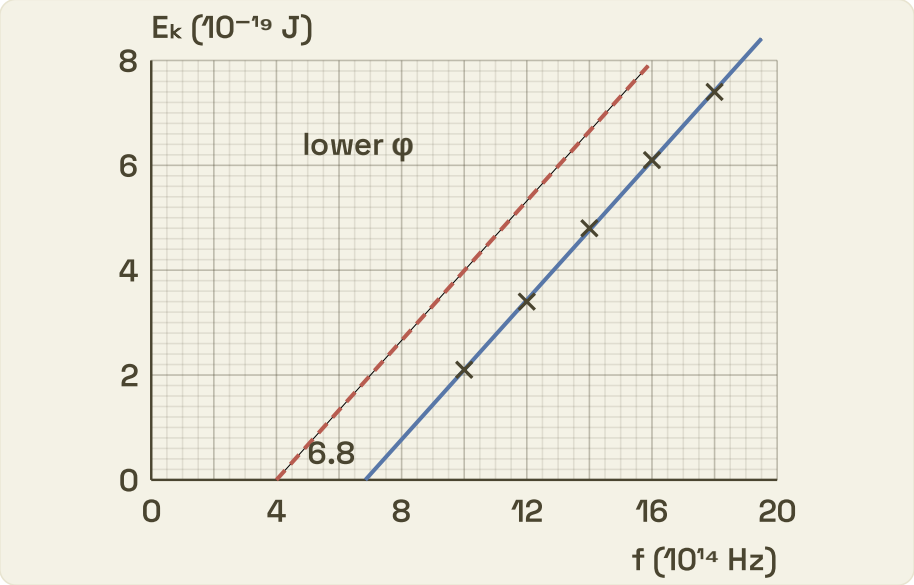


Figure 21.13 *The answer to question 17: the best line through the points, meeting the frequency axis at about 6.8×10^{14} Hz, and a parallel line (dashed) for a metal of lower work function.*

19.

a. Explain what is meant by the photoelectric effect.

b. One wavelength of the radiation from a mercury vapor lamp is 436 nm. Calculate the photon energy for this wavelength.

c. Light from the lamp falls, separately, on surfaces of cesium and tungsten. Their work functions are given in the table. Calculate the threshold wavelength for photoelectric emission from (i) cesium and (ii) tungsten.

Metal	Work function (eV)
Cesium	1.4
Tungsten	4.5

d. Use your answers to (c) to say, with reasons, whether light of wavelength 436 nm from the lamp will cause photoelectric emission from each metal.

CAREFUL

Big energy, tiny packets

One photon of visible light carries about 3.5×10^{-19} J, far too little to notice. A lamp giving out 1 W of green light sends out about 3×10^{18} photons every second. That is why light looks smooth and continuous: the packets are too small, and too many, to see one at a time.

IN SHORT

The photon's properties

a. Its energy depends only on its frequency (or wavelength): $E = hf = hc/\lambda$.

b. In a vacuum it travels in a straight line at the speed of light, $c = 3 \times 10^8$ m/s.

c. Its rest mass is zero ($m_0 = 0$).

d. Its momentum is $p = h/\lambda$.

e. Its frequency does not change when it passes from one medium into another.

f. Its speed and wavelength do change from one medium to another (both get smaller in glass).

g. Light shows both wave and particle properties.

h. Photons exert a force, and so a pressure, on a surface they strike (Exercise question 16).

Check 1

Blue light has a shorter wavelength than red light. Compared with a photon of red light, a photon of blue light has:

a less energy and less momentum

b more energy and more momentum

c more energy but the same momentum

d the same energy, since both travel at the speed of light

21.2 Photoelectric effect

In 1887 the German physicist **Heinrich Hertz**, busy proving that radio waves exist, noticed that sparks jumped across a gap more easily when ultraviolet light shone on it. A year later **Wilhelm Hallwachs** made it clear: a clean zinc plate on an electroscope, **negatively** charged, slowly lost its charge when ultraviolet light fell on it. A positively charged plate did not. Visible light, however bright, did nothing. The ultraviolet light was knocking negative charges out of the zinc; once J. J. Thomson had found the electron (1897), it was clear that these were electrons. **Philipp Lenard** then studied the effect carefully, with light of different frequencies and brightness.

The ejection of electrons from a metal surface when light of high enough frequency falls on it is called the **photoelectric effect**. The electrons ejected are called **photoelectrons**, and the current they carry a **photoelectric current**.

As you will see, the effect shows that light gives up its energy in packets, photons, each carrying its own amount of energy.



Figure 21.3 *A photocell made in the 1950s: in the vacuum, light falls on the curved cathode, coated with cesium, and the electrons it knocks out are collected by the thin wire in front.*
Photo: Mister rf, CC BY-SA 4.0

21.3 Experimental study of photoelectric effect

Lenard's apparatus, in the form found in school laboratories, is drawn in Figure 21.4. An evacuated glass tube holds two electrodes: **C**, the **emitter**, a metal plate on which light falls, and **A**, the **collector**. (They are often called the cathode and the anode.) The air is pumped out so that the electrons are not stopped by collisions with air molecules.

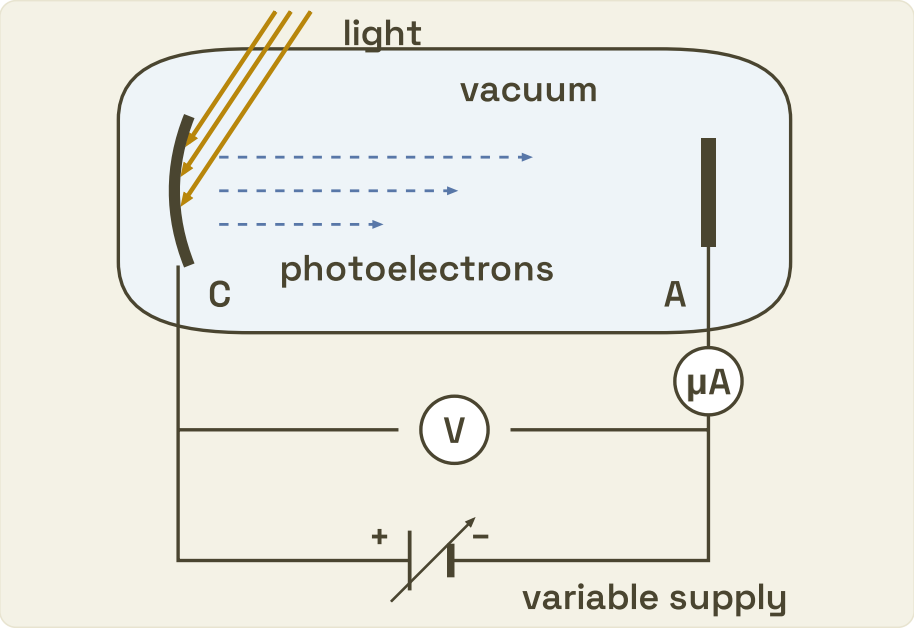


Figure 21.4 *A photocell circuit. Light on the emitter C knocks out electrons; those that reach the collector A make a current, read on the microammeter (μA). A variable supply sets the potential of A against C, read on the voltmeter V; joined as drawn, A is negative, and it slows the electrons down.*

When light of a suitable frequency falls on C (the **incident** light), electrons come out of it in all directions, with all kinds of speeds up to some largest one. Those that reach A make the current read on the microammeter.

