Unit 20

Electrons

This unit online, with its interactives:
science-playground.com/s/12f20

Your phone is charging. In the cable, more than ten billion billion electrons pass any point every second. In the screen, each tiny pixel lights because electrons are pushed through it. On the Sun's glowing surface, sunlight is given out as electrons are knocked free of atoms and caught again. Yet no one has ever seen an electron, and until 1897 nobody even knew it existed.

This unit is about how we found out. You will follow two of the most famous experiments in physics. **J. J. Thomson** bent a beam of "cathode rays" with electric and magnetic fields and showed it was made of particles far lighter than any atom. **Robert Millikan** watched single drops of oil float between two charged plates and weighed, in effect, the charge of one electron. On the way you will work out how an electron moves in an electric field (along a parabola, like a thrown ball) and in a magnetic field (round a circle, like a stone on a string).

A STORY FROM SCIENCE

J. J. Thomson and the "corpuscles"

Joseph John Thomson (1856–1940) was born near Manchester, in England, the son of a bookseller. He meant to be an engineer, but his family could not afford the apprenticeship, so he studied mathematics and physics instead, at Owens College in Manchester and then at Trinity College, Cambridge. In 1884, at only 28, he was made head of Cambridge's Cavendish Laboratory.

For thirty years, physicists had argued about **cathode rays**, the glow that streams from the negative plate (the cathode) of a glass tube when most of the air is pumped out and a high voltage is applied. Were they a kind of light, or a stream of particles? In 1897 Thomson bent the rays with both an electric field and a magnetic field, and from the two bendings worked out their **charge-to-mass ratio**. It was more than a thousand times larger than that of a hydrogen ion, the lightest thing then known. The rays were particles, all alike, far smaller than atoms, and the same whatever metal the cathode was made of: a piece of every atom. Thomson called them **corpuscles**; the name **electron**, which the Irish physicist **George Johnstone Stoney** had proposed in 1891 for the unit of charge, soon took over.

Thomson won the Nobel Prize in Physics in 1906 for his work on how electricity passes through gases. In 1913, studying beams of positive ions with **Francis Aston**, he found that neon came as atoms of two different masses: the first evidence of **isotopes** in an element that isn't radioactive. Aston built on the method to make the mass spectrograph. Several of the young physicists who worked in his laboratory, and his own son George, went on to win Nobel Prizes too.



Figure 20.1 Sir Joseph John Thomson (18 December 1856 – 30 August 1940), in 1923.
Photo: Georges Chevalier, CC BY 4.0

20.1 Introduction

The **electron** is one of the smallest pieces of matter we know. Its mass is $m_e = 9.1 \times 10^{-31}$ kg, about 1840 times smaller than a proton's, and it carries a **negative charge** of 1.6×10^{-19} C. Every atom has electrons, held round its tiny, heavy nucleus in definite **energy levels** (shells). How those electrons are arranged decides how the atom bonds with others, and so the whole of chemistry.

Electrons also do most of the work in physics around us:

- A current in a metal wire is a drift of free electrons.
- Magnetism in iron comes from the electrons' own spin.
- Electrons behave as waves too, the key idea of quantum mechanics; the electron microscope uses this.

The electron's discovery in 1897 began modern atomic physics. In this unit you will see how its **charge** e and its **specific charge** e/m were measured, and how a beam of electrons moves in electric and magnetic fields.

Check 1

A proton and an electron have charges of the same size. Which is true?

a The electron is about 1840 times heavier than the proton.
b The proton is about 1840 times heavier than the electron.
c They have the same mass, since their charges are equal.
d The electron has no mass.

20.2 Millikan's oil drop experiment

By 1900 physicists knew the electron was negative and knew its e/m, but not e itself. In 1909 the American physicist **Robert A. Millikan**, working with his student **Harvey Fletcher** at the Ryerson Physical Laboratory of the University of Chicago, found a way to measure it. The experiment also showed that charge always comes in whole-number multiples of one smallest amount: $e = 1.6 \times 10^{-19}$ C. Millikan received the Nobel Prize in Physics in 1923.



Figure 20.2 *Millikan's own oil-drop apparatus. The two plates are inside the metal cylinder; the atomizer's rubber bulb is on the left.*
Photo: Unknown author, Public domain

Principle

A drop of oil a micrometer across, falling through air, quickly stops speeding up: the air's drag grows with its speed until it balances the drop's weight, and from then on the drop falls at a steady **terminal velocity**. **Stokes' law** gives that drag on a small sphere,

$$F_v = 6\pi\eta r v$$

where η is the viscosity of the air, r the drop's radius and v its speed. If the drop carries a charge and sits in an electric field, a fourth force joins its weight, the air's upthrust and the drag. Measuring the drop's steady speeds with and without the field gives first its size, then its charge.

Apparatus

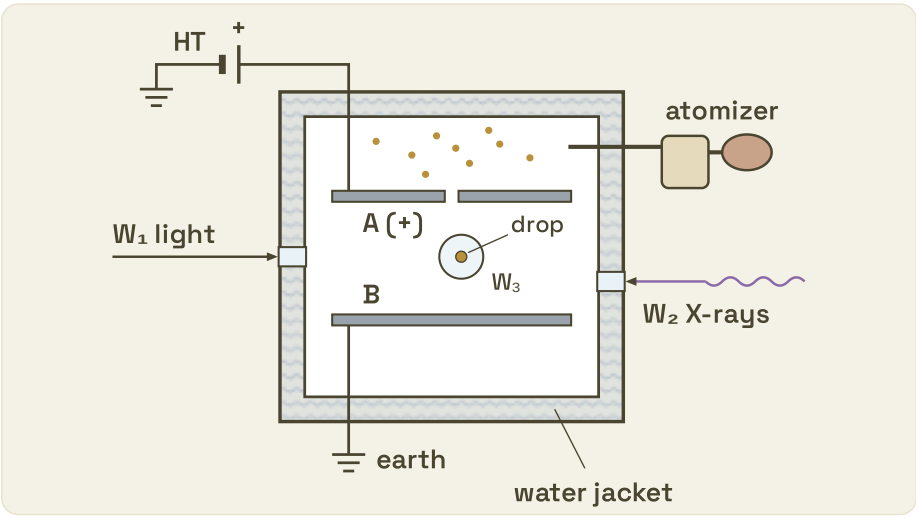


Figure 20.3 *Millikan's apparatus (a sketch, not to scale). Oil from an atomizer drifts through a small hole in the top plate A into the space between two horizontal metal plates. A is joined to the positive terminal of a high-voltage (HT) supply and B is earthed. Light enters through window W_1 and X-rays through W_2; a microscope watches the drops through W_3, a window in the front wall (drawn round the drop), at right angles to the light. The whole chamber sits in a water bath that keeps its temperature steady.*

- A closed chamber holds two large, round, horizontal metal plates, **A** on top and **B** below, about 22 cm across and a centimeter or two apart. A has a small hole in its middle.
- **A** is connected to the **positive** terminal of a high-voltage supply (a few thousand volts) and **B** is **earthed**, so between them there is a uniform field $E = V/d$, pointing down.

- An **atomizer** sprays fine drops of clock oil (an oil that hardly evaporates) above A. A few drift through the hole.
- Window $\mathbf{W_1}$ lets in light to make the drops shine against a dark background; window $\mathbf{W_2}$ lets in **X-rays**, which ionize the air between the plates, so the drops pick up (or lose) electrons; through window $\mathbf{W_3}$, at right angles to the light, a **microscope** with a scale in its eyepiece watches one drop, a bright point against the dark.
- The chamber is surrounded by a **water jacket** that keeps the temperature constant: the viscosity of air changes with temperature, and uneven warming would stir the air with currents.

A drop that has picked up extra electrons is **negative**. In the field, which points down from A to B, the force on it is **up**, towards the positive plate.

Theory

The experiment has two stages. First the drop falls with the field switched off; then it moves with the field on.

