Unit 21

Answers: Photons

Quick checks

1 b $E = hc/\lambda$ and $p = h/\lambda$: a shorter wavelength means more of both. All photons travel at the same speed in a vacuum, but their energies differ.

2 b Three times the brightness at the same frequency is three times as many photons, each still of energy hf. Each photon frees at most one electron with up to $hf - \phi$: three times the number, the same energies.

3 b At 450 nm, $hf \approx 1240/450 = 2.76$ eV. That is more than the work functions of cesium (1.9 eV) and lithium (2.5 eV), but far less than copper's 4.5 eV, which needs ultraviolet shorter than 276 nm.

4 c The slope is h/e for every metal, so the lines are parallel. A larger work function means a larger threshold frequency $f_0 = \phi/h$, so the line crosses the frequency axis further to the right.

5 c $E - W$ is what is left of the photon's energy after the work function is paid. Only the electrons at the very surface keep all of it; the rest lose some on the way out. So E_k is the kinetic energy of the emitted electrons: no electron has more.

6 b Light ("photo") frees the electrons: photoelectric emission. Thermionic emission is by heating, as in an old TV's electron gun; cold (field) emission by a very strong electric field; "electronic emission" is not the name of any process.

7 c Zinc's threshold is in the ultraviolet ($\lambda_0 \approx 1240/4.3 \approx 290$ nm): infrared and visible photons have too little energy to free any electrons at all, whatever the intensity. Only ultraviolet gives a current. (For a metal from which all three could free electrons, the same intensity of the lowest frequency would bring the most photons each second, and the comparison would come out differently: the question names the metal for that reason.)

8 a A photon is one packet (quantum) of electromagnetic energy, hf. It has no charge and no rest mass.

9 b The current depends on the number of photons arriving each second, so on the intensity. A higher frequency (shorter wavelength) gives faster electrons, not more of them.

10 a The photoelectric effect shows that light's energy arrives in quanta, hf. Quantization of charge was shown by Millikan's oil drops (Unit 20).

11 c Maximum kinetic energy $= 2 - 0.6 = 1.4$ eV, so $V_0 = 1.4$ V.

12 c $f_0 = \dfrac{\phi}{h} = \dfrac{2.51 \times 1.6 \times 10^{-19}}{6.6 \times 10^{-34}} = 6.08 \times 10^{14}$ Hz (6.06×10^{14} Hz with $h = 6.63 \times 10^{-34}$).

13 d $\frac{1}{2}mv_{\max}^2 = hf - hf_0 = 4hf_0 - hf_0 = 3hf_0$.

14 b $6.2 - 4.2 = 2.0$ eV $= 2.0 \times 1.6 \times 10^{-19} = 3.2 \times 10^{-19}$ J, about 3×10^{-19} J.

11. a. f_0: the least frequency of light that can eject electrons from a given metal; below it none are ejected, however intense the light.

V_0: the least reverse p.d. between emitter and collector that stops even the fastest photoelectrons, so that the photocurrent becomes zero; $eV_0 = \frac{1}{2}mv_{\max}^2$.

ϕ: the least energy needed to free an electron from the surface of a given metal; $\phi = hf_0$.

b. On the , light's energy is spread evenly over the wave front and grows with the intensity. Any frequency, made bright enough or shone long enough, should then give an electron the energy to escape: there should be no threshold frequency.

On the , light is absorbed as photons, each of energy hf, and one photon gives all its energy to one electron. The electron escapes only if $hf \geq \phi$, that is $f \geq \phi/h = f_0$. Below f_0, every photon has too little energy, and more photons only mean more electrons each getting too little. So there must be a threshold frequency, fixed by the metal's work function. Experiments find exactly such a threshold, so they support the quantum theory, and the wave theory can't explain them.

12. a. Radiation is absorbed as photons, each of energy hf, and one photon gives all its energy to one electron. An electron needs at least the work function ϕ to escape. Below the frequency $f_0 = \phi/h$, each photon has less energy than ϕ, so no electron can escape, however many photons arrive.

b. Any two of: emission starts , even in very dim light (a wave would need time to pour in enough energy); the of the electrons depends on the frequency and ; the maximum kinetic energy rises in a with frequency, as $hf - \phi$.

c. (i) $f_0 = \dfrac{\phi}{h} = \dfrac{3.8 \times 1.6 \times 10^{-19}}{6.63 \times 10^{-34}} = \dfrac{6.08 \times 10^{-19}}{6.63 \times 10^{-34}} =$ (ultraviolet).