14.

a. What is the photoelectric effect? Obtain Einstein's photoelectric equation, and state and explain the laws of photoelectric emission from it.

b. A sheet of silver is lit by monochromatic ultraviolet radiation of wavelength 1810 Å. The threshold wavelength of silver is 2640 Å. What is the maximum kinetic energy of the emitted electrons?

15.

a. Explain Einstein's theory of photoelectricity, and use it to arrive at Einstein's photoelectric equation.

b. Photosynthesis starts in certain plants when they are put in sunlight, but not if they get only infrared light. Explain.

16. A parallel beam of monochromatic light of wavelength 660 nm has an intensity of 0.25 mW/m^2. It falls normally on a surface that absorbs it totally. For a cross-sectional area of 1.2 cm^2 of the beam, find (a) the number of photons passing through it each second, (b) the momentum of one photon, and (c) the force exerted on the surface by the light. ($h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m/s.)

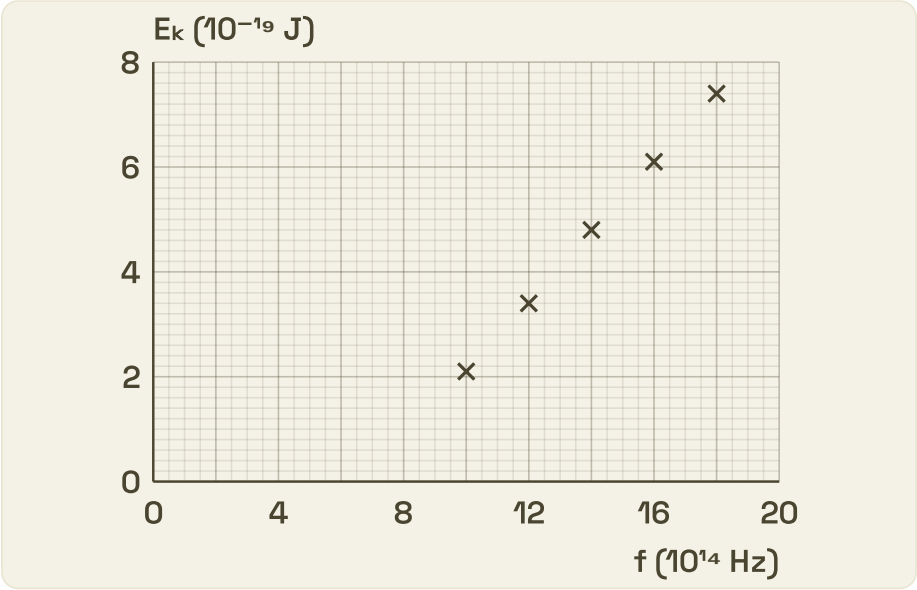


Figure 21.12 *For question 17: the maximum kinetic energy E_k of the emitted electrons (in units of 10^{-19} J) against the frequency f of the radiation (in units of 10^{14} Hz).*

Check 10

6. The study of the photoelectric effect is useful in understanding:

a quantization of energy
b quantization of charge
c conservation of charge
d conservation of kinetic energy

Check 11

7. A metal with a work function of 0.6 eV is lit by light of photon energy 2 eV. The stopping potential is:

a 4.6 V
b 0.8 V
c 1.4 V
d 3.6 V

Check 12

8. The work function of a metal is 2.51 eV. Its threshold frequency is:

a 7.5×10^{13} Hz
b 10.4×10^{12} Hz
c 6.08×10^{14} Hz
d 6.8×10^{11} Hz

Check 13

9. The threshold frequency of a metal is f_0. Light of frequency $4f_0$ falls on it. The maximum kinetic energy of the photoelectrons is:

a $5hf_0$
b $hf_0/2$
c hf_0
d $3hf_0$

Check 14

10. Ultraviolet radiation of photon energy 6.2 eV falls on an aluminum surface of work function 4.2 eV. The kinetic energy of the fastest electrons is about:

a 5.2×10^{-17} J
b 3×10^{-19} J
c 2.4×10^{-18} J
d 2.65×10^{-16} J

Answer these questions

11.
a. Define threshold frequency, stopping potential and work function in photoelectric emission.
b. Show that the existence of a threshold frequency is evidence in favor of the quantum theory.

12. When electromagnetic radiation falls on the surface of a metal, electrons leave the surface. But when the radiation's frequency is below a certain value, no electrons are emitted.
a. Explain why no electrons are emitted if the frequency is below a certain value.
b. State two other observations of the photoelectric effect which suggest that electromagnetic radiation has particle properties.
c. The work function of the metal is 3.8 eV.
i. Calculate the least frequency of electromagnetic radiation that causes photoelectric emission.
ii. An electron at the surface of the metal is emitted with a kinetic energy of 4.5×10^{-19} J. Calculate the energy of the photon that freed it, in eV.
d. Mention two uses of the photoelectric effect in everyday life.

13.
a. Describe Millikan's experiment to find the value of Planck's constant.
b. The work function of potassium is 3.8×10^{-19} J. Light of wavelength 5000 Å falls on it. Find (i) the stopping potential, (ii) the kinetic energy of the most energetic electrons, and (iii) their speed.

Now make A **negative** compared with C. The field between them pushes the electrons back: the p.d. is now a **retarding potential**. Only electrons that start with enough kinetic energy can climb the "hill" and reach A, so the current falls. Make A more and more negative, and at a certain p.d. not even the fastest electron gets there: the current drops to zero. This reverse p.d. is the **stopping potential**, V_0.

An electron that just fails to reach A has lost all its kinetic energy climbing a p.d. of V_0: it has done work eV_0 against the field. So the fastest electron's kinetic energy is

$$eV_0 = \tfrac{1}{2}mv_{\max}^2 \tag{21.1}$$

Since e is the same for every electron, $V_0 \propto \tfrac{1}{2}mv_{\max}^2$: measuring the stopping potential, in volts, measures the maximum kinetic energy, in electronvolts. A stopping potential of 1.5 V means the fastest electrons left the metal with 1.5 eV.

Laws of photoelectric emission

Experiments like this gave four results, often called the **laws of photoelectric emission**.

(i) There is a threshold frequency. Photoelectrons come out only when the frequency of the light is above a certain value, the **threshold frequency** f_0, which is different for each metal. Lower the frequency step by step, and the stopping potential needed gets smaller and smaller (Figure 21.5); at f_0 the current stops even with no reverse p.d. at all. Below f_0, no electrons come out, however bright the light and however long it shines.