First stage: the drop falls under gravity alone

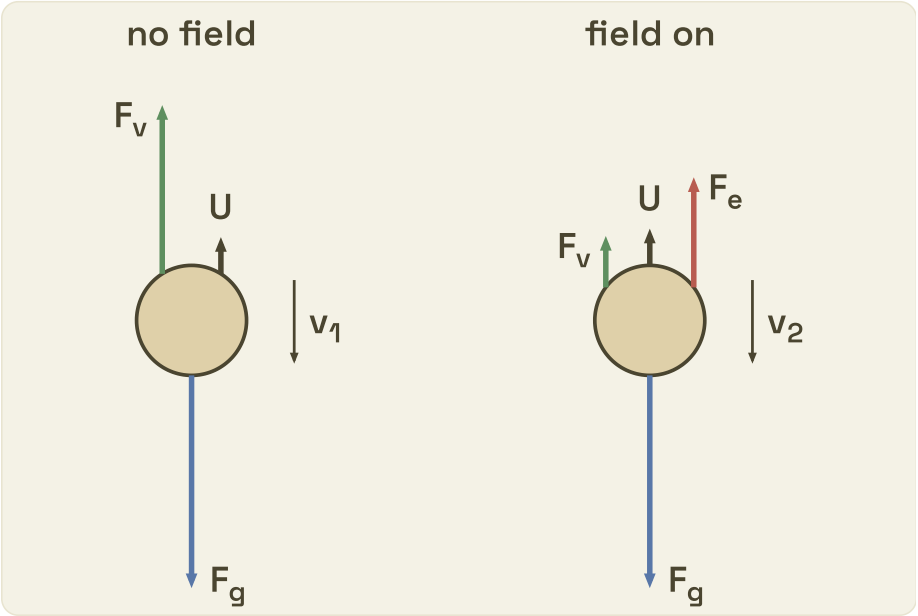


Figure 20.4 *The forces on an oil drop at its terminal velocity. Left: with no field, its weight F_g is balanced by the upthrust U and the viscous drag F_v as it falls at v_1. Right: with the field on, the electric force $F_e = qE$ also pushes up, and the drop falls more slowly, at v_2, with a smaller drag. Arrow lengths are to scale for $v2 = v1/3$, except the upthrust, really about a thousandth of the weight, drawn larger so that it shows.*

Let the drop have radius r and density ρ, and let the air have density σ and viscosity η. Three forces act on it:

- its **weight**, down: $F_g = \frac{4}{3}\pi r^3 \rho g$
- the **upthrust** of the air, up (the weight of the air it pushes aside): $U = \frac{4}{3}\pi r^3 \sigma g$
- the **viscous drag**, up, against its motion (Stokes' law): $F_v = 6\pi\eta r v$

Within a fraction of a millisecond the drop reaches its terminal velocity v_1, when the net force is zero:

1. Downward force = upward force:

$$F_g - U = F_v \qquad \text{(i)}$$

2. Put in the three forces:

$$\tfrac{4}{3}\pi r^3(\rho-\sigma)g = 6\pi\eta r v_1 \qquad \text{(ii)}$$

3. Divide by r and solve for r^2: $r^2 = \dfrac{9\eta v_1}{2(\rho-\sigma)g}$, so

$$\boxed{r = \sqrt{\frac{9\eta v_1}{2(\rho-\sigma)g}}} \qquad \text{(iii)}$$

Timing the drop as it falls between two lines of the eyepiece's scale gives v_1, and equation (iii) gives its radius: far too small to measure with a ruler, but measured all the same.

Second stage: the drop moves in the field

Now the field is switched on. The **electric force** on the drop, with charge $q = ne$ (n extra electrons), is

$$F_e = qE, \qquad E = \frac{V}{d}$$

where V is the p.d. between the plates and d their separation. It points up for a negative drop. The drop soon reaches a new, smaller terminal velocity v_2, still falling:

1. Downward force = upward forces (drag and electric force both up):

$$F_g - U = F_v + F_e$$
$$\tfrac{4}{3}\pi r^3(\rho-\sigma)g = 6\pi\eta r v_2 + qE$$

2. The left side is $6\pi\eta r v_1$, by (ii):

$$6\pi\eta r v_1 = 6\pi\eta r v_2 + qE$$

3. So

$$q = \frac{6\pi\eta r(v_1 - v_2)}{E}$$

4. Put in r from (iii):

$$\boxed{q = \frac{6\pi\eta(v_1 - v_2)}{E}\sqrt{\frac{9\eta v_1}{2(\rho-\sigma)g}}} \qquad \text{(iv)}$$

Every quantity on the right is known or measured, so q is found.

If the electric force is bigger than the drop's weight, the drop **rises** at a steady speed v_2. The drag now acts **down**, against its motion:

$$F_g - U + F_v = F_e \quad \Rightarrow \quad 6\pi\eta r v_1 + 6\pi\eta r v_2 = qE$$
$$q = \frac{6\pi\eta(v_1 + v_2)}{E}\sqrt{\frac{9\eta v_1}{2(\rho-\sigma)g}} \qquad \text{(v)}$$

If the drop is held still (the p.d. adjusted until it hangs motionless), there is no drag at all, and (leaving out the upthrust, which is about a thousand times smaller than the weight for oil in air)

$$\boxed{qE = mg} \qquad \text{(vi)}$$

Millikan watched single drops for hours, changing their charge again and again with X-rays and measuring it each time. Thousands of measurements on hundreds of drops all came out as **whole-number multiples** of one value,

$$e = 1.602 \times 10^{-19}\ \text{C}$$

the **elementary charge**: the size of the charge on one electron.

Check 2

In the second stage, a negatively charged drop falls more slowly than it did with the field off. Which way does the electric field between the plates point?

a Down, from the positive top plate to the earthed bottom plate

b Up, from the bottom plate to the top plate

c Horizontally, along the plates

d There is no field; only the drag has changed

CAREFUL

Upthrust or no upthrust?

Some questions say "neglect the upthrust of air". Then put $\sigma = 0$: $r = \sqrt{9\eta v_1/(2\rho g)}$. For oil (about 900 kg/m³) in air (1.3 kg/m³) the difference is about 0.1%, so either answer is fine unless the question gives σ.

Significance of Millikan's oil drop experiment

1. **It measured the charge of the electron.** It was the first direct and accurate measurement of the elementary charge, $e \approx 1.6 \times 10^{-19}$ C.
2. **It showed that charge is quantized.** Every charge measured was a whole-number multiple of e: $q = \pm ne$, with $n = 1, 2, 3, \ldots$. No drop ever carried $1.5e$ or $0.3e$.
3. **It supported the atomic theory.** Charge, like matter, comes in separate, countable pieces.
4. **It gave the electron's mass.** Thomson had measured $e/m = 1.76 \times 10^{11}$ C/kg (Section 20.5); with e known,

$$m = \frac{e}{e/m} = \frac{1.6 \times 10^{-19}}{1.76 \times 10^{11}} = 9.1 \times 10^{-31} \text{ kg}$$

DID YOU KNOW?

Quarks have a third of a charge, but you never see one alone

Protons and neutrons are made of **quarks**, which carry $+\frac{2}{3}e$ or $-\frac{1}{3}e$. Doesn't that break the rule? No: quarks are always locked together in groups whose charges add up to a whole number of e (two up quarks and a down quark make a proton, $+e$). No experiment has ever found a free particle with a fractional charge, though people have searched through tonnes of matter, some of them with Millikan's own method.

A STORY FROM SCIENCE

Millikan's notebooks

Millikan's 1913 paper said its results came from "all of the drops experimented upon". When historians read his notebooks in the 1970s, they found he had kept 58 drops and left out others, some because his apparatus was not working well, some for reasons he didn't write down. His value for e was about 0.6% too small, because the value of air's viscosity he used was a little wrong; it took physicists years to drift up to the right value, perhaps because each was reluctant to disagree with the famous result. Choosing data is sometimes fair (a drop that was evaporating gives bad numbers), but a scientist must say what was left out and why. That is now a firm rule of science.

Worked examples

WORKED EXAMPLE

Example 20.1: why X-rays?

What is the use of X-rays in Millikan's oil drop experiment?

Answer

The drops pick up a little charge by friction as they are sprayed from the atomizer, but many carry too little, or none. The **X-rays ionize the air** between the plates, knocking electrons off its molecules; the drops collide with these electrons and ions and so **gain or change their charge**. By switching the X-rays on for a moment, Millikan could change the charge on the same drop many times and measure each new value, which is what showed that the charge always changes in steps of e.

WORKED EXAMPLE

Example 20.2: the quantum nature of charge

How did Millikan's experiment lead to the idea that electric charge is quantized?

Answer

Every charge Millikan measured, on every drop, was a whole-number multiple of the same smallest value, $e = 1.6 \times 10^{-19}$ C:

$$q = ne, \qquad n = 1, 2, 3, \ldots$$

where n is the number of extra (or missing) electrons. Charge never came in fractions of e. A quantity that comes only in whole multiples of a smallest unit is said to be **quantized**; the experiment showed that electric charge is.

