Unit 20

Answers: Electrons

Quick checks

1 b $m_p/m_e = 1.67 \times 10^{-27}/9.1 \times 10^{-31} \approx 1840$. Equal charges do not mean equal masses: the size of a particle's charge and its mass are quite separate properties.

2 a The drop slows, so the electric force on it is **up**. A negative charge is pushed **against** the field, so the field points **down**, from the + plate A to the earthed plate B. (The drag is smaller in stage two, but only because the drop is slower; it is the electric force that slowed it.)

3 a The electric force eE doesn't depend on speed; the magnetic force evB does. A slower electron feels a smaller magnetic force, so the **electric force wins** and bends it that way. (A faster electron is bent the other way, by the magnetic force.)

4 b $K = eV = e \times 100 = 100$ eV, which is $100 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-17}$ J. Neither joule option equals that: 1.6×10^{-18} J is only 10 eV.

5 c The crossed fields, balanced, give $v = E/B$, and with $\frac{1}{2}mv^2 = eV_a$ this gives e/m. Parallel fields would push the beam in directions at right angles to each other, and could never cancel.

6 b Millikan measured the **charge**, $e = 1.6 \times 10^{-19}$ C, and showed it is quantized. (The mass then followed from Thomson's e/m.)

7 a Undeflected needs $v = E/B$. Doubling B with E the same halves v.

8 b Held still: $qE = mg$, so $q = mg/E$. For the same drop (same mg), a stronger field needs a **smaller** charge to hold it: the drop must have lost some of its charge.

9 c Only the magnetic force is left. It is always at right angles to the velocity, so the electron goes round a circle of radius $mv/(Be)$, at the same speed.

10 c More deflection by the electric field means $eE > evB$, so $v < E/B$: the electrons are slow compared with the speed the two fields would balance. The other options say the opposite of what was seen, or compare fields of different kinds, which can't be compared directly.

11 b Held still: $q\dfrac{V}{d} = mg$, so $q = \dfrac{mgd}{V}$, and $m \propto r^3$. Half the radius gives $1/8$ of the mass:

$$\frac{q_2}{Q} = \frac{m_2}{m_1} \cdot \frac{V_1}{V_2} = \frac{1}{8} \times \frac{2400}{600} = \frac{1}{2}$$

12 d $\frac{1}{2}mv^2 = eV$, so $v \propto \sqrt{V}$. Four times the voltage gives $\sqrt{4} = 2$ times the speed: 1.2×10^7 m/s.

13 d The p.d. is $4.0 - (-4.0) = 8.0$ V, so $E = \dfrac{8.0}{0.020} = 400$ V/m, pointing down. $F = eE = 1.6 \times 10^{-19} \times 400 = 6.4 \times 10^{-17}$ N, **upwards**, towards the positive plate. (3.2×10^{-17} N comes from using 4.0 V instead of the whole 8.0 V.)

11. a. A magnetic field exerts a force $F = evB\sin\theta$ on a moving electron, at right angles to both its velocity and the field. Moving **along** the field ($\theta = 0°$ or $180°$), it feels no force and goes straight; at **another angle**, it moves along a helix; at **right angles**, the force $F = evB$ has a constant size and is always at right angles to the velocity. Such a force changes only the direction of motion, never the speed, and always points towards one fixed point: it is a centripetal force, and the path is a **circle**. Its radius follows from

$$evB = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{Be}$$

The period is

$$T = \frac{2\pi r}{v} = \frac{2\pi}{v}\cdot\frac{mv}{Be} = \frac{2\pi m}{Be}$$

in which v does not appear: a faster electron goes round a proportionally larger circle in the same time.

b. Let the beam enter with speed v along x, at right angles to the uniform field $E = V/d$. There is no force along x, so $x = vt$. Across the plates the force eE is constant, so the acceleration $a = eE/m$ is constant, and from rest in that direction $y = \tfrac{1}{2}at^2$. Putting $t = x/v$:

$$y = \frac{eE}{2mv^2}\,x^2 = kx^2$$

with $k = eE/2mv^2$ constant: the equation of a **parabola** (as in 20.3).

c. $E = \dfrac{V}{d} = \dfrac{90}{0.02} = 4500$ V/m, and $a = \dfrac{e}{m}E = 1.8\times10^{11}\times4500 = 8.1\times10^{14}$ m/s².

Time between the plates: $t = \dfrac{D}{v} = \dfrac{0.05}{10^7} = 5.0\times10^{-9}$ s.

Deflection at the end: $y = \tfrac{1}{2}at^2 = \tfrac{1}{2}\times8.1\times10^{14}\times(5.0\times10^{-9})^2 = 1.01\times10^{-2}$ m, which is $d/2 = 1.0$ cm (to the data's accuracy): the beam just grazes the edge of the positive plate.