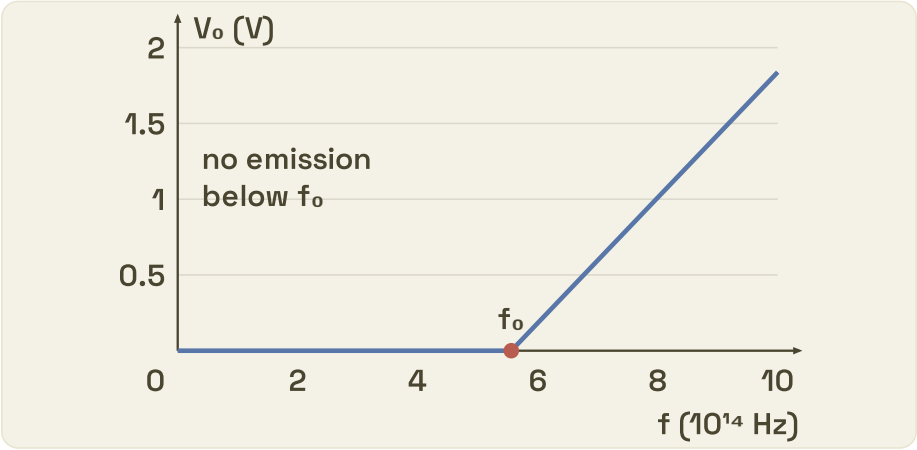


Figure 21.5 *Stopping potential against frequency, for sodium. Below the threshold frequency f_0 no electrons come out at all; above it, the stopping potential rises in a straight line.*

(ii) There is no delay. Emission begins the instant the light falls on the metal, even when the light is very dim: the delay has been measured to be less than 10^{-9} s. (Many books quote "less than 10^{-8} s", which is also true and fine in an exam.)

(iii) The maximum kinetic energy depends on the frequency, not on the brightness. Light of one frequency at two intensities L_1 and L_2 gives the curves of current against p.d. in Figure 21.6. Both cut off at the same reverse p.d., $-V_0$: the fastest electrons are no faster in the brighter light. Light of two frequencies f_1 and f_2, with $f_2 > f_1$, gives different stopping potentials (Figure 21.7): the higher frequency gives the larger V_0, and so the faster electrons.

(iv) The number of electrons depends on the brightness. The photoelectric current (with A positive, so that every electron emitted is collected: the flat part of the curves, the **saturation current**) is proportional to the intensity of the light. In Figure 21.6 the brighter light L_2 gives the larger saturation current.

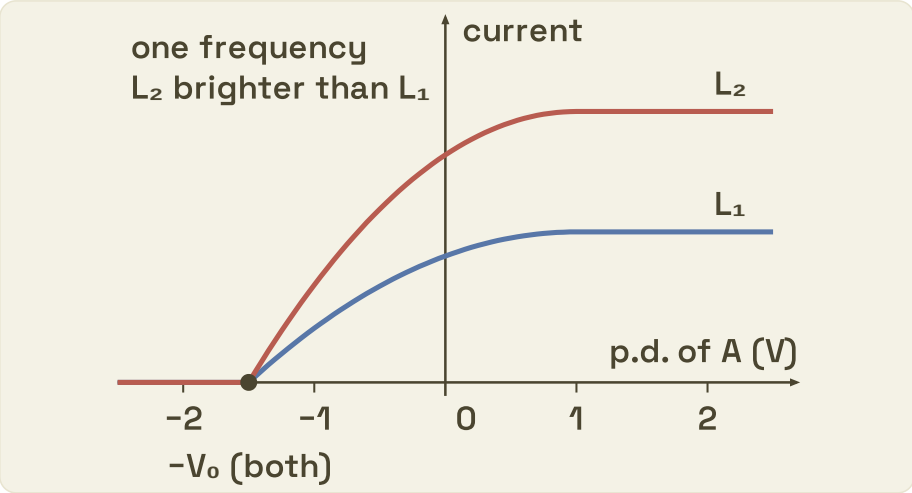


Figure 21.6 Current against the p.d. of A, for light of one frequency at two intensities: L_2 is brighter than L_1. Both currents stop at the same reverse p.d., $-V_0$.

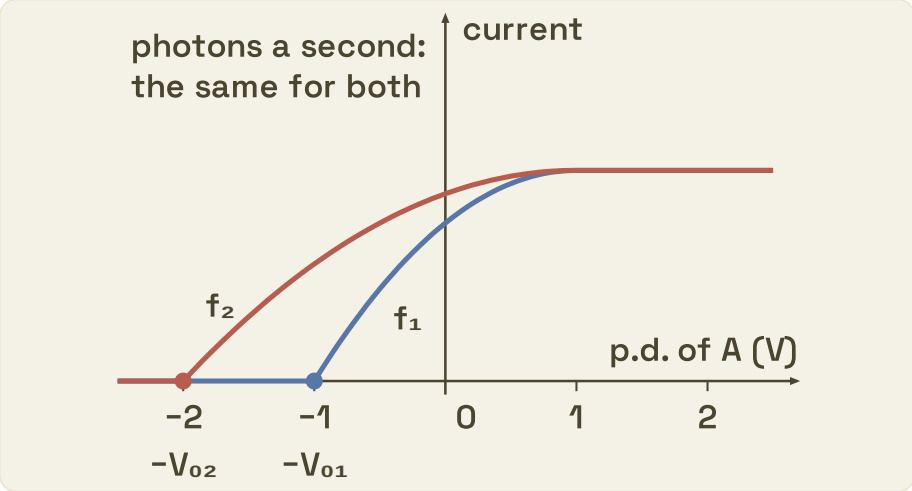


Figure 21.7 Current against the p.d. of A, for two frequencies, f_2 higher than f_1, sending the same number of photons each second. The saturation current is the same; the stopping potential is larger for f_2.

Worked examples

IN SHORT

- **Quantum:** radiation of frequency f is given out and absorbed in packets of energy $E = hf$; $h = 6.63 \times 10^{-34}$ J s.
- **Photon:** $E = hf = hc/\lambda$, no rest mass, momentum $p = h/\lambda = E/c$. Handy: E (eV) $\approx 1240/\lambda$ (nm).
- **Photoelectric effect:** light above a threshold frequency ejects electrons from a metal. Stopping potential: $eV_0 = \frac{1}{2}mv_{\max}^2$.
- **Laws:** (i) no emission below f_0, whatever the brightness; (ii) no delay; (iii) the maximum kinetic energy rises with frequency, not brightness; (iv) the number of electrons, and the current, rise with brightness.
- **Einstein's equation:** $hf = \phi + \frac{1}{2}mv_{\max}^2$, with $\phi = hf_0 = hc/\lambda_0$; also $\frac{1}{2}mv_{\max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$ and $hf = \phi + eV_0$.
- **Millikan:** $V_0 = \frac{h}{e}f - \frac{\phi}{e}$, a straight line of slope h/e; it cuts the frequency axis at f_0 and the V_0 axis at $-\phi/e$.