WORKED EXAMPLE

Example 20.3: counting the electrons on a drop

An oil drop of radius 10^{-6} m, carrying a charge, stays stationary between two horizontal plates in an electric field of 1.7×10^{5} V/m. The density of the oil is 2.0×10^{3} kg/m³. How many extra electrons are on the drop? (Take g = 9.8 m/s²; neglect the upthrust.)

Answer

Given: $r = 10^{-6}$ m, $E = 1.7 \times 10^{5}$ V/m, $\rho = 2.0 \times 10^{3}$ kg/m³.
The drop is still, so weight = electric force:

$$mg = qE \quad \Rightarrow \quad \tfrac{4}{3}\pi r^{3}\rho g = neE$$

$$n = \frac{4\pi r^{3}\rho g}{3eE} = \frac{4 \times 3.14 \times (10^{-6})^{3} \times 2.0 \times 10^{3} \times 9.8}{3 \times 1.6 \times 10^{-19} \times 1.7 \times 10^{5}} = \frac{2.46 \times 10^{-13}}{8.16 \times 10^{-14}} = 3.0$$

The drop carries **3 extra electrons**.

YOUR TURN

Test yourself 20.3: a drop with 20 electrons

An oil drop of mass 3×10^{-11} g and radius 2×10^{-4} cm carries 20 extra electrons. The viscosity of air is 1.8×10^{-5} N s/m². Find its terminal velocity:

a. when it falls under gravity alone;

b. in an electric field of 3×10^{5} V/m directed downward.

(Neglect the upthrust; take g = 9.8 m/s².)

Hint

Change to SI first: $m = 3 \times 10^{-14}$ kg, $r = 2 \times 10^{-6}$ m. Which way does a downward field push extra electrons?

Answer

$mg = 3 \times 10^{-14} \times 9.8 = 2.94 \times 10^{-13}$ N, and $6\pi\eta r = 6\pi \times 1.8 \times 10^{-5} \times 2 \times 10^{-6} = 6.79 \times 10^{-10}$ N s/m.

a. Weight = drag: $v_1 = \dfrac{mg}{6\pi\eta r} = \dfrac{2.94 \times 10^{-13}}{6.79 \times 10^{-10}} = \mathbf{4.3 \times 10^{-4}\ m/s}$, downward.

b. The drop is negative and the field points down, so the electric force is **up**:

$$qE = 20 \times 1.6 \times 10^{-19} \times 3 \times 10^{5} = 9.6 \times 10^{-13}\ \text{N}$$

This is more than the weight, so the drop **rises**. At its terminal velocity the drag points down: $qE = mg + 6\pi\eta r v_2$,

$$v_2 = \frac{9.6 \times 10^{-13} - 2.94 \times 10^{-13}}{6.79 \times 10^{-10}} = \frac{6.66 \times 10^{-13}}{6.79 \times 10^{-10}} = 9.8 \times 10^{-4}\ \text{m/s}$$

9.8×10^{-4} m/s, upward. (If the drop carried 20 *positive* charges, the field would push it down, and it would fall at $(2.94 + 9.6) \times 10^{-13} / 6.79 \times 10^{-10} = 1.85 \times 10^{-3}$ m/s.)

20.3 Motion of an electron beam in an electric field

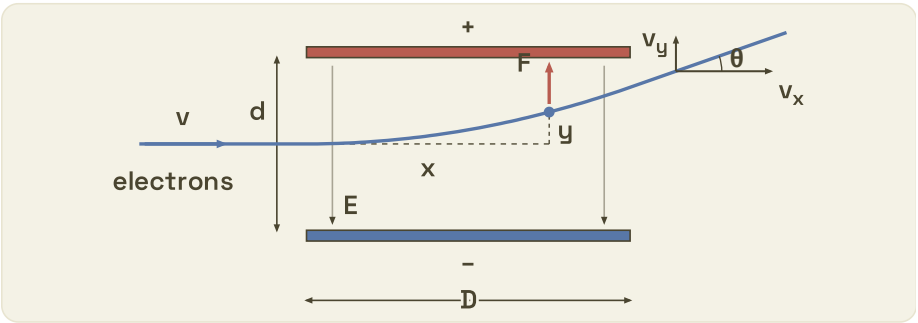


Figure 20.5 *An electron beam enters the uniform field between two plates at right angles to the field. Inside, the force $F = eE$ is constant and towards the positive plate, so the beam curves along a parabola, reaching (x, y). It leaves the plates at an angle θ, with velocity components $v_x = v$ and v_y, and goes on in a straight line. (The deflection is drawn larger than in the examples.)*

A horizontal beam of electrons, moving with speed v, enters the space between two parallel plates at right angles to their field E. The plates are a distance d apart with a p.d. V between them, so $E = V/d$, pointing down from the + plate to the − plate. The field acts over the plates' length D.

The field is vertical, so there is **no horizontal force**: the beam keeps its horizontal speed v all the way. Vertically, the force on each electron is constant,

$$F = eE \quad \text{(up, towards the + plate)}$$

so it has a constant upward acceleration

$$a = \frac{F}{m} = \frac{eE}{m} \qquad \text{(i)}$$

This is the same as a ball thrown horizontally off a cliff, with the electric force in place of gravity, and the same mathematics follows.

The path is a parabola

Let the electron enter at the origin, and reach a point (x, y) after a time t.

1. Horizontally, constant speed:

$$x = vt \quad \Rightarrow \quad t = \frac{x}{v}$$

2. Vertically, it starts with no vertical velocity ($u = 0$) and constant acceleration a:

$$y = ut + \tfrac{1}{2}at^2 = 0 + \tfrac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{x}{v}\right)^2$$

3. So

$$y = \frac{eE}{2mv^2}\,x^2 = kx^2, \qquad k = \frac{eE}{2mv^2} = \frac{eV}{2mdv^2} \qquad \text{(ii)}$$

Since e, E, m and v are constant, k is a constant, and $y = kx^2$ is the equation of a **parabola**. Once the electron leaves the plates, no force acts on it, and it travels on in a **straight line** along the parabola's tangent at the exit.

The velocity and direction as it leaves

After a horizontal distance x, the vertical velocity is

$$v_y = at = \frac{eE}{m}\cdot\frac{x}{v} = \frac{eVx}{mdv} \qquad \text{(iii)}$$

while the horizontal velocity is still $v_x = v$. The speed there is

$$v' = \sqrt{v_x^2 + v_y^2} = \sqrt{v^2 + \left(\frac{eVx}{mdv}\right)^2} \qquad \text{(iv)}$$

and the beam's direction makes an angle θ with the plates, where

$$\tan\theta = \frac{v_y}{v_x} = \frac{eVx}{mdv^2}, \qquad \theta = \tan^{-1}\left(\frac{eVx}{mdv^2}\right) \qquad \text{(v)}$$

At the far end of the plates, put $x = D$: the beam leaves deflected by $y = \dfrac{eVD^2}{2mdv^2}$, at the angle $\theta = \tan^{-1}\left(\dfrac{eVD}{mdv^2}\right)$.

> CAREFUL
> **The beam can hit a plate**
>
> A beam that enters **midway** between the plates has only $d/2$ of room on each side. If the formula gives $y > d/2$ at $x = D$, the beam doesn't come out at all: it strikes the plate before the end. Always compare your deflection with $d/2$.

In the widget the plates are 5.0 cm long and 2.0 cm apart, and the screen is 4.0 cm past them, all drawn to scale. Check your grazing setting with $y = \dfrac{eVD^2}{2mdv^2} = \dfrac{d}{2}$: it should hold to within the widget's steps.

WORKED EXAMPLE

Example 20.4: does the beam gain energy?

Will an electron beam gain kinetic energy when it enters a uniform electric field?

Answer

Yes, in general. The electric force does work on the electrons whenever they move along the direction of the force. A beam entering at right angles to the field is pushed sideways, towards the + plate, and moves that way, so the force does positive work, eEy, and its speed grows from v to $\sqrt{v^2 + v_y^2}$ (equation iv).

(A beam fired **against** the force, from the + plate towards the − plate, would instead be slowed down and lose kinetic energy. Only a magnetic force never changes a particle's energy; see 20.4.)

WORKED EXAMPLE

Example 20.5: a stronger field

How does increasing the strength of the electric field affect the acceleration of the electron beam?

Answer

The acceleration is $a = \dfrac{F}{m} = \dfrac{eE}{m}$, so $a \propto E$: doubling the field doubles the acceleration. The beam then bends more sharply ($k = eE/2mv^2$ doubles).

