Unit 19

Answers: Alternating Current

Quick checks

1 b Compare with $V = V_0 \sin \omega t$: $\omega = 100\pi$ rad/s, and $f = \dfrac{\omega}{2\pi} = \dfrac{100\pi}{2\pi} = 50$ Hz, Nepal's mains frequency. 325 V is the peak value, not a frequency; 100π is ω, not f.

2 c In each cycle the current reverses twice: once as it goes from + to −, and once from − back to +. 50 cycles a second give 100 reversals a second. That is also why a transformer hums at 100 Hz, and a fluorescent tube flickers 100 times a second: twice per cycle.

3 c A.c. meters read rms values, and $I_0 = \sqrt{2}\, I = 1.414 \times 2.0 = 2.8$ A. 1.4 A would be $2.0/\sqrt{2}$, the mistake of treating 2.0 A as the peak.

4 a Phasors turn anticlockwise, so the one ahead in that direction gets to the top (its peak) first: the voltage leads by 90°, a quarter period. The angle between phasors is a phase difference; their lengths, not their angles, give the peak values.

5 b $X_L = 2\pi f L$ doubles with f, so $I = V/X_L$ halves. Doubling the current would be a capacitor's behavior, and a resistor's current wouldn't change.

6 b An inductor's reactance $2\pi f L$ is zero for d.c. and grows with frequency. A capacitor does the opposite: it blocks d.c. ($X_C \to \infty$) and passes high frequencies.

7 c $Z = \sqrt{30^2 + 40^2} = 50\ \Omega$, so $I = 100/50 = 2.0$ A. 1.4 A comes from adding 30 + 40 = 70 Ω, which forgets that the two p.d.s are 90° apart.

8 b $X_C > X_L$, so it is capacitive (the e.m.f. lags). $Z = \sqrt{30^2 + (30 - 70)^2} = \sqrt{900 + 1600} = 50\ \Omega$. 130 Ω adds everything as numbers; 76 Ω forgets that X_L and X_C cancel in part.

9 c At resonance V_L and V_C are equal and opposite, so they cancel **each other**, but each on its own is not zero; in fact each can be many times the supply voltage (Section 19.12). The other three are all true at resonance.

10 b Apparent power = 230 × 2 = 460 VA. Power factor = real power ÷ apparent power = 92 ÷ 460 = 0.2. The current is mostly wattless: the coil is mostly reactance.

11 c D.c.: $R = 100/1 = 100\ \Omega$. A.c.: $Z = 100/0.5 = 200\ \Omega$. $X_L = \sqrt{200^2 - 100^2} = 173.2\ \Omega$, so $L = \dfrac{173.2}{2\pi \times 50} = 0.55$ H (Example 19.5).

12 b $X_L = X_C$: the circuit is at resonance, so $Z = R = 2\ \Omega$ and $I = 12/2 = 6$ A.

13 b A good inductor is mostly reactance, with only the small resistance of its wire: $X_L \gg R$ (for an ideal one, R = 0). $X_L = \infty$ only at infinite frequency.

14 a $f_0 = \dfrac{1}{2\pi\sqrt{LC}} \propto \dfrac{1}{\sqrt{C}}$. Four times C gives $\dfrac{1}{\sqrt{4}} = \dfrac{1}{2}$: the frequency halves.

15 b $\dfrac{I_m}{I} = \dfrac{2I_0/\pi}{I_0/\sqrt{2}} = \dfrac{2\sqrt{2}}{\pi}$ (= 0.90).

16 a $V = \sqrt{V_R^2 + (V_L - V_C)^2} = \sqrt{40^2 + 30^2} = 50$ V. 130 V adds them as numbers.

17 d The capacitor's energy all passes to the coil when the current is greatest (the LC circuit's energy swap, Section 19.11):

$$\tfrac{1}{2}CV^2 = \tfrac{1}{2}LI_{\max}^2 \quad \Rightarrow \quad I_{\max} = V\sqrt{\frac{C}{L}}$$

$$I_{\max} = 1 \times \sqrt{\frac{10^{-6}}{10^{-6}}} = 1 = 1000$$

√100 mA is 10 mA, which would need an inductance of 10 mH.