Exercise

Choose the correct option

Check 5

1. Einstein's photoelectric equation is $E_k = E - W$. Here E_k is the:

a kinetic energy of all the emitted electrons
b average kinetic energy of the emitted electrons
c maximum kinetic energy of the emitted electrons
d least kinetic energy of the emitted electrons

Check 6

2. The emission of electrons from the surface of a metal by the action of light on it is called:

a thermionic emission
b photoelectric emission
c electronic emission
d cold emission

Check 7

3. Infrared, visible and ultraviolet radiation of the same intensity fall in turn on a clean zinc plate (work function about 4.3 eV). Which gives the largest photocurrent?

a infrared
b visible
c ultraviolet
d all give the same

Check 8

4. A photon is a:

a quantum of light energy
b quantum of matter
c positively charged particle
d instrument for measuring light intensity

Check 9

5. The photoelectric current can be increased by:

a increasing the frequency
b increasing the intensity
c decreasing the intensity
d decreasing the wavelength

YOUR TURN

Test yourself 21.10: cesium in violet light

Light of wavelength 400 nm, at the violet end of the visible spectrum, falls on a cesium surface of work function 1.9 eV. Find the maximum speed of the emitted electrons. ($m_e = 9.1 \times 10^{-31}$ kg, $c = 3 \times 10^8$ m/s, $h = 6.62 \times 10^{-34}$ J s.)

Answer

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{400 \times 10^{-9}} = 4.97 \times 10^{-19}\ \text{J}$$

$\phi = 1.9 \times 1.6 \times 10^{-19} = 3.04 \times 10^{-19}$ J, so $\frac{1}{2}mv_{\max}^2 = 4.97 \times 10^{-19} - 3.04 \times 10^{-19} = 1.93 \times 10^{-19}$ J.

$$v_{\max} = \sqrt{\frac{2 \times 1.93 \times 10^{-19}}{9.1 \times 10^{-31}}} = \sqrt{4.24 \times 10^{11}} = 6.5 \times 10^5\ \text{m/s}$$

YOUR TURN

Test yourself 21.11: two frequencies at once

Light whose electric field at a place is $E = (100\ \text{V/m})\,[\sin(5 \times 10^{15}\,t) + \sin(8 \times 10^{15}\,t)]$, with t in seconds, falls on a metal surface with a work function of 2.0 eV. Find the maximum kinetic energy of the photoelectrons. ($h = 6.63 \times 10^{-34}$ J s.)

Hint

The light is a mix of two frequencies. The numbers multiplying t are angular frequencies, $\omega = 2\pi f$. Which photons give the fastest electrons?

Answer

The fastest electrons come from the higher-frequency photons, $\omega = 8 \times 10^{15}$ rad/s, so $f = \frac{\omega}{2\pi} = \frac{8 \times 10^{15}}{2\pi} = 1.27 \times 10^{15}$ Hz.

$$hf = 6.63 \times 10^{-34} \times 1.273 \times 10^{15} = 8.44 \times 10^{-19}\ \text{J} = 5.28\ \text{eV}$$

Maximum kinetic energy: $5.28 - 2.0 =$ **3.28 eV** (3.27 eV with $e = 1.602 \times 10^{-19}$ C). (The other frequency gives photons of 3.30 eV, and electrons of at most 1.30 eV.)



Figure 21.11 *Small solar panels on a lodge's roof at Mongla, in Solukhumbu, in the Everest region. Across Nepal's hills such panels light homes and charge phones where no power lines reach.*

Photo: Gerd Eichmann, CC BY-SA 4.0

CAREFUL

What "the same intensity" means here

Intensity is energy per second per square meter. Blue light of the same intensity as red light delivers the same energy each second in **fewer** photons, because each blue photon carries more. So, strictly, the saturation current is set by the number of **photons** arriving each second (and by how many of them free an electron), not by the energy. In Figure 21.7 the two lights send the same number of photons each second; that is why their saturation currents are equal.

CAREFUL

Why waves can't explain it

On the wave theory, a brighter light is a stronger wave, so it should shake electrons out **faster** (law iii fails); any frequency, given time, should pile up enough energy to free an electron (law i fails); and very dim light should need seconds or minutes to build up the energy, not 10^{-9} s (law ii fails). Only law (iv) fits the wave picture. The next section shows how photons explain all four.

WORKED EXAMPLE

Example 21.1: brighter light, faster electrons?

Does increasing the intensity of light increase the kinetic energy of the photoelectrons?

Answer

No. The maximum kinetic energy of the photoelectrons depends only on the **frequency** of the light. A brighter light of the same frequency brings more photons each second, each with the same energy hf, so it frees **more** electrons, each with the same range of energies.

WORKED EXAMPLE

Example 21.2: doubling the intensity

What happens to the number of photoelectrons if the intensity of the light is doubled?

Answer

As long as the frequency is above the threshold, the number of photoelectrons emitted each second **doubles**: twice the intensity of the same light means twice as many photons each second, and each photon can free at most one electron. (Below the threshold frequency, doubling the intensity still frees none.)

21.4 Einstein's photoelectric equation

Einstein's explanation takes three ideas.

1. Light of frequency f arrives as photons, each of energy hf.
2. A metal's free electrons move about inside it, but they can't leave on their own: the metal pulls them back at its surface. The least energy needed to free an electron from the surface is the metal's **work function**, ϕ.
3. One photon gives **all** its energy to **one** electron, at once. (Never half a photon, never two photons adding up for one electron, in ordinary light.)

If hf is more than ϕ, the electron gets out, and what is left over, $hf - \phi$, is its kinetic energy. The electrons right at the surface leave with all of that; electrons from deeper inside lose some energy in collisions on the way out, so they come out slower (Figure 21.8). The fastest therefore have

$$\tfrac{1}{2}mv_{\max}^2 = hf - \phi$$

or, in words, energy of the photon = work function + maximum kinetic energy:

$$hf = \phi + \tfrac{1}{2}mv_{\max}^2 \tag{21.2}$$

This is **Einstein's photoelectric equation**. Here m is the electron's mass and $v_{\max}$ the maximum speed of the photoelectrons.