WORKED EXAMPLE

Example 20.6: does it get out?

In a vacuum, two flat metal plates, each 5.0 cm long, lie one above the other with a 3.0 cm gap. The top plate is kept at 400 V and the bottom one is earthed. A beam of electrons moving at 1.0×10^7 m/s is fired in horizontally, halfway between them and parallel to them. Does the beam come out from between the plates? If so, find its deflection there; if not, find where it strikes. ($e/m = 1.76 \times 10^{11}$ C/kg.)

The textbook asks for the deflection as the beam emerges, but with these numbers the beam strikes the plate (as shown below), so the book's numbers don't work. Exercise question 19b is the same question with numbers that do: 4.0 cm plates at 300 V.

Answer

Given: $D = 5.0 \times 10^{-2}$ m, $d = 3.0 \times 10^{-2}$ m, $V = 400$ V, $v = 1.0 \times 10^7$ m/s, $e/m = 1.76 \times 10^{11}$ C/kg.

Deflection the beam would have at the end of the plates:

$$y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{D}{v}\right)^2 = \frac{e}{m} \cdot \frac{VD^2}{2dv^2} = \frac{1.76 \times 10^{11} \times 400 \times (5.0 \times 10^{-2})^2}{2 \times 3.0 \times 10^{-2} \times (1.0 \times 10^7)^2} = 2.9 \times 10^{-2} \text{ m}$$

But the beam starts midway, only $d/2 = 1.5$ cm below the upper plate. It cannot be deflected 2.9 cm: it **strikes the upper (positive) plate**. It does so where $y = 1.5 \times 10^{-2}$ m:

$$x = \sqrt{\frac{2dv^2\, y}{(e/m)V}} = \sqrt{\frac{2 \times 3.0 \times 10^{-2} \times 10^{14} \times 1.5 \times 10^{-2}}{1.76 \times 10^{11} \times 400}} = \sqrt{1.28 \times 10^{-3}} = 3.6 \times 10^{-2} \text{ m}$$

The beam hits the upper plate **3.6 cm** from the entrance, before reaching the end.

YOUR TURN

Test yourself 20.6: just grazing

A beam of electrons moving at 1.0×10^7 m/s enters midway between two horizontal plates, parallel to them. The plates are 5.0 cm long and 3.0 cm apart, with a p.d. V between them. Find V if the beam just grazes the edge of the upper plate. ($e/m = 1.8 \times 10^{11}$ C/kg.)

Hint

"Just grazes the edge" means the deflection at $x = D$ is exactly $d/2$.

Answer

$$\frac{d}{2} = \frac{e}{m} \cdot \frac{VD^2}{2dv^2} \quad \Rightarrow \quad V = \frac{d^2v^2}{(e/m)D^2} = \frac{(3.0 \times 10^{-2})^2 \times (1.0 \times 10^7)^2}{1.8 \times 10^{11} \times (5.0 \times 10^{-2})^2} = \frac{9.0 \times 10^{10}}{4.5 \times 10^8} = \mathbf{200\ V}$$

20.4 Motion of an electron beam in a magnetic field

A charge moving through a magnetic field feels a force, the **magnetic Lorentz force**. For an electron (charge $-e$) moving with velocity $\vec{v}$ in a field $\vec{B}$,

$$\vec{F} = -e\left(\vec{v} \times \vec{B}\right), \qquad F = evB\sin\theta$$

where θ is the angle between $\vec{v}$ and $\vec{B}$. The force is always **at right angles to both** the velocity and the field. Its direction is given by **Fleming's left-hand rule** (for the conventional current, which flows the **opposite** way to the electrons) or by the right-hand palm rule.

CAREFUL

Electrons: point the current backwards

Fleming's left-hand rule gives the force on a positive charge moving in the direction your second finger points. For an electron, point the second finger **opposite** to the electron's motion, because a stream of electrons moving right is a conventional current to the left. Forgetting this sends the electron the wrong way round its circle.

Because the force is always at right angles to the velocity, it **changes the electron's direction but never its speed**: it does no work, so the electron's kinetic energy stays the same.

Entering at right angles: a circle

Let the beam enter with speed v at right angles to a uniform field B. The force on each electron has the constant size

$$F = evB \qquad \text{(i)}$$

and is always at right angles to its velocity. A force of constant size always at right angles to the motion is exactly what keeps a stone on a string going round in a circle (Grade 11, 6.4 Centripetal force). So the electron moves in a **circle** of radius r, and the Lorentz force provides the centripetal force:

1. Lorentz force = centripetal force:

$$Bev = \frac{mv^2}{r}$$

2. Cancel one v and solve for r:

$$\boxed{r = \frac{mv}{Be}} \qquad \text{(ii)}$$

A faster electron goes round a bigger circle; a stronger field makes a smaller one.

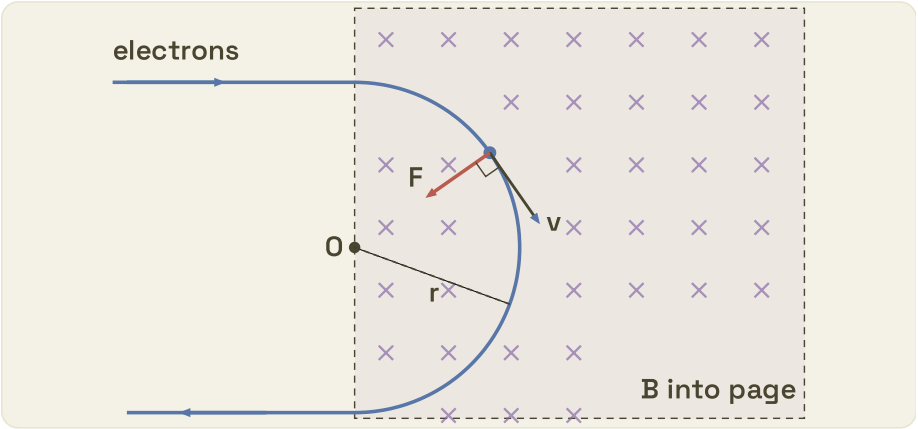


Figure 20.6 *An electron beam enters a uniform magnetic field (into the page, ×) at right angles to it. The force $F = evB$ is always at right angles to the velocity, towards the center, so the beam goes round a circle of radius r. Outside the field, it goes straight on.*

The period. The time for one full turn is the circumference divided by the speed:

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v} \cdot \frac{mv}{Be} = \boxed{\frac{2\pi m}{Be}} \qquad \text{(iii)}$$

The speed has canceled: **the period does not depend on the speed or on the radius**. A faster electron goes round a bigger circle in exactly the same time. The **frequency** of revolution and the **angular frequency** are

$$f = \frac{1}{T} = \frac{Be}{2\pi m} \qquad \text{(iv)} \qquad \omega = 2\pi f = \frac{Be}{m}$$

This frequency, the **cyclotron frequency**, is what a cyclotron (a machine that speeds up protons for treating cancer, among other uses) is tuned to: it pushes its particles once each half turn, and because the period stays the same as they speed up, one fixed frequency keeps in step with all of them.

At other angles: a straight line or a helix

- **Velocity along the field** ($\theta = 0°$) or exactly against it ($\theta = 180°$): $\sin\theta = 0$, so $F = 0$. The beam goes straight through, **undeflected**.
- **Any other angle**: split the velocity into a part along the field, $v\cos\theta$, and a part across it, $v\sin\theta$. The part along the field feels no force and stays the same; the part across it makes a circle of radius $r = \dfrac{mv\sin\theta}{Be}$. Together the electron moves along a **helix** (a spiral like a spring's), going round at $v\sin\theta$ while it moves along the field at $v\cos\theta$. In each period it moves forward a distance (the **pitch**) $p = v\cos\theta \times T = \dfrac{2\pi mv\cos\theta}{Be}$.