ii. An electron at the surface loses nothing on the way out, so photon energy = work function + kinetic energy:

$$hf = 6.08 \times 10^{-19} + 4.5 \times 10^{-19} = 10.58 \times 10^{-19} = \frac{10.58 \times 10^{-19}}{1.6 \times 10^{-19}} = 6.6$$

d. Any two of: photocells in automatic doors and burglar alarms; the photocell that read the sound track of old film projectors; photomultiplier tubes that count single photons in scientific instruments and medical scanners; night-vision devices. (Solar cells, light meters, the switches that turn street lights on at dusk and phone camera sensors use the inner photoelectric effect, in which the electrons are freed inside a semiconductor; they are usually accepted as uses too.)

13. a. Millikan put three cylinders of alkali metal (sodium, potassium, lithium) on a wheel inside an evacuated chamber, with a knife worked from outside to shave a clean, untarnished surface on each. Monochromatic light from a mercury lamp, one spectral line at a time, fell through a window onto a clean surface; the photoelectrons crossed to a gauze collector. With a potentiometer he made the collector more and more negative compared with the metal, until a sensitive electrometer showed no current: that p.d. is the stopping potential V_0, and $eV_0 = \frac{1}{2}mv_{\max}^2$.

He repeated this for several frequencies and plotted V_0 against f. Einstein's equation, $hf = hf_0 + eV_0$, gives $V_0 = (h/e)f - hf_0/e$: a straight line, as he found. Its slope is h/e, so $h = e \times$; he got $h \approx 6.57 \times 10^{-34}$ J s. The line cuts the frequency axis at f_0, giving the work function $\phi = hf_0$.

b. Photon energy:

$$E = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^8}{5000 \times 10^{-10}} = 3.972 \times 10^{-19}$$

ii. Maximum kinetic energy: $3.972 \times 10^{-19} - 3.8 \times 10^{-19} = 0.172 \times 10^{-19} =$.

i. Stopping potential: $V_0 = \dfrac{1.72 \times 10^{-20}}{1.6 \times 10^{-19}} =$, about 0.11 V.

iii. Speed: $v_{\max} = \sqrt{\dfrac{2 \times 1.72 \times 10^{-20}}{9.1 \times 10^{-31}}} = \sqrt{3.78 \times 10^{10}} =$.

14. a. The is the ejection of electrons from a metal surface when light (or other electromagnetic radiation) of high enough frequency falls on it.

Light of frequency f consists of photons of energy hf. A photon gives all its energy to one electron. Part of it, at least the work function ϕ, frees the electron from the metal; an electron at the surface keeps the rest as kinetic energy. So

$$hf = \phi + \tfrac{1}{2}mv_{\max}^2, \qquad \phi = hf_0$$

(i) Since the kinetic energy can't be negative, emission needs $hf \geq \phi$, $f \geq f_0$: a threshold frequency, whatever the intensity. (ii) $\frac{1}{2}mv_{\max}^2 = h(f - f_0)$: the maximum kinetic energy rises in a straight line with frequency and doesn't involve the intensity at all. (iii) A more intense light of the same frequency has more photons each second; each frees at most one electron, so the number of electrons, and the current, are proportional to the intensity. (iv) A photon's energy is handed over in one go, so emission starts the moment the light arrives.

b.

$$\tfrac{1}{2}mv_{\max}^2 = hc\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = 6.62 \times 10^{-34} \times 3 \times 10^8 \left(\frac{1}{1810 \times 10^{-10}} - \frac{1}{2640 \times 10^{-10}}\right)$$

$$= 1.986 \times 10^{-25} \times (5.525 \times 10^6 - 3.788 \times 10^6) = 1.986 \times 10^{-25} \times 1.737 \times 10^6 = 3.45 \times 10^{-19}$$

In electronvolts: $3.45 \times 10^{-19}/1.6 \times 10^{-19} =$ (2.155 eV unrounded).

15.

a. As in question 14(a): light is a stream of photons of energy hf; one photon gives all its energy to one electron; at least the work function ϕ is needed to free it, and the rest is its kinetic energy, so $hf = \phi + \frac{1}{2}mv_{\max}^2$, with $\phi = hf_0$.

b. Photosynthesis begins when a photon absorbed by a chlorophyll molecule lifts one of its electrons to a higher energy, enough to start the chain of chemical reactions. That needs a photon of at least a certain energy, which red and blue light have (about 1.8 eV and 2.8 eV). An infrared photon has much less energy, too little to excite the electron. As in the photoelectric effect, the energy of one photon is what counts: piling on more infrared photons, however intense the light, cannot add their energies on one electron, so photosynthesis doesn't start.