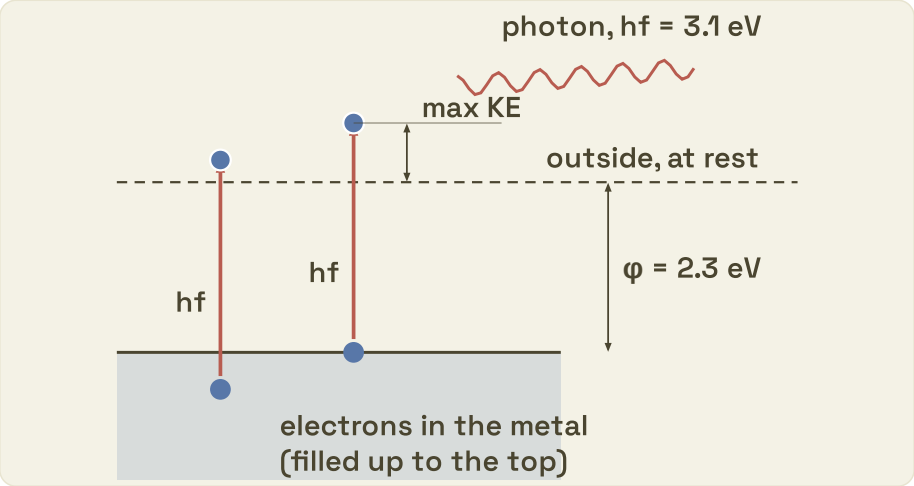


Figure 21.8 *Einstein's idea on an energy ladder, for sodium ($\phi = 2.3$ eV) and violet light ($hf = 3.1$ eV). One photon lifts one electron by hf. An electron from the top of the metal's filled levels ends $3.1 - 2.3 = 0.8$ eV above "outside, at rest": the maximum kinetic energy. An electron from deeper inside, given the same hf, ends with less.*

The work function is a property of the metal (and of how clean its surface is), not of the light. A photon with exactly $hf = \phi$ frees an electron with no kinetic energy to spare; its frequency is the threshold frequency f_0:

$$\phi = hf_0$$

So the equation can also be written

$$hf = hf_0 + \tfrac{1}{2}mv_{\max}^2$$

and, with $f = c/\lambda$ and $f_0 = c/\lambda_0$,

$$\tfrac{1}{2}mv_{\max}^2 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0} = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right)$$

All three forms are Einstein's photoelectric equation. With $\tfrac{1}{2}mv_{\max}^2 = eV_0$ (equation 21.1) it also reads $hf = \phi + eV_0$.

The laws, explained

1. **Threshold.** The kinetic energy $hf - \phi$ can be positive only if $hf > \phi$, that is $f > f_0$. Below f_0, no single photon has enough energy, and since one electron can't collect the energy of two photons, no number of photons helps.
2. **Faster electrons for higher frequency.** $\tfrac{1}{2}mv_{\max}^2 = hf - hf_0$ rises in a straight line with f.
3. **More electrons for brighter light.** Brighter light of the same frequency has more photons each second; one photon frees at most one electron, so more photons free more electrons, each with the same energies.
4. **No delay.** The photon's energy is handed over in one go, the moment it is absorbed; nothing needs to build up.

WORKED EXAMPLE

Example 21.15: molybdenum

The work function of molybdenum is 5 eV. Ultraviolet light of wavelength 1000 Å falls on it. Find the maximum speed of the ejected photoelectrons. ($h = 6.62 \times 10^{-34}$ J s, $m = 9 \times 10^{-31}$ kg, 1 eV $= 1.6 \times 10^{-19}$ J.)

Answer

$\phi = 5 \times 1.6 \times 10^{-19} = 8.0 \times 10^{-19}$ J; $\lambda = 1000\ \text{Å} = 1.0 \times 10^{-7}$ m.

Photon energy:

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{1.0 \times 10^{-7}} = 19.86 \times 10^{-19}\ \text{J}$$

Maximum kinetic energy:

$$\tfrac{1}{2}mv_{\max}^2 = 19.86 \times 10^{-19} - 8.0 \times 10^{-19} = 11.86 \times 10^{-19}\ \text{J}$$

$$v_{\max} = \sqrt{\frac{2 \times 11.86 \times 10^{-19}}{9 \times 10^{-31}}} = \sqrt{2.64 \times 10^{12}} = 1.62 \times 10^{6}\ \text{m/s}$$

(In the older CGS units, with h in erg s and m in grams, the same working gives 1.62×10^{8} cm/s.)

WORKED EXAMPLE

Example 21.16: the light's frequency

The work function of potassium is 2.3 eV. Photoelectrons leave it with a maximum speed of 10^4 m/s. Find the frequency of the light. ($m = 9.1 \times 10^{-31}$ kg, $h = 6.62 \times 10^{-34}$ J s.)

Answer

$\phi = 2.3 \times 1.6 \times 10^{-19} = 3.68 \times 10^{-19}$ J.

Maximum kinetic energy: $\tfrac{1}{2}mv_{\max}^2 = \tfrac{1}{2} \times 9.1 \times 10^{-31} \times (10^4)^2 = 4.55 \times 10^{-23}$ J.

$$hf = \phi + \tfrac{1}{2}mv_{\max}^2 = 3.68 \times 10^{-19} + 0.000455 \times 10^{-19} = 3.680 \times 10^{-19}\ \text{J}$$

$$f = \frac{3.680 \times 10^{-19}}{6.62 \times 10^{-34}} = 5.56 \times 10^{14}\ \text{Hz}$$

These electrons are very slow (10^4 m/s gives only 0.0003 eV), so the light is almost exactly at potassium's threshold.

Test yourself

YOUR TURN

Test yourself 21.9: a stopping potential

Light of wavelength 5400 Å falls on a metal whose work function is 1.9 eV. Find the maximum kinetic energy of the photoelectrons and their stopping potential. ($h = 6.62 \times 10^{-34}$ J s.)

Hint

Find the photon energy in eV first. The stopping potential in volts equals the maximum kinetic energy in electronvolts.

Answer

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5400 \times 10^{-10}} = 3.68 \times 10^{-19}\ \text{J} = 2.30\ \text{eV}$$

Maximum kinetic energy: $2.30 - 1.9 =$ **0.40 eV** (6.4×10^{-20} J). Stopping potential: $V_0 = \dfrac{\tfrac{1}{2}mv_{\max}^2}{e} =$ **0.40 V**.

WORKED EXAMPLE

Example 21.12: not every wavelength works

Why can't the photoelectric effect be seen with every wavelength of light?

Answer

Light gives up its energy as photons, each of energy $hf = hc/\lambda$, and one photon frees at most one electron. A photon frees an electron only if its energy is more than the metal's work function, that is, only if its wavelength is **shorter** than the metal's threshold wavelength $\lambda_0 = hc/\phi$. Light of longer wavelength has photons of too little energy, and no number of them helps. For sodium, potassium and cesium, λ_0 lies in the visible, so green, blue and violet light work; for most other metals it lies in the ultraviolet, and no visible light works at all.

WORKED EXAMPLE

Example: the stopping potential

What is meant by stopping potential?

Answer

The **stopping potential** V_0 is the least reverse p.d. between the emitter and the collector that stops even the fastest photoelectrons from reaching the collector, so that the photoelectric current becomes zero. The work done against it equals the fastest electrons' kinetic energy:

$$eV_0 = \tfrac{1}{2}mv_{\max}^2$$

WORKED EXAMPLE

Example 21.13: will it work?