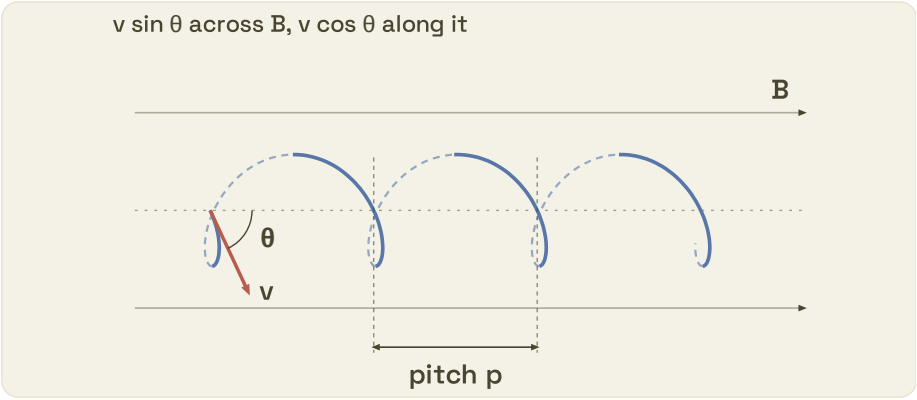


Figure 20.7 *An electron entering at an angle θ to a uniform field B goes round a helix: it circles at v sin θ while it drifts along the field at v cos θ, one pitch p per turn.*

DID YOU KNOW?

The northern lights

Electrons from the Sun's wind, trapped and speeded up in the space round the Earth, spiral in helices along the Earth's magnetic field lines, down towards the poles. When they hit the air 100 to 300 km up, they make it glow green and red: the **aurora**. Near the poles the field lines point almost straight down into the ground, which is why the aurora is seen mostly there. In May 2024, during the strongest solar storm in twenty years, a red aurora was photographed low on the northern horizon from Hanle in Ladakh, India, only a few degrees north of Nepal: an extremely rare sight so far south.

WORKED EXAMPLE

Example 20.7: an electron at rest

An electron at rest in a uniform magnetic field feels no force. Explain.

Answer

The magnetic force on an electron is $F = evB\sin\theta$. An electron at rest has $v = 0$, so $F = 0$, however strong the field. A magnetic field acts only on **moving** charges. (An electric field, by contrast, pushes on a charge whether it moves or not.)

WORKED EXAMPLE

Example 20.8: whose circle is smaller?

An electron and a proton moving at the same speed enter the same magnetic field at right angles to it. Which goes round the smaller circle?

Answer

$r = \dfrac{mv}{Be}$. Both have charges of size e, the same v and the same B, so $r \propto m$. The electron's mass is about 1840 times smaller, so the **electron** goes round the smaller circle, about 1840 times smaller. (They also go round in **opposite** directions, since their charges are opposite.)

WORKED EXAMPLE

Example 20.9: speed, period and angular frequency

An electron beam moves in a circle of radius 4 cm after entering a uniform magnetic field of 0.01 T at right angles. Find its orbital speed, its period of revolution and its angular frequency.

Answer

Given: $r = 4 \times 10^{-2}$ m, $B = 0.01$ T, $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg.
Speed, from $r = mv/(Be)$:

$$v = \frac{eBr}{m} = \frac{1.6 \times 10^{-19} \times 0.01 \times 4 \times 10^{-2}}{9.1 \times 10^{-31}} = 7.0 \times 10^{7} \text{ m/s}$$

Period:

$$T = \frac{2\pi m}{Be} = \frac{2\pi \times 9.1 \times 10^{-31}}{0.01 \times 1.6 \times 10^{-19}} = 3.6 \times 10^{-9} \text{ s}$$

Angular frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{3.6 \times 10^{-9}} = 1.7 \times 10^{9} \text{ rad/s}$$

(Check: $\omega = v/r = 7.0 \times 10^{7}/0.04 = 1.75 \times 10^{9}$ rad/s.) The electron goes round almost 300 million times a second.

YOUR TURN

Test yourself 20.9: energy, momentum and frequency

An electron moves at 3×10^{6} m/s at right angles to a magnetic field of 0.1 T. Calculate a. its kinetic energy, b. its momentum, c. its frequency of revolution, and d. the radius of its path.

The textbook also gives the radius as 0.02 m, but an electron at this speed in 0.1 T goes round a circle of about 0.17 mm (part d), so the book's numbers don't fit together; the radius is worked out here from the speed instead.

Answer

a. $K = \frac{1}{2}mv^2 = \frac{1}{2} \times 9.1 \times 10^{-31} \times (3 \times 10^{6})^2 = \mathbf{4.1 \times 10^{-18}\ J}$ (about 26 eV).

b. $p = mv = 9.1 \times 10^{-31} \times 3 \times 10^{6} = \mathbf{2.7 \times 10^{-24}\ kg\ m/s}$.

c. $f = \dfrac{Be}{2\pi m} = \dfrac{0.1 \times 1.6 \times 10^{-19}}{2\pi \times 9.1 \times 10^{-31}} = \mathbf{2.8 \times 10^{9}\ Hz}$.

d. $r = \dfrac{mv}{Be} = \dfrac{2.7 \times 10^{-24}}{0.1 \times 1.6 \times 10^{-19}} = \mathbf{1.7 \times 10^{-4}\ m}$ (0.17 mm).

YOUR TURN

Activity 20.3: the radius of a heavy ion's path

A charged particle of mass 4×10^{-25} kg and charge -1.6×10^{-16} C moves at 7.5×10^{6} m/s at right angles to a magnetic field of 2 T. Find the radius of its path. Then use the widget above to see how the radius would change if the speed were doubled, or the field doubled.

Answer

$$r = \frac{mv}{qB} = \frac{4 \times 10^{-25} \times 7.5 \times 10^{6}}{1.6 \times 10^{-16} \times 2} = \frac{3.0 \times 10^{-18}}{3.2 \times 10^{-16}} = 9.4 \times 10^{-3} \text{ m} \approx 9.4 \text{ mm}$$

Doubling the speed doubles the radius ($r \propto v$); doubling the field halves it ($r \propto 1/B$).

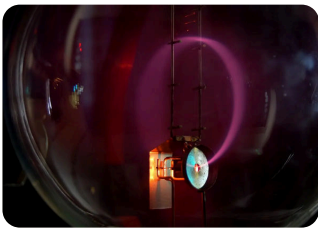


Figure 20.8 *A beam of electrons bent into a circle by a magnetic field, in a "fine beam tube". The tube holds a trace of gas, which glows where the electrons pass.*
Photo: Marcin Białek, CC BY-SA 4.0

20.5 Thomson's experiment to find the specific charge of the electron

The **specific charge** of a particle is its **charge per unit mass**, e/m, in C/kg. In 1897 J. J. Thomson measured it for the electron.

Principle. A beam of electrons is passed through an electric field and a magnetic field at right angles to each other and to the beam (**crossed fields**). Each field alone deflects the beam; the fields are adjusted until their forces **cancel**, and the beam goes straight through as if no force acted. That balance gives the electrons' speed; the voltage that accelerated them then gives e/m.

Apparatus

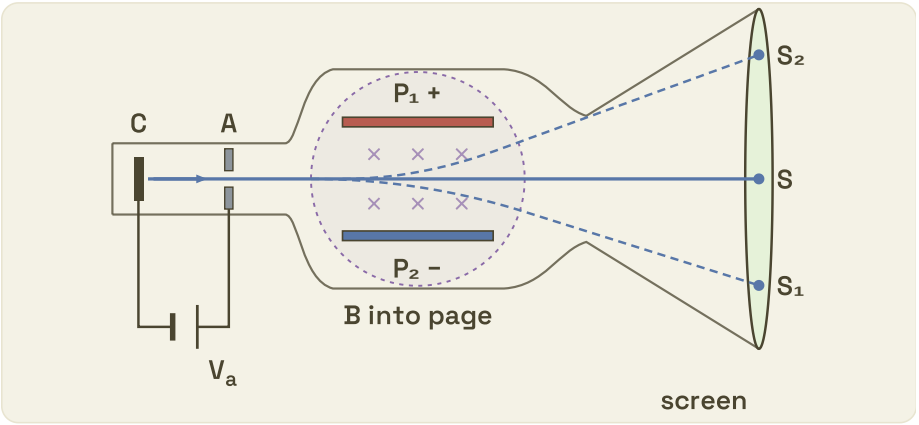


Figure 20.9 *Thomson's apparatus (a sketch). Electrons from the cathode C are accelerated to the anode A by the p.d. V_a and pass through a hole in it as a narrow beam. Between the plates P_1 (+) and P_2 (−), a p.d. V_p makes an electric field E; coils outside the tube (dotted circle) make a magnetic field B into the page over the same region. The beam strikes the fluorescent screen at S with no field, at S_1 with the magnetic field alone, and at S_2 with the electric field alone.*

- A glass tube is pumped down to a very low pressure (about 0.01 mm of mercury), so the electrons rarely hit a gas molecule.
- At one end, a high p.d. V_a between the **cathode C** and the **anode A** (from an induction coil in Thomson's day) accelerates electrons from the cathode. A narrow beam passes through a hole in the anode (and a second slit, to make it narrower).
- The beam passes between two parallel plates $\mathbf{P_1}$ and $\mathbf{P_2}$, a distance d apart, with a p.d. V_p: a uniform electric field $E = V_p/d$, from P_1 (+) to P_2 (−).