The textbook prints the inductance as 10 $^{-3}$ μH, which gives $I_{\max} = \sqrt{10^{-6}/10^{-9}} = 31.6$ A, and none of its options is that, so the book's numbers don't work. With 1 μH, the inductance changed and nothing else, the answer is 1000 mA, one of the book's own options.

18 c With no loss, power in = power out, so the currents go inversely as the turns: $I_p = I_s \times \dfrac{N_s}{N_p} = 80 \times \dfrac{1}{20} = 4$ A. (The secondary voltage is 2400/20 = 120 V; 120 × 80 = 2400 × 4 = 9600 W.)

19 b $\omega = 2\pi f = 2\pi \times \dfrac{400}{\pi} = 800$ rad/s. $C = \dfrac{1}{\omega X_C} = \dfrac{1}{800 \times 25} = 5 \times 10^{-5}$ F = 50 μF.

20 c $P = VI\cos\phi = 0$ needs $\cos\phi = 0$: φ = 90°.

11.

a. The highest point of the curve: **I_0 = 17 A**.

b. The current starts at zero and rises, so $I = 17\sin\left(\dfrac{2\pi t}{0.1}\right)$. At t = 0.23 s, $\dfrac{2\pi \times 0.23}{0.1} = 4.6\pi$, which is the same angle as $0.6\pi = 108°$:

$$I = 17\sin 108° = 17 \times 0.951 = 16.2$$

(On the graph, 0.23 s is just after the peak at 0.225 s.)

c. The current rises through zero at t = 0, 0.10 s, 0.20 s and 0.30 s, so the **period T = 0.10 s** and the **frequency f = 1/T = 10 Hz**.

d. $I = \dfrac{I_0}{\sqrt{2}} = \dfrac{17}{1.414} =$ **12.0 A**. Mean over half a cycle: $I_m = \dfrac{2I_0}{\pi} = 0.637 \times 17 =$ **10.8 A** (over a whole cycle the mean is zero).

12. a. In a pure capacitor the current leads the voltage by 90°, so the power factor is $\cos 90^\circ = \mathbf{0}$: no power is used.

b. Let $I = I_0 \sin \omega t$. The charge is $q = \int I\,dt = -\dfrac{I_0}{\omega}\cos \omega t$, so the p.d. is

$$V = \frac{q}{C} = -\frac{I_0}{\omega C}\cos \omega t = -V_0 \cos \omega t = V_0 \sin\left(\omega t - \frac{\pi}{2}\right)$$

with $V_0 = I_0 X_C$, $X_C = \dfrac{1}{\omega C}$. The voltage lags the current by $\pi/2$ (90°); the current leads (Section 19.7).

c. For d.c., f = 0, so $X_C = \dfrac{1}{2\pi f C}$ is infinite. Physically, the plates are separated by an insulator: once the capacitor has charged to the supply's p.d., no more charge flows, so no steady current passes. It acts as a break in the circuit, an open circuit.

d. As Example 19.14: $X_L = 628.3\ \Omega$, $Z = \sqrt{500^2 + 628.3^2} = 803\ \Omega$, **I = 200/803 = 0.25 A**; $\tan \phi = 628.3/500 = 1.257$, **φ = 51.5°** (the current lags the supply voltage); the coil's impedance $\sqrt{50^2 + 628.3^2} = 630.3\ \Omega$, so the **p.d. across the coil = 0.249 × 630.3 = 157 V**.

13. a. The rms value of an alternating current is the steady (d.c.) current that, passing through the same resistance for the same time, produces the same amount of heat as the alternating current. For a sine wave, $I = I_0/\sqrt{2}$.

b. The peaks come at about 2.5 ms and 17.5 ms, so the period T = 15 ms, and $f = \dfrac{1}{0.015} = \mathbf{66.7\ Hz}$.

c. 0 to 30 ms is exactly two whole cycles, in which the current is positive and negative equally: the **mean current is zero**. (The mean over a half cycle, the "mean value" of Section 19.3, is $2I_0/\pi = 0.637 \times 0.75 = 0.48$ A.)

d. The peak is 0.75 A, so $I = \dfrac{0.75}{\sqrt{2}} = \mathbf{0.53\ A}$.

e. $E = I^2 R t = 0.53^2 \times 450 \times 0.030 = \mathbf{3.8\ J}$.