The work function of a metal surface is 3.5 eV. If photons of energy 3 eV strike it, will electrons be emitted?

Answer

No. Each photon's energy, 3 eV, is less than the 3.5 eV needed to free an electron; emission is possible only when the photon's energy is more than the work function.

WORKED EXAMPLE

Example 21.14: sodium in red light

Light of frequency 4.7×10^{14} Hz ejects photoelectrons from a sodium surface whose work function is 2.88×10^{-19} J. Find the maximum kinetic energy of the ejected electrons. ($h = 6.62 \times 10^{-34}$ J s.)

Answer

$$\tfrac{1}{2}mv_{\max}^2 = hf - \phi = 6.62 \times 10^{-34} \times 4.7 \times 10^{14} - 2.88 \times 10^{-19}$$
$$= 3.11 \times 10^{-19} - 2.88 \times 10^{-19} = 0.23 \times 10^{-19}\ \text{J} = 2.3 \times 10^{-20}\ \text{J}$$

only about 0.14 eV: this red light is barely above this sodium surface's threshold. (The work function given here, 1.8 eV, is lower than the 2.3 eV in the table: measured work functions depend on how the surface is prepared.)

Check 2

Light of a certain frequency ejects electrons from a metal. The light is made three times as bright, its frequency unchanged. What happens?

a The electrons come out with three times the kinetic energy.
b Three times as many electrons come out each second, with the same energies.
c The electrons come out faster, and more of them.
d Nothing changes at all.

Work function (φ)

The **work function** of a metal, also called its **threshold energy**, is the least energy that must be given to an electron to free it from the metal's surface. It is different for different metals, and it is usually given in electronvolts:

Metal	Work function (eV)	Metal	Work function (eV)
Cesium	1.9	Calcium	3.2
Potassium	2.2	Copper	4.5
Sodium	2.3	Silver	4.7
Lithium	2.5	Platinum	5.6

(Measured values vary a little with how the surface is prepared; tables differ by a few tenths of an electronvolt.)

If a photon of energy E falls on a metal of work function ϕ:

- $E < \phi$: no photoelectric effect;
- $E = \phi$: electrons are just freed, with no kinetic energy;
- $E > \phi$: electrons are freed, with kinetic energy up to $E - \phi$.

CAREFUL

The work function is not the ionization energy

The **ionization energy** is the energy needed to take an electron from a single, free atom. The **work function** is the energy to take a free electron out of a solid metal, where the atoms share their outer electrons. They are different, and the work function is always the smaller: sodium's is 2.3 eV, but freeing an electron from one sodium atom takes 5.1 eV.

The alkali metals, cesium to lithium, have the lowest work functions, so visible light frees electrons from them. Copper, silver and platinum need ultraviolet light. That is why the photocells used with visible light have cathodes coated with cesium compounds.

Threshold frequency (f_0)

The **threshold frequency** $f_0 = \phi/h$ is the least frequency of light that can eject electrons from a metal; at f_0 they just come out, with no kinetic energy.

- $f < f_0$: no electrons ejected, no photoelectric effect;
- $f = f_0$: electrons are just ejected, with no kinetic energy;
- $f > f_0$: electrons are ejected with kinetic energy.

Threshold wavelength (λ_0)

The **threshold wavelength** λ_0 is the longest wavelength that can eject electrons from a metal. Longer waves mean lower frequency and less energy per photon:

$$\lambda_0 = \frac{c}{f_0} = \frac{hc}{\phi}$$

- $\lambda < \lambda_0$: electrons are ejected, with kinetic energy;
- $\lambda = \lambda_0$: electrons are just ejected, with no kinetic energy;
- $\lambda > \lambda_0$: no photoelectric effect.

DID YOU KNOW?

A handy shortcut

With $h = 6.63 \times 10^{-34}$ J s, $c = 3.00 \times 10^{8}$ m/s and 1 eV $= 1.6 \times 10^{-19}$ J,

$$E\ (\text{in eV}) \approx \frac{1240}{\lambda\ (\text{in nm})}$$

Red light at 700 nm: 1.8 eV. Violet at 400 nm: 3.1 eV. Ultraviolet at 250 nm: 5.0 eV. So sodium (2.3 eV) needs light shorter than $1240/2.3 \approx 540$ nm, green; copper (4.5 eV) needs shorter than 276 nm, well into the ultraviolet. In an exam, do the full working, and use the shortcut to check your answer.

Check 3

Light of wavelength 450 nm falls in turn on cesium ($\varphi = 1.9$ eV), lithium (2.5 eV) and copper (4.5 eV). Which give photoelectrons?

a copper only
b cesium and lithium
c cesium only
d all three

Worked examples

WORKED EXAMPLE

Example 21.3: below the threshold

What happens if the frequency of the light is below the threshold frequency?

Answer

No photoelectrons are emitted, however intense the light and however long it shines. Each photon has $hf < hf_0 = \phi$, too little to free an electron, and an electron can't add up the energy of several photons.

WORKED EXAMPLE

Example 21.4: why a cutoff?

Why is there a cutoff frequency below which no photoelectrons are emitted?

Answer

Because one photon gives its energy to one electron. Below the cutoff frequency each photon's energy hf is less than the metal's work function, so it can't free an electron from the surface, and its energy only warms the metal.

WORKED EXAMPLE

Example 21.5: infrared and ultraviolet

Infrared light does not cause electrons to be emitted from a metal, but ultraviolet light does. Why?

Answer

Light is absorbed as photons of energy hf. Ultraviolet has a much higher frequency than infrared, so each ultraviolet photon carries much more energy. An ultraviolet photon's energy is more than the metal's work function, and it frees an electron; an infrared photon's is less, and it can't, however many arrive.

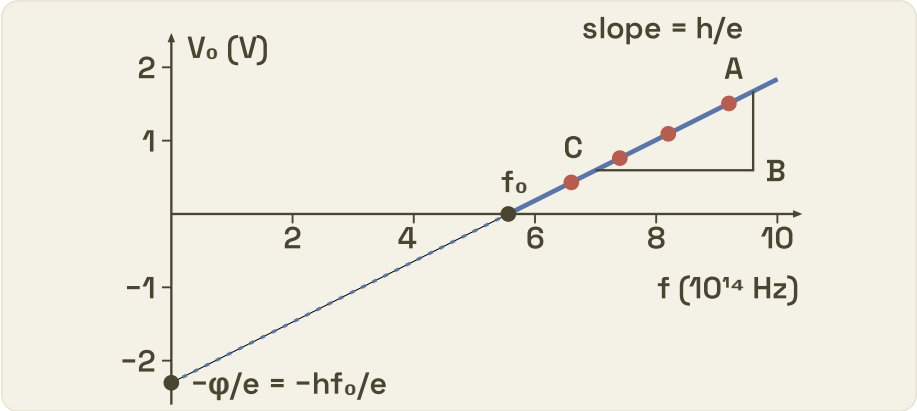


Figure 21.10 *Millikan's graph for sodium: stopping potential against frequency is a straight line. Its slope, AB/BC, is h/e. It cuts the frequency axis at f_0, and, drawn back, the stopping-potential axis at $-\phi/e$.*

For different metals the lines are **parallel** (same slope h/e) but cross the frequency axis at different places (different f_0): exactly what Einstein's equation says.