- Coils outside the tube make a uniform **magnetic field** B over the same region, at right angles to the electric field and to the beam (into the page in the figure).
- The beam strikes a **fluorescent screen** at the far end, which glows where it is hit.

Theory

1. **No fields.** The beam goes straight and hits the screen at **S**.
2. **Magnetic field alone.** The beam is bent **down** and hits the screen at $\mathbf{S_1}$. The magnetic force on each electron is

$$F_m = evB$$

3. **Electric field alone.** The beam is bent **up**, towards the positive plate, and hits at $\mathbf{S_2}$. The electric force is

$$F_e = eE, \qquad E = \frac{V_p}{d}$$

4. **Both fields together.** Thomson adjusted the fields until the spot came back to **S**: the two forces are equal and opposite.

$$F_m = F_e \quad \Rightarrow \quad evB = eE \quad \Rightarrow \quad \boxed{v = \frac{E}{B} = \frac{V_p}{dB}} \qquad \text{(i)}$$

Notice that e cancels: the balance gives the speed of **any** charged particle, whatever its charge or mass. A pair of crossed fields used like this is called a **velocity selector**.

5. **The accelerating p.d.** In falling through the p.d. V_a from cathode to anode, each electron loses potential energy eV_a and gains the same kinetic energy (it starts almost at rest):

$$\tfrac{1}{2}mv^2 = eV_a \quad \Rightarrow \quad \frac{e}{m} = \frac{v^2}{2V_a}$$

6. Put in $v = E/B$:

$$\boxed{\frac{e}{m} = \frac{E^2}{2V_aB^2} = \frac{V_p^2}{2V_a d^2 B^2}} \qquad \text{(ii)}$$

Knowing V_p, d, B and V_a, Thomson found e/m. Today's value is

$$\frac{e}{m} = 1.76 \times 10^{11}\ \text{C/kg}$$

(Thomson's own measurements in 1897 were rougher, but already showed e/m about a thousand times larger than for a hydrogen ion.)

CAREFUL

Two different voltages

There are two voltages in Thomson's experiment: V_a, which **accelerates** the electrons from cathode to anode, and V_p, across the **deflecting plates**, which makes $E = V_p/d$. Some books write both as V and end with "$e/m = V/2d^2B^2$", which is right only if one and the same p.d. both accelerates the electrons and sits across the plates. Keep the two apart:

$$\frac{e}{m} = \frac{E^2}{2V_aB^2}$$

Check 3

In a velocity selector, electrons moving at exactly $v = E/B$ go straight through. A slower electron, entering the same crossed fields, is bent:

a towards the side the electric force pushes it
b towards the side the magnetic force pushes it
c not at all, since the forces still balance
d round a full circle

WORKED EXAMPLE

Example 20.10: why crossed fields?

In Thomson's method, why are the electric and magnetic fields at right angles to each other? What does the ratio of the fields give?

Answer

With the fields at right angles to each other (and both at right angles to the beam), the electric force and the magnetic force on an electron lie along the **same line, in opposite directions**. So they can be made to cancel exactly: the beam then goes straight, and the balance $eE = evB$ gives the electrons' speed. (Parallel fields would push the beam in two different directions, at right angles, and could never cancel.)

The ratio of the fields gives the **speed** of the electrons:

$$v = \frac{E}{B}$$

WORKED EXAMPLE

Example 20.11: negative particles

How does Thomson's experiment show that cathode rays are made of negatively charged particles?

Answer

The rays are bent by an electric field and by a magnetic field, which a beam of light is not: so they are **charged particles**. In the electric field they bend **towards the positive plate**, and in the magnetic field the way Fleming's left-hand rule gives for a current flowing **backwards** along the beam: so the charge is **negative**. Thomson also found the same e/m whatever the cathode was made of, so the particles are the same in every metal.

WORKED EXAMPLE

Example 20.12: e/m from the circle

In a Thomson-type experiment, electrons move at 2×10^7 m/s. In a magnetic field of 1×10^{-2} T alone, they go round a circle of radius 0.01 m. Find the specific charge of the electron.

Answer

Given: $v = 2 \times 10^7$ m/s, $B = 1 \times 10^{-2}$ T, $r = 0.01$ m.

From $r = \dfrac{mv}{eB}$:

$$\frac{e}{m} = \frac{v}{Br} = \frac{2 \times 10^7}{1 \times 10^{-2} \times 0.01} = 2 \times 10^{11} \text{ C/kg}$$

(The speed comes from the crossed fields, $v = E/B$; the circle in B alone then gives e/m. This is another way of finding it. The rounded data give 2×10^{11}, close to the true 1.76×10^{11} C/kg.)

YOUR TURN

Test yourself 20.12: straight through?

An electron moves through perpendicular electric and magnetic fields, $E = 5 \times 10^3$ V/m and $B = 5 \times 10^{-3}$ T. Is it undeflected if its speed is 1×10^6 m/s?

Answer

The speed that goes straight through is

$$v = \frac{E}{B} = \frac{5 \times 10^3}{5 \times 10^{-3}} = 1 \times 10^6 \text{ m/s}$$

which is exactly the electron's speed: **yes, it goes through undeflected**.

DID YOU KNOW?

The old television and the electron gun

Until about 2010, almost every television in Nepal was a **cathode ray tube** (CRT): a big glass bottle with electron guns at its narrow end, firing beams at a screen coated with phosphors that glowed red, green or blue where hit. Coils round the tube's neck swept the beams across the screen 15,625 times a second, line by line, painting a whole picture 25 times a second. Thomson's tube, deflection and all, sat in every living room for half a century.

IN SHORT

- **Electron:** charge $-e = -1.6 \times 10^{-19}$ C, mass 9.1×10^{-31} kg, specific charge $e/m = 1.76 \times 10^{11}$ C/kg.
- **Millikan:** with no field, $r = \sqrt{\dfrac{9\eta v_1}{2(\rho - \sigma)g}}$; falling in the field, $q = \dfrac{6\pi\eta r(v_1 - v_2)}{E}$; rising, $q = \dfrac{6\pi\eta r(v_1 + v_2)}{E}$; held still, $qE = mg$. Every charge is $q = ne$: **charge is quantized**.
- **Electric field, beam entering at right angles:** $a = eE/m$; path $y = \dfrac{eE}{2mv^2}x^2$, a **parabola**; leaves at $\tan\theta = \dfrac{eVx}{mdv^2}$, then straight.
- **Magnetic field:** $F = evB\sin\theta$, at right angles to v: changes direction, never speed. At right angles: a **circle**, $r = \dfrac{mv}{Be}$, $T = \dfrac{2\pi m}{Be}$ (independent of speed), $f = \dfrac{Be}{2\pi m}$. Along the field: straight. Other angles: a **helix**.
- **Thomson:** crossed fields balanced, $v = E/B$; with $\frac{1}{2}mv^2 = eV_a$, $\dfrac{e}{m} = \dfrac{E^2}{2V_aB^2}$.

Exercise

Choose the correct option

Check 4

1. What is the kinetic energy of an electron accelerated through a p.d. of 100 V?

a 1 eV
b 100 eV
c 1.6×10^{-18} J
d 100×10^{-18} J

Check 5

2. In J. J. Thomson's experiment, the charge-to-mass ratio of the electron was found using:

a only an electric field
b only a magnetic field
c crossed electric and magnetic fields
d electric and magnetic fields parallel to each other

Check 6

3. Which property of the electron was established by Millikan's oil drop experiment?

a mass
b charge
c speed
d spin

Check 7

4. In Thomson's experiment, the magnetic field is doubled while the electric field is kept the same. For the beam still to go undeflected, the speed of the electrons must be:

a halved
b doubled
c unchanged
d reduced to one-fourth

Check 8

5. In Millikan's experiment, the electric field is made stronger, and a drop still stays suspended. Compared with a drop held still in the weaker field, its charge must be:

a larger
b smaller
c the same, but its mass has increased
d larger than any charge measured before

Check 9

6. An electron moves in a straight line through perpendicular electric and magnetic fields. If the electric field is suddenly switched off, the electron will:

a stop moving
b continue in a straight line
c move in a circular path
d oscillate back and forth

Check 10

7. In a CRT, the deflection on the screen is greater with the electric field alone than with the magnetic field alone. What can you conclude?

a The magnetic field is stronger.
b The electric field is weaker.
c The electrons' speed is less than E/B.
d The magnetic force is larger.