Its vertical velocity there: $v_y = at = 8.1\times10^{14}\times5.0\times10^{-9} = 4.05\times10^6$ m/s. Its speed:

$$v' = \sqrt{v^2 + v_y^2} = \sqrt{(1.0\times10^7)^2 + (4.05\times10^6)^2} = \sqrt{1.164\times10^{14}} =$$

12. a. The **specific charge** of the electron is its charge per unit mass, e/m (in C/kg; 1.76×10^{11} C/kg). Thomson's experiment: electrons from the cathode of an evacuated tube are accelerated through a p.d. V_a and pass, as a narrow beam, between two plates with a p.d. V_p (field $E = V_p/d$) and through a magnetic field B at right angles to both the beam and E, onto a fluorescent screen. The magnetic field alone bends the beam one way; the electric field alone bends it the other way; the fields are adjusted until the spot returns to its undeflected position. Then $eE = evB$, so $v = E/B$. The electrons gained their kinetic energy from the accelerating p.d., $\frac{1}{2}mv^2 = eV_a$, so

$$\frac{e}{m} = \frac{v^2}{2V_a} = \frac{E^2}{2V_aB^2}$$

(Figure Figure 20.9 shows the apparatus; Section 20.5 gives the details.)
b. Weight: $mg = \frac{4}{3}\pi r^3 \rho g = \frac{4}{3}\pi \times (10^{-7})^3 \times 960 \times 9.8 = 3.94 \times 10^{-17}$ N.
Drag: $6\pi\eta r v = 6\pi \times 1.8 \times 10^{-5} \times 10^{-7} \times 3.5 \times 10^{-4} = 1.19 \times 10^{-14}$ N.
Rising steadily, the electric force balances both (drag acts down):

$$qE = mg + 6\pi\eta r v = 3.94 \times 10^{-17} + 1.19 \times 10^{-14} = 1.19 \times 10^{-14}$$

$$q = \frac{1.19 \times 10^{-14}}{1.95 \times 10^{4}} = 6.1 \times 10^{-19}, \qquad n = \frac{q}{e} = \frac{6.1 \times 10^{-19}}{1.6 \times 10^{-19}} = 3.8 \approx$$

(For so small a drop, the weight is a tiny part of the balance: the drag is 300 times larger.)

13. a. The diagrams are Figure 20.4. With no field: the weight F_g down; the upthrust U and the viscous drag $F_v = 6\pi\eta r v_1$ up; $F_g = U + F_v$. In the field (drop negative, field down): the weight down; the upthrust, the smaller drag $6\pi\eta r v_2$ and the electric force qE up; $F_g = U + F_v + F_e$.

b. Without the field: $\frac{4}{3}\pi r^3(\rho - \sigma)g = 6\pi\eta r v_1$. With it: $\frac{4}{3}\pi r^3(\rho - \sigma)g = 6\pi\eta r v_2 + qE$. Subtracting,

$$q = \frac{6\pi\eta r(v_1 - v_2)}{E}$$

and putting in $r = \sqrt{9\eta v_1 / 2(\rho - \sigma)g}$ gives equation (iv) of 20.2.
c.

$$q = \frac{6\pi\eta r(v_1 - v_2)}{E} = \frac{6\pi \times 1.8 \times 10^{-5} \times 1.5 \times 10^{-6} \times (0.25 - 0.10) \times 10^{-3}}{10^5} = 7.6 \times 10^{-19}$$

That is about $4.8e$: with speeds given to only two figures, it is close to $5e = 8.0 \times 10^{-19}$ C.

14. a. **Quantization of charge** means that charge exists only in whole-number multiples of a smallest amount, the elementary charge $e = 1.6 \times 10^{-19}$ C: $q = \pm ne$, with $n = 1, 2, 3, \ldots$. No body can carry $1.5e$.

b. i. The upward electric force must equal the drop's weight (no drag, since it isn't moving; upthrust neglected): $qE = mg$, that is $q\dfrac{V}{d} = mg$. The sketch is Figure 20.11.

ii.

$$q = \frac{mg\,d}{V} = \frac{1.5 \times 10^{-14} \times 10 \times 10^{-3}}{470} = 3.19 \times 10^{-19}$$

which is $2e$: two extra electrons.

iii. An alpha particle (charge $+2e$) knocks electrons off the drop or off nearby air molecules, or is itself captured. Either way, the drop's charge changes. Then qE no longer equals mg, and the drop starts to move: if its negative charge decreases (for example if it captures the alpha particle and becomes neutral), the electric force becomes smaller and it **falls**; if it picks up extra electrons, it **rises**. In either case it soon reaches a steady speed, and the p.d. must be changed to hold it still again.