Check 4

On a graph of stopping potential against frequency, a second metal with a larger work function gives a line that is:

a steeper, crossing the frequency axis at the same place
b less steep, starting from the origin
c parallel to the first, crossing the frequency axis at a higher frequency
d parallel to the first, crossing the frequency axis at a lower frequency

DID YOU KNOW?

Photons at work

- A **photomultiplier tube** uses the photoelectric effect to count single photons: each electron knocked out is multiplied a million times by a chain of electrodes. Thousands of them line the giant tank of Japan's Super-Kamiokande, catching the faint flashes made by neutrinos.
- Old film projectors read the sound track printed beside the pictures with a **photocell**, and automatic doors and burglar alarms long used a beam of light falling on one.
- **Night-vision goggles** turn the few photons from a dark scene into electrons, multiply them and turn them back into a bright picture.
- **Solar cells** and the camera sensor in your phone work by an inner photoelectric effect: a photon frees an electron inside a semiconductor without throwing it out of the material (Unit 22 explains semiconductors).

Worked examples

der of wire gauze, whose potential against the metal was set with a potentiometer; a sensitive electrometer **G** showed whether any current flowed.

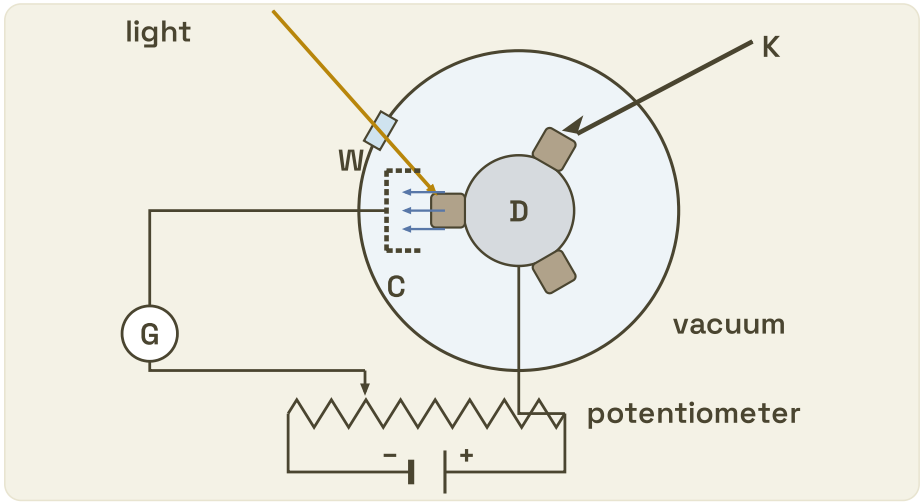


Figure 21.9 *Millikan's apparatus, simplified. A knife K shaves a clean surface on one of the alkali-metal cylinders on the wheel D; light through the window W falls on it; the gauze cylinder C collects the electrons, and its potential is set with the potentiometer until the electrometer G shows no current.*

For each frequency, Millikan made C more and more negative until even the fastest electrons were turned back and G showed no current. That reverse p.d. is the stopping potential V_0, and

$$\tfrac{1}{2}mv_{\max}^2 = eV_0$$

where e is the electron's charge.

Theory

Put this into Einstein's equation, $hf = \phi + \tfrac{1}{2}mv_{\max}^2$, with $\phi = hf_0$:

$$hf = hf_0 + eV_0$$

$$V_0 = \left(\frac{h}{e}\right) f - \frac{hf_0}{e}$$

This has the form $y = mx + c$: a graph of V_0 (up) against f (across) must be a straight line with

- slope $m = \dfrac{h}{e}$, the same for every metal;
- intercept on the V_0 axis $c = -\dfrac{hf_0}{e} = -\dfrac{\phi}{e}$.

Millikan's graph was exactly such a straight line (Figure 21.10).

To find h. Measure the slope from a triangle ABC under the line: $\tan\theta = \dfrac{AB}{BC} = \dfrac{h}{e}$, so

$$h = e \times \text{slope} = e\,\frac{AB}{BC}$$

Millikan found $h = 6.57 \times 10^{-34}$ J s, within about 1% of today's value, 6.626×10^{-34} J s.

To find the work function. Where the line cuts the **frequency** axis, $V_0 = 0$: electrons just come out with no energy to spare, so that frequency is the threshold frequency f_0. Multiply by h: $\phi = hf_0$. Or read where the line, drawn back, cuts the V_0 axis: that intercept is $-\phi/e$, and in volts it gives the work function in electronvolts directly.

WORKED EXAMPLE

Example 21.6: infrared or ultraviolet?

Which has more energy, a photon of infrared or one of ultraviolet? Give a reason.

Answer

The energy of a photon is $E = hf$: the higher the frequency, the more the energy. Ultraviolet has a higher frequency than infrared, so **an ultraviolet photon has more energy** than an infrared photon.

WORKED EXAMPLE

Example 21.7: sodium and orange light

The energy needed to remove an electron from sodium is 2.3 eV. Does sodium show the photoelectric effect with orange light of wavelength 680 nm? ($h = 6.625 \times 10^{-34}$ J s.)

Answer

The work function is the least energy needed, so the longest wavelength that can eject electrons is

$$\lambda_0 = \frac{hc}{\phi} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{2.3 \times 1.6 \times 10^{-19}} = 5.40 \times 10^{-7}\ \text{m} = 540\ \text{nm}$$

The orange light's 680 nm is longer than 540 nm, so its photons have too little energy: **sodium shows no photoelectric effect with this light**.

WORKED EXAMPLE

Example 21.8: photon energies

Find the energy of a photon of (i) frequency 1000 kHz, (ii) wavelength 5890 Å, (iii) wavelength 1 Å, in joules and in electronvolts. ($h = 6.62 \times 10^{-34}$ J s, $c = 3 \times 10^{8}$ m/s, 1 eV $= 1.6 \times 10^{-19}$ J.)