16.

a. Power through the area: $P = 0.25 \times 10^{-3} \times 1.2 \times 10^{-4} = 3.0 \times 10^{-8}$ W.

Energy of one photon: $E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3.00 \times 10^{8}}{660 \times 10^{-9}} = 3.01 \times 10^{-19}$ J.

Photons each second: $n = \dfrac{3.0 \times 10^{-8}}{3.01 \times 10^{-19}} =$.

b. $p = \dfrac{h}{\lambda} = \dfrac{6.63 \times 10^{-34}}{660 \times 10^{-9}} =$.

c. Each photon absorbed gives the surface its momentum; force = momentum delivered each second:

$$F = np = 1.0 \times 10^{11} \times 1.0 \times 10^{-27} = 1.0 \times 10^{-16}$$

(The same as $P/c = 3.0 \times 10^{-8}/3.0 \times 10^{8}$.) A tiny force; spread over the area, a pressure of only about 10^{-12} Pa. Yet sunlight's much larger pressure, over a huge thin sheet, can push a spacecraft: Japan's IKAROS sailed by sunlight in 2010.

17.

a. The .

b. Draw the best straight line through the five points (Figure 21.13).

i. It meets the frequency axis ($E_k = 0$) at about 6.8×10^{14} Hz: the threshold frequency $f_0 \approx$.

ii. Its slope is h. From two points on the line far apart, say (10, 2.1) and (18, 7.4):

$$h = \frac{\Delta E_k}{\Delta f} = \frac{(7.4 - 2.1) \times 10^{-19}}{(18 - 10) \times 10^{14}} = \frac{5.3 \times 10^{-19}}{8 \times 10^{14}} = 6.6 \times 10^{-34}$$

(Any value from about 6.4 to 6.8×10^{-34} J s, carefully read, is fine.)

c. A lower work function means a lower threshold frequency, but the same slope h: a line to the first, to its , meeting the frequency axis at a lower frequency (dashed in Figure 21.13).

d. Only electrons right at the surface leave with all of $hf - \phi$. Electrons freed deeper in the metal use some of their energy in collisions on the way to the surface, and electrons that start below the metal's top energy level need more than ϕ to escape, so they come out with less kinetic energy, anywhere from zero up to the maximum.

18. a. A is a quantum (packet) of electromagnetic radiation, of energy $E = hf$, with no charge and no rest mass, traveling at the speed of light.

b. $E = 0.51 = 0.51 \times 10^6 \times 1.6 \times 10^{-19} = 8.16 \times 10^{-14}$ J.

i. $\lambda = \dfrac{hc}{E} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{8.16 \times 10^{-14}} =$.

ii. $p = \dfrac{h}{\lambda} = \dfrac{E}{c} = \dfrac{8.16 \times 10^{-14}}{3 \times 10^8} =$.

c. The photon's energy comes from a loss of mass, $E = \Delta m\, c^2$:

$$\Delta m = \frac{E}{c^2} = \frac{8.16 \times 10^{-14}}{(3 \times 10^8)^2} = 9.1 \times 10^{-31}$$

the mass of an electron, as it happens. (The nucleus also recoils, with momentum equal and opposite to the photon's, but its recoil energy is tiny.)

19. a. The emission of electrons from a metal surface when electromagnetic radiation of high enough frequency falls on it, each photon giving its energy to one electron.

b. $E = \dfrac{hc}{\lambda} = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{436 \times 10^{-9}} =$ (2.85 eV).

c. $\lambda_0 = \dfrac{hc}{\phi}$.

i. Cesium: $\lambda_0 = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{1.4 \times 1.6 \times 10^{-19}} = 8.9 \times 10^{-7}$ m = .

ii. Tungsten: $\lambda_0 = \dfrac{6.63 \times 10^{-34} \times 3 \times 10^8}{4.5 \times 1.6 \times 10^{-19}} = 2.8 \times 10^{-7}$ m = .

d. 436 nm is shorter than its threshold, 890 nm, so each photon (2.85 eV) has more than the 1.4 eV needed.

436 nm is longer than its threshold, 280 nm, so each photon has less than the 4.5 eV needed, and no emission happens however bright the lamp.
(The table in Section 21.4 gives 1.9 eV for cesium; measured work functions depend on how the surface is prepared. With 1.9 eV the threshold is 650 nm, and 436 nm light still frees electrons from cesium.)