Check 11

8. In Millikan's experiment, an oil drop carrying a charge Q is held stationary by a p.d. of 2400 V between the plates. To hold a drop of half the radius stationary, the p.d. has to be 600 V. The charge on the second drop is:

a 3Q/2
b Q/2
c Q/4
d Q

Check 12

9. A Thomson tube run at 2 kV gives an electron beam a speed of 6×10^6 m/s. What is the speed when it is run at 8 kV?

a 2.4×10^7 m/s
b 3.6×10^7 m/s
c 4.8×10^7 m/s
d 1.2×10^7 m/s

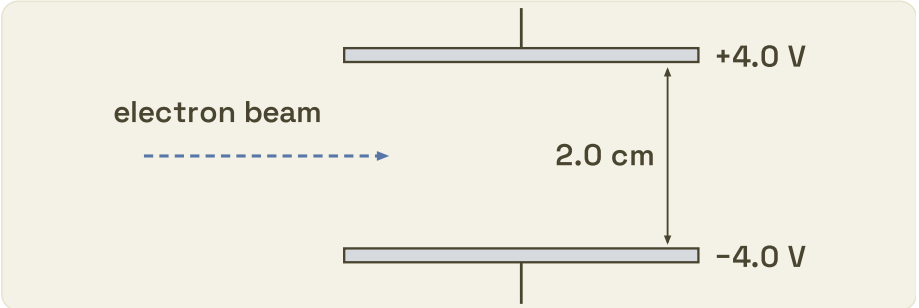


Figure 20.10 *For question 10: a horizontal electron beam between two horizontal plates 2.0 cm apart, the upper at +4.0 V and the lower at −4.0 V.*

Check 13

10. A horizontal beam of electrons passes between two horizontal parallel plates 2.0 cm apart, as shown. The upper plate is at +4.0 V and the lower plate at −4.0 V. What is the force on each electron between the plates?

a 3.2×10^{-17} N downwards
b 3.2×10^{-17} N upwards
c 6.4×10^{-17} N downwards
d 6.4×10^{-17} N upwards

Answer these questions

11.

a. Discuss the motion of an electron in a uniform magnetic field, and show that if a free electron moves at right angles to the field, its path is a circle and its period of revolution does not depend on its speed.

b. Prove that the path of a horizontal beam of electrons entering midway between two oppositely charged plates is a parabola.

c. A beam of electrons moving at 10^7 m/s enters midway between two horizontal parallel plates, in a direction parallel to the plates. Each plate is 5 cm long; they are 2 cm apart, with a p.d. of 90 V between them. Show that the beam just grazes the edge of the positive plate, and find its speed as it does. ($e/m = 1.8 \times 10^{11}$ C/kg.)

12.

a. What is the specific charge of an electron? Describe J. J. Thomson's experiment to find it.

b. An oil drop of density 960 kg/m^3 and radius 10^{-7} m is driven vertically upward through air by an electric field of 1.95×10^4 V/m, at a constant velocity of 3.5×10^{-4} m/s. The viscosity of air is 1.8×10^{-5} N s/m^2. Find the number of electronic charges on the drop. (Neglect the upthrust of the air.)

13.

a. Using a sketch of Millikan's apparatus (Figure 20.3), draw free-body diagrams for an oil drop moving down at a constant speed, first with no electric field and then in an electric field, showing the forces on it.

b. Find an expression for the charge on an oil drop of radius r which falls at a constant velocity v_1 with no field and at a constant velocity v_2 in a field E.

c. Find the charge on an oil drop of radius 1.5×10^{-6} m whose terminal velocity changes from 0.25 mm/s to 0.10 mm/s (still falling) when an electric field of 10^5 V/m is switched on. The viscosity of air is 1.8×10^{-5} N s/m².

The textbook gives the radius as 1.5×10^{-6} mm, which would give a charge of 7.6×10^{-22} C, smaller than one electron's, so the book's numbers don't work. A drop that falls at 0.25 mm/s has a radius of about 1.5×10^{-6} m (equation iii), which is used here.

14.

a. What is meant by quantization of charge?

b. In Millikan's experiment, an oil drop of weight 1.5×10^{-14} N is held stationary between plates 10 mm apart by a p.d. of 470 V.

i. State the condition for the drop to stay stationary, and sketch the forces on it.

ii. Calculate the charge on the drop.

iii. Explain what would happen if the drop were suddenly struck by a stray alpha particle.

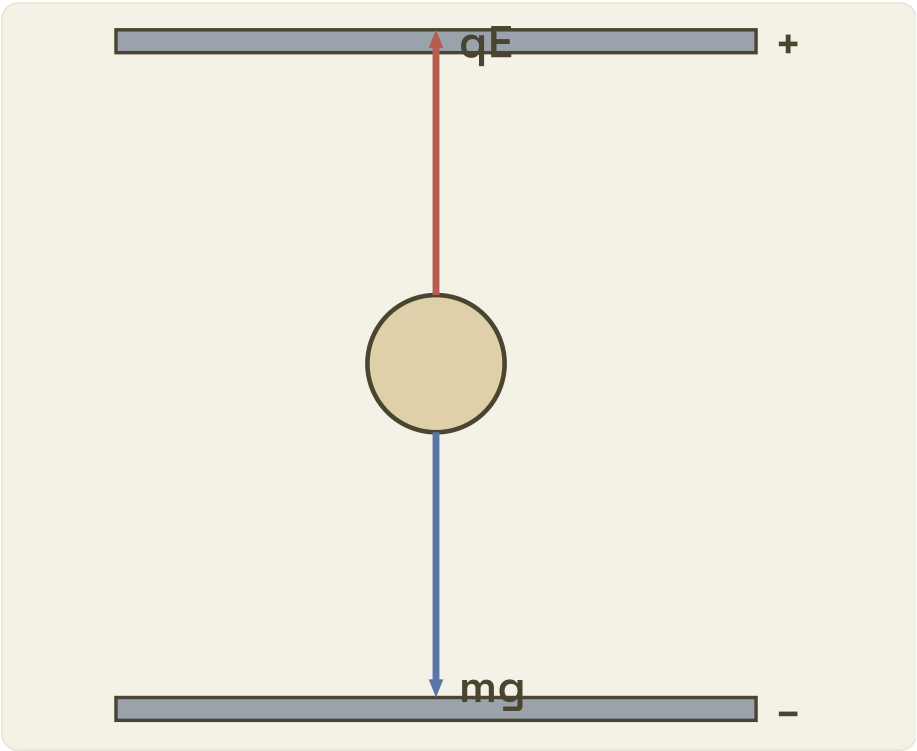


Figure 20.11 *For question 14: a drop held still. Its weight mg is balanced by the electric force qE; with no motion there is no drag.*

15. A beam of electrons passes through crossed fields: a p.d. of 1.36×10^{4} V across two parallel plates 4 cm apart, and a magnetic field of 2×10^{-3} T, at right angles to each other. (Mass of the electron: 9.1×10^{-31} kg.)

    a. State the condition for the beam to go undeflected, and sketch the forces on an electron.

    b. If the two fields produce no deflection, find the speed of the electrons.

    c. Explain what happens to the beam if the electric field is made zero.

16. In Millikan's apparatus, a charged oil drop is balanced between two parallel plates by adjusting the electric field.

    a. What is the use of X-rays in this experiment? Also discuss the physical principle involved.

    b. What happens if a falling drop suddenly experiences zero net force?

    c. Name the forces that play an important part in the drop's motion through the air, and sketch them.

    d. A drop falls with a terminal velocity of 2.0×10^{-4} m/s in air. The density of the oil is 920 kg/m³ and the viscosity of air 1.8×10^{-5} N s/m². Find the radius of the drop. ($g = 9.8$ m/s²; neglect the upthrust.)