14. a. In a pure inductor the voltage leads the current by 90°, so the power factor is $\cos 90^\circ = \mathbf{0}$.

b. Let $I = I_0 \sin \omega t$. With no resistance, the source's e.m.f. balances the back e.m.f.: $V = L\dfrac{dI}{dt} = \omega L I_0 \cos \omega t = V_0 \sin\left(\omega t + \dfrac{\pi}{2}\right)$, with $V_0 = I_0 X_L$, $X_L = \omega L$. The voltage leads the current by $\pi/2$ (90°) (Section 19.6).

c. For d.c., f = 0, so $X_L = 2\pi f L = 0$: a steady current induces no back e.m.f., and an ideal coil is then just a wire of zero resistance, a short circuit.

d. As Example 19.9: $Z_1 = 5\ \Omega$, $Z_2 = 8\ \Omega$; $L^2 = \dfrac{8^2 - 5^2}{4\pi^2(100^2 - 50^2)} = 1.32 \times 10^{-4}$ H², so **L = 0.0115 H = 11.5 mH**.

15.

a. $I = \dfrac{V}{Z} = \dfrac{V}{\sqrt{R^2 + (X_L - X_C)^2}}$, with the e.m.f. leading the current by φ, where $\tan\phi = \dfrac{X_L - X_C}{R}$.

b. Draw I along the horizontal; V_R along it; V_L straight up and V_C straight down; their difference $V_L - V_C$ up (if $X_L > X_C$); and V the hypotenuse from the origin, at φ above I (Figure 19.14). If $X_C > X_L$, $V_L - V_C$ points down and V is below I.

c. $X_L = 2\pi \times 50 \times 0.030 = 9.42\ \Omega$; $X_C = \dfrac{1}{2\pi \times 50 \times 10 \times 10^{-6}} = 318.3\ \Omega$.

$$Z = \sqrt{25^2 + (9.42 - 318.3)^2} = \sqrt{625 + 95\,407} = 309.9\ \Omega$$

$$I = \frac{240}{309.9} = , \qquad V_C = IX_C = 0.774 \times 318.3 =$$

The voltmeter across the capacitor reads more than the 240 V supply: the coil's p.d. (7.3 V), in the opposite direction, cancels part of it. The circuit is capacitive: the current leads by $\tan^{-1}(308.9/25) = 85.4°$.

16.

a. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$.

b. Z is least, and the current greatest, when $X_L = X_C$, that is $\omega L = \dfrac{1}{\omega C}$, which gives the resonant frequency $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$. Then Z = R and the current is in phase with the e.m.f.

c. $X_C = \dfrac{1}{2\pi \times 50 \times 2 \times 10^{-6}} = 1592\ \Omega$.

$$Z = \sqrt{1000^2 + 1592^2} = 1880\ \Omega, \qquad I = \frac{12}{1880} = 6.38 \times 10^{-3} =$$

$$V_C = IX_C = 6.38 \times 10^{-3} \times 1592 =$$

$$\tan\phi = \frac{X_C}{R} = \frac{1592}{1000} = 1.59, \qquad \phi = 57.9°$$

the e.m.f. lagging behind the current. (Check: $V_R = 6.4$ V, and $\sqrt{6.4^2 + 10.2^2} = 12$ V.)

17. i. The e.m.f. reaches each peak **before** the current does (its peaks are to the left of the current's), so the e.m.f. leads the current: φ is **positive**.

ii. Draw the current phasor I, and the e.m.f. phasor V, longer, an angle φ **anticlockwise** from it (ahead of it), with φ between 0 and 90°. (The p.d. V_R lies along I, and $V_L - V_C$ points at 90° ahead of I.) This is the diagram of Figure 19.14.

iii. The e.m.f. leads, so $\tan\phi = \dfrac{X_L - X_C}{R} > 0$: $X_L > X_C$. The circuit is **more inductive**.

iv. The power is I^2R, largest at resonance, where $X_L = X_C$. Here X_L is too large, so **decrease** L: X_L comes down towards X_C, Z falls, the current rises and more energy is delivered to R each second. (Decreasing L also moves the resonant frequency $1/(2\pi\sqrt{LC})$ up towards the supply's frequency.)