15. a. The electric force and the magnetic force must be equal and opposite: $eE = evB$. The electric force eE points towards the positive plate, the magnetic force evB the opposite way, both at right angles to the beam, as in Figure 20.9 with the beam going straight to S.

b.

$$E = \frac{V}{d} = \frac{1.36 \times 10^{4}}{0.04} = 3.4 \times 10^{5}, \qquad v = \frac{E}{B} = \frac{3.4 \times 10^{5}}{2 \times 10^{-3}} =$$

c. Only the magnetic force is left, always at right angles to the velocity: the beam goes round an **arc of a circle**, at the same speed, of radius

$$r = \frac{mv}{eB} = \frac{9.1 \times 10^{-31} \times 1.7 \times 10^{8}}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 0.48$$

(At 1.7×10^{8} m/s, more than half the speed of light, the electron's motion needs relativity: the true radius is about 20% larger. The balance $v = E/B$ in b stays exact.)

16. a. The X-rays **ionize the air** between the plates; the drops pick up electrons and ions, so they become charged, and their charge can be changed during an experiment (Example 20.1). **Principle:** a small drop moving through air soon reaches a terminal velocity, where the forces on it balance. Stokes' law, $F_v = 6\pi\eta r v$, gives the drag. The terminal velocity with no field gives the drop's radius; the velocity with the field on (or the p.d. that holds it still) then gives the electric force qE, and so the charge q.

b. By Newton's first law, it does not stop: it goes on moving **at a constant velocity**. That is exactly what happens at the terminal velocity: once the drag has grown to balance the other forces, the drop falls steadily.

c. Its **weight** (down), the **upthrust** of the air (up), the **viscous drag** (opposite to its motion), and, with the field on, the **electric force** qE (up for a negative drop in a downward field). The sketch is Figure 20.4.

d. With no upthrust, $r = \sqrt{\dfrac{9\eta v}{2\rho g}}$:

$$r = \sqrt{\frac{9 \times 1.8 \times 10^{-5} \times 2.0 \times 10^{-4}}{2 \times 920 \times 9.8}} = \sqrt{\frac{3.24 \times 10^{-8}}{1.80 \times 10^{4}}} = \sqrt{1.80 \times 10^{-12}} =$$

17. a. Charge is **quantized**: it comes only in whole-number multiples of a smallest charge, e.

b. i. **Parallel**, so that the field between them is uniform: the force on the drop is the same wherever it is, and $E = V/d$. **Horizontal**, so that the field, and the electric force, is **vertical**, along the same line as the weight, so the two can balance.

ii. Held still: $q\dfrac{V}{d} = mg$,

$$q = \frac{mg\,d}{V} = \frac{7.7 \times 10^{-15} \times 9.8 \times 5.4 \times 10^{-3}}{850} = 4.8 \times 10^{-19}$$

Q is positive, so the field points down; the electric force must be **up** to hold the drop: the charge is **negative**, -4.8×10^{-19} C.

c. The four charges are 3.2, 6.4, 3.2 and 4.8×10^{-19} C, which are 2, 4, 2 and 3 times 1.6×10^{-19} C. They are all whole-number multiples of 1.6×10^{-19} **C**, the largest value that divides them all; so the data suggest $e = 1.6 \times 10^{-19}$ C. (Strictly, four drops can't rule out a smaller unit such as 0.8×10^{-19} C; it took Millikan's thousands of measurements, none ever between the steps of 1.6×10^{-19} C, to show that this is the smallest.)

18. a. The proton moves to the right with the field into the page. By Fleming's left-hand rule, the magnetic force on it is **up** (towards the top of the page). The electric force must be **down**, and on a positive charge the electric force is along the field: **E points down the page**, at right angles to the proton's path, as in Figure 20.14.

b. i. **No deviation.** For any charge q: $qE = qvB$ when $v = E/B$. The balance does not depend on the charge or the mass, so an alpha particle ($+2e$, four times the proton's mass) with the same speed v also goes straight through. (Both forces on it are twice as big, and still equal.)

ii. **It is deviated, downwards.** For the electron, both forces reverse: the electric force eE is now **up**, and the magnetic force **down**. At speed $2v$, the magnetic force is $e(2v)B = 2eE$, twice the electric force. The net force is eE downwards, so the electron curves **down**, towards the bottom of the page.

19. a. See Figure 20.16. i. The beam goes straight until it reaches the plates, curves **upward** along a parabola between them, then goes on in a **straight line** (along the tangent where it left the plates) to P. ii. The beam bends towards the upper plate, so the **upper plate is positive** and the **lower plate negative**. iii. Conventional current flows the **opposite way** to the electrons: the arrow C points back along the beam, towards where the electrons came from.

b. Given: $D = 4.0 \times 10^{-2}$ m, $d = 3.0 \times 10^{-2}$ m, $V = 300$ V, $v = 1.0 \times 10^{7}$ m/s.

$$y = \frac{e}{m} \cdot \frac{VD^2}{2dv^2} = \frac{1.8 \times 10^{11} \times 300 \times (4.0 \times 10^{-2})^2}{2 \times 3.0 \times 10^{-2} \times (1.0 \times 10^{7})^2} = \frac{8.64 \times 10^{10}}{6.0 \times 10^{12}} =$$

That is 1.44 cm, just less than the 1.5 cm to the upper plate, so the beam does get out.