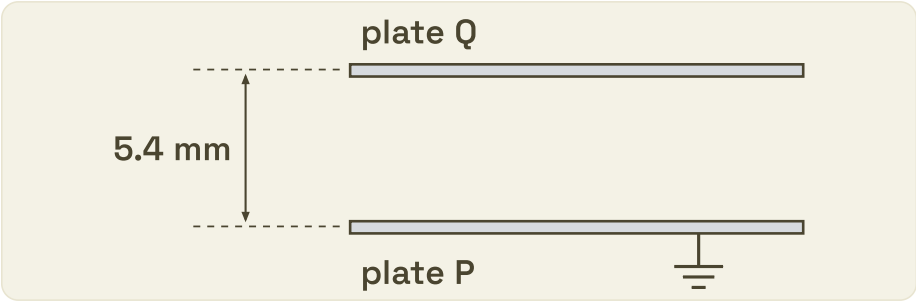


Figure 20.12 *For question 17: two horizontal plates in a vacuum, Q above and P below, 5.4 mm apart; P is earthed.*

17. Millikan's oil drop experiment found the charge of the electron.

a. State a fundamental property of charge that this experiment suggested.

b. Two parallel metal plates P and Q are in a vacuum, horizontal, 5.4 mm apart, as in the figure. The lower plate P is earthed; the p.d. between the plates can be varied. An oil droplet of mass 7.7×10^{-15} kg stays stationary between the plates when Q is at +850 V.

i. Suggest why the plates P and Q must be parallel and horizontal.

ii. Calculate the charge, with its sign, on the droplet.

c. The procedure in b was repeated for three more droplets. The sizes of their charges were 3.2×10^{-19} C, 6.4×10^{-19} C and 3.2×10^{-19} C. What value do these data and your answer to b ii suggest for the charge of the electron? Explain.

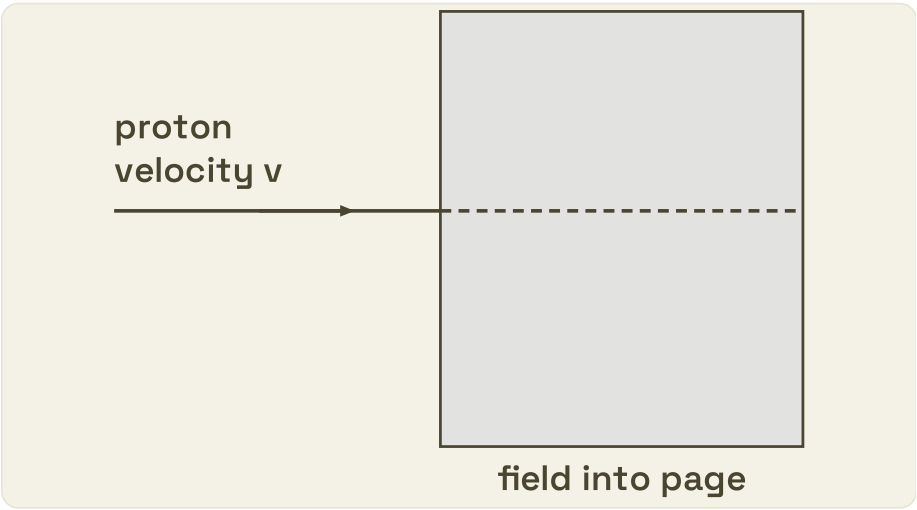


Figure 20.13 *For question 18: a proton moving with velocity v enters a uniform magnetic field directed into the plane of the page (shaded), at right angles to the field.*

18. A proton moving with constant velocity v enters a uniform magnetic field that is at right angles to its motion, as in the figure. A uniform electric field is applied in the same region so that the proton passes through undeviated.

    a. On a copy of the figure, draw an arrow labeled E to show the direction of the electric field.

    b. The proton is replaced by other particles; the fields stay the same. State and explain the deviation, if any, of:

    i. an alpha particle with initial velocity v;

    ii. an electron with initial velocity $2v$.

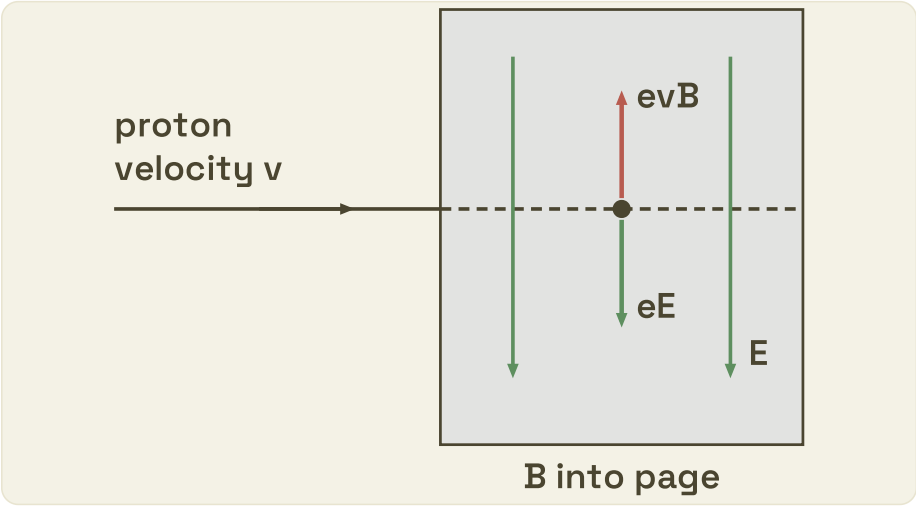


Figure 20.14 *Answer to question 18a: the magnetic force on the proton is up, so the electric field E points down the page, making an equal electric force down.*

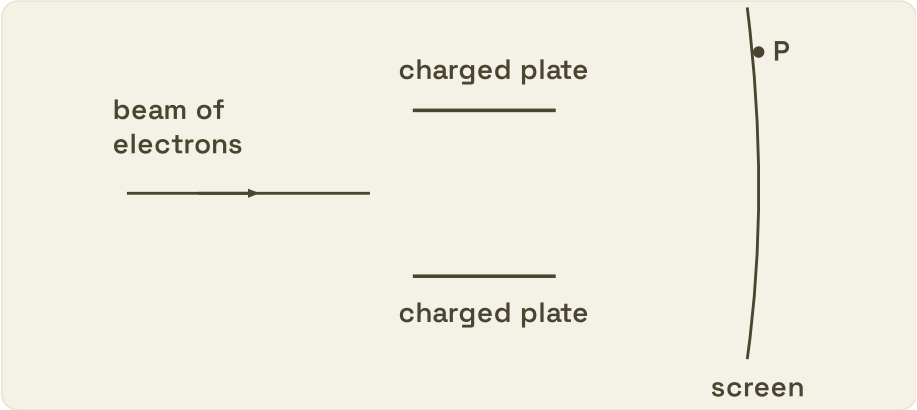


Figure 20.15 *For question 19: a beam of electrons passes between two charged plates and hits a screen at P, above the beam's starting line.*

19. The figure shows a beam of electrons, two charged plates and a screen, inside an electron tube (not drawn). The beam hits the screen at the point P.

a. On a copy of the figure: i. complete the path of the electron beam; ii. mark the charges on the two plates; iii. mark with an arrow labeled C the direction of the conventional current in the beam.

b. Two plane metal plates 4.0 cm long are held horizontally 3.0 cm apart in a vacuum, one above the other. The upper plate is at a potential of 300 V and the lower plate is earthed. Electrons with a speed of 1.0×10^7 m/s are injected horizontally midway between the plates, along their length. Calculate the vertical deflection of the beam as it leaves the plates. ($e/m = 1.8 \times 10^{11}$ C/kg.)

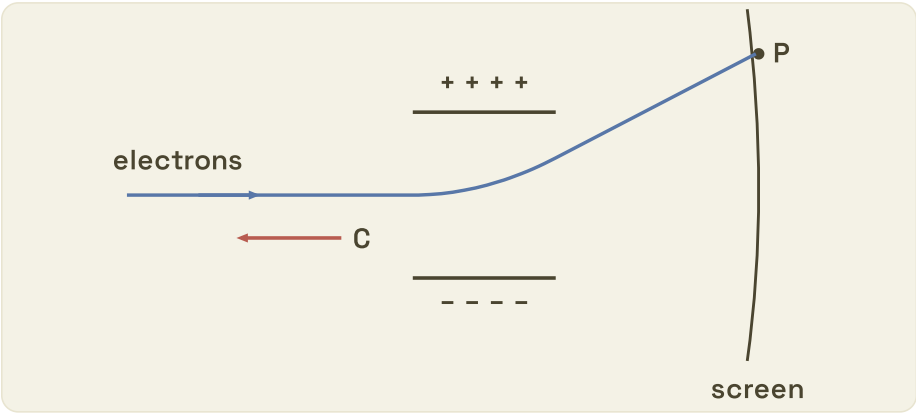


Figure 20.16 *Answer to question 19a: the beam curves up between the plates (upper +, lower −) along a parabola, then goes straight to P. The conventional current C flows back along the beam.*

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.