18. a. i. T = 15 ms, so f = 1/0.015 = **66.7 Hz**. ii. $I = 0.75/\sqrt{2} = 0.53$ A; $E = I^2Rt = 0.53^2 \times 450 \times 0.030 =$ **3.8 J**.

b. The same current I flows through R and C. $V_R = IR$ is in phase with I; $V_C = IX_C$ lags I by 90°. The supply's e.m.f. is their phasor sum, so from the right-angled triangle, $V^2 = V_R^2 + V_C^2 = I^2(R^2 + X_C^2)$. The impedance is $Z = \dfrac{V}{I} = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \dfrac{1}{\omega^2 C^2}}$, and the e.m.f. lags the current by φ with $\tan\phi = X_C/R$ (Section 19.9).

c. Adding the capacitor brings in X_C, which cancels part of the coil's X_L. If $X_C < 2X_L$ the impedance $\sqrt{R^2 + (X_L - X_C)^2}$ becomes smaller, the current larger and the bulb **brighter** (brightest at resonance, $X_C = X_L$); if $X_C > 2X_L$, the bulb becomes **dimmer** (Test yourself 19.3).

19. a. The impedance of an LCR circuit is its total opposition to a.c., the ratio of the rms (or peak) voltage to the rms (or peak) current: $Z = \dfrac{V}{I} = \sqrt{R^2 + (X_L - X_C)^2}$, in ohms.

b. i. $X_L = 2\pi \times 50 \times 0.060 = 18.8\ \Omega$; $X_C = \dfrac{1}{2\pi \times 50 \times 0.50 \times 10^{-6}} = 6366\ \Omega$.

$$Z = \sqrt{300^2 + (18.8 - 6366)^2} = \sqrt{90\,000 + 40\,288\,000} = 6354\ \Omega \approx 6.35\ \Omega$$

(The current is only 50/6354 = 7.9 mA, leading the voltage by 87°.)

ii. Resonance occurs when the supply's frequency makes $X_L = X_C$, at $f_0 = \dfrac{1}{2\pi\sqrt{LC}} = \dfrac{1}{2\pi\sqrt{0.060 \times 0.50 \times 10^{-6}}} = 919$ Hz. At 50 Hz the circuit is far from resonance; raising the source's frequency to 919 Hz (or changing L or C so that $f_0 = 50$ Hz) would bring it to resonance, with Z = 300 Ω and the current in phase with the voltage.

c. i. The current leads the voltage by π/2 (90°): X is a **capacitor**, with reactance $X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$.

ii. Over one cycle, the voltage is a sine wave starting at zero and rising; the current is a sine wave a quarter of a cycle ahead, starting at its peak $+I_0$ at t = 0, falling to zero when the voltage peaks (as Figure 19.10 shows).

20. a. A current that flows but uses no power, because it is 90° out of phase with the e.m.f., so $P = VI\cos 90^\circ = 0$: the current in a pure inductor or a pure capacitor.

b. i. **a. Resistor:** R does not depend on frequency, so the current **stays the same**. **b. Inductor:** $X_L = 2\pi f L$ rises with f, so the current **decreases**. **c. Capacitor:** $X_C = 1/(2\pi f C)$ falls as f rises, so the current **increases**.

ii. In **a**, the opposition is R for d.c. as for a.c.: **no change**. In **b**, the coil's reactance disappears for d.c. ($X_L = 0$ at f = 0), leaving only the small resistance of its wire (zero for an ideal coil): the opposition becomes **much smaller**, and the current much larger, dangerously so for a real circuit without a resistor.

21.

a. **Electrical resonance** happens in a series LCR circuit when the frequency of the applied e.m.f. equals the circuit's natural frequency: the inductive and capacitive reactances are equal and cancel, the impedance is least (Z = R), and the current is greatest and in phase with the e.m.f. Example: tuning a radio, whose LC circuit is set to resonate at the chosen station's frequency (Activity 19.2).

b. $Z = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$ is least when the bracket is zero: $\omega L = \dfrac{1}{\omega C}$, so $\omega^2 = \dfrac{1}{LC}$, $\omega = \dfrac{1}{\sqrt{LC}}$. With $\omega = 2\pi f$, $f = \dfrac{1}{2\pi\sqrt{LC}}$ (Section 19.11).

c. $L = \dfrac{1}{4\pi^2 f^2 C} = \dfrac{1}{4\pi^2 \times (810 \times 10^3)^2 \times 33 \times 10^{-12}} = 1.17 \times 10^{-3}$ H = **1.17 mH**.