Answer

(i) $f = 1000$ kHz $= 10^{6}$ Hz, a medium-wave radio signal:

$$E = hf = 6.62 \times 10^{-34} \times 10^{6} = 6.62 \times 10^{-28}\ \text{J} = \frac{6.62 \times 10^{-28}}{1.6 \times 10^{-19}} = 4.14 \times 10^{-9}\ \text{eV}$$

(ii) $\lambda = 5890\ \text{Å} = 5890 \times 10^{-10}$ m, sodium's yellow light:

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{5890 \times 10^{-10}} = 3.37 \times 10^{-19}\ \text{J} = 2.11\ \text{eV}$$

(iii) $\lambda = 1\ \text{Å} = 10^{-10}$ m, an X-ray:

$$E = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{10^{-10}} = 1.99 \times 10^{-15}\ \text{J} = \frac{1.99 \times 10^{-15}}{1.6 \times 10^{-19}} = 1.24 \times 10^{4}\ \text{eV}$$

About 12 400 eV. A radio photon carries a billionth of an electronvolt, an X-ray photon over ten thousand: that is why X-rays can damage the cells they pass through and radio waves can't.

WORKED EXAMPLE

Example 21.9: photons from a bulb

5% of the energy supplied to a 100 W filament bulb is given out as visible light. How many visible photons does it emit each second? Take all of them to have wavelength 5600 Å; $h = 6.625 \times 10^{-34}$ J s.

Answer

Energy of one photon:

$$E = \frac{hc}{\lambda} = \frac{6.625 \times 10^{-34} \times 3 \times 10^{8}}{5600 \times 10^{-10}} = 3.549 \times 10^{-19} \text{ J}$$

Visible light given out each second: $5\% \times 100 \text{ J} = 5$ J. Number of photons each second:

$$n = \frac{5}{3.549 \times 10^{-19}} = 1.41 \times 10^{19}$$

The other 95 J each second leaves as heat and infrared, which is why filament bulbs have been replaced by LEDs.

WORKED EXAMPLE

Example 21.10: nickel's threshold

The work function of nickel is 5.9 eV. Find its threshold frequency and threshold wavelength. ($h = 6.62 \times 10^{-34}$ J s.)

Answer

$\phi = 5.9 \times 1.6 \times 10^{-19} = 9.44 \times 10^{-19}$ J.

$$f_0 = \frac{\phi}{h} = \frac{9.44 \times 10^{-19}}{6.62 \times 10^{-34}} = 1.43 \times 10^{15} \text{ Hz}$$

$$\lambda_0 = \frac{c}{f_0} = \frac{3 \times 10^{8}}{1.43 \times 10^{15}} = 2.10 \times 10^{-7} \text{ m} = 210 \text{ nm}$$

Only ultraviolet shorter than 210 nm frees electrons from nickel.

WORKED EXAMPLE

Example 21.11: a photocurrent

Light of wavelength 4560 Å and power 1 mW falls on a cesium surface (work function 1.92 eV). Find the photoelectric current, if the quantum efficiency is 0.5%, that is, only 0.5% of the photons free an electron. ($h = 6.6 \times 10^{-34}$ J s, $c = 3 \times 10^{8}$ m/s.)

Answer

First, do these photons free electrons at all? $hc/\lambda = 6.6 \times 10^{-34} \times 3 \times 10^{8} / 4560 \times 10^{-10} = 4.34 \times 10^{-19}$ J $= 2.7$ eV, more than 1.92 eV: yes.

Photons arriving each second: power = (number per second) × (energy of each), so

$$n = \frac{P\lambda}{hc} = \frac{10^{-3} \times 4560 \times 10^{-10}}{6.6 \times 10^{-34} \times 3 \times 10^{8}} = 2.30 \times 10^{15} \text{ per second}$$

Electrons freed each second: 0.5% of these, $0.005 \times 2.30 \times 10^{15} = 1.15 \times 10^{13}$. Current = charge per second:

$$I = 1.15 \times 10^{13} \times 1.6 \times 10^{-19} = 1.84 \times 10^{-6} \text{ A}$$

about 1.8 μA.

Test yourself

YOUR TURN

Test yourself 21.4–21.6: three photons

Photons of energies 1.0 eV, 2.0 eV and 3.0 eV strike a metal surface whose work function is 1.8 eV.

a. Which of these photons can release an electron from the metal?

b. Find the maximum kinetic energy of the electrons released in each case.

Answer

a. Only photons with more energy than 1.8 eV: **the 2.0 eV and the 3.0 eV photons**. The 1.0 eV photon can't.

b. Maximum kinetic energy = $hf - \phi$:

2.0 eV photon: $2.0 - 1.8 =$ **0.2 eV**;
3.0 eV photon: $3.0 - 1.8 =$ **1.2 eV**.

YOUR TURN

Test yourself 21.7: the bulb again

If 5% of the energy supplied to a 100 W filament bulb is radiated as visible light, how many visible photons does it emit each second? Take every visible photon's wavelength to be 5600 Å; $h = 6.6 \times 10^{-34}$ J s.

Answer

This is Example 21.9 with $h = 6.6 \times 10^{-34}$ J s. One photon: $E = hc/\lambda = 6.6 \times 10^{-34} \times 3 \times 10^{8} / 5.6 \times 10^{-7} = 3.54 \times 10^{-19}$ J. Visible energy each second: 5 J. Photons each second: $5 / 3.54 \times 10^{-19} =$ **1.41×10^{19}**.

YOUR TURN

Test yourself 21.8: platinum

The work function of platinum is 9.0×10^{-19} J. Find (a) the threshold frequency for photoelectric emission from platinum, and (b) the maximum kinetic energy of a photoelectron when radiation of frequency 2.0×10^{15} Hz falls on platinum. ($h = 6.63 \times 10^{-34}$ J s.)

Answer

a. $f_0 = \frac{\phi}{h} = \frac{9.0 \times 10^{-19}}{6.63 \times 10^{-34}} =$ **1.4×10^{15} Hz** (ultraviolet).

b. $\frac{1}{2}mv_{\max}^2 = hf - \phi = 6.63 \times 10^{-34} \times 2.0 \times 10^{15} - 9.0 \times 10^{-19} = 13.3 \times 10^{-19} - 9.0 \times 10^{-19} =$ **4.3×10^{-19} J** (about 2.7 eV).

21.5 Experimental verification of Einstein's equation

Einstein's equation makes a sharp prediction: a graph of stopping potential against frequency must be a **straight line**, whose slope is the same for every metal, h/e. In 1914–1916 **Robert Millikan**, the same Millikan as the oil drops of Unit 20, tested it with great care. He expected to prove Einstein wrong. Instead he found the equation right in every detail, and the slope of his line gave Planck's constant directly.

Apparatus

The difficulty was the metal's surface: in air, alkali metals tarnish within seconds, and a tarnished surface has a different work function. So Millikan built what he called a "machine shop in a vacuum" (Figure 21.9). Inside an evacuated chamber, a wheel **D** carried three cylinders of the alkali metals sodium, potassium and lithium. A knife **K**, worked from outside by a magnet, shaved a fresh, clean surface on a cylinder; the wheel then turned it to face the window **W**. **Monochromatic** light (one wavelength at a time, from a mercury lamp's lines picked out by a spectrometer) shone through W onto the clean surface. The photoelectrons crossed to a collector **C**, a cylin-