Unit 18

Electromagnetic Induction

This unit online, with its interactives:
science-playground.com/s/12f18

Why must you swipe a card through a card reader instead of holding it still in the slot? The magnetic stripe on an old bank card is a row of tiny magnets. Held still, they do nothing to the reader. Swept past the reader's little coil, their field through the coil changes, and a changing field makes a voltage in the coil: the reader "hears" the stripe only while it moves.

That is **electromagnetic induction**, and it is how almost all the world's electricity is made, from Kulekhani to the bicycle dynamo. This unit gives you its two laws (Faraday's, for the size of the induced e.m.f., and Lenz's, for its direction), the e.m.f. of a rod sliding through a field and of a coil spinning in one (the a.c. generator), the eddy currents that brake trains and cook food, the self and mutual induction of coils, the energy a coil stores, and the transformer that carries power from a dam to your home.

A STORY FROM SCIENCE

Michael Faraday (1791–1867)

Michael Faraday was born in London to a poor blacksmith's family and left school at 13 to work for a bookbinder. He read the books he bound, went to public lectures by the chemist Humphry Davy, wrote them up neatly and sent them to Davy, who took him on as an assistant at the Royal Institution in 1813.
He started as a chemist, and became one of the greatest experimental scientists of the 1800s:

- In 1821 he made the first **electric motor**: a wire carrying a current that went round and round a magnet.
- In 1831 he was the first to **make electricity using a magnet** (electromagnetic induction), and built the first **dynamo**, a copper disc turning between the poles of a magnet.
- His laws of **electrolysis** showed that electricity and chemical bonding are connected.
- In 1845 he found that a magnetic field turns the plane of polarized light (the **Faraday effect**): magnetism can affect light.
- He discovered **diamagnetism**: every material is weakly pushed away by a strong magnet.

He had almost no mathematics. He pictured fields as "lines of force" filling space, and James Clerk Maxwell later turned his pictures into equations. The unit of capacitance, the farad, is named after him.

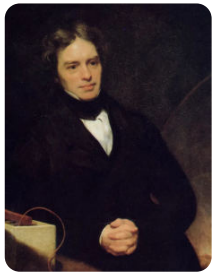


Figure 18.1 *Michael Faraday in 1842, painted by Thomas Phillips.*
Photo: Thomas Phillips, Public domain

DID YOU KNOW?

Nepal's first power station still runs on this

Most of the electricity we use is generated by electromagnetic induction. Nepal's first hydropower station, **Chandra Jyoti** at Pharping, south of Kathmandu, began work in 1911 with two 250 kW generators, and was one of the first in Asia. It is now kept as a living museum. In its powerhouse, as in every power station since, coils of wire turn in a magnetic field (or magnets turn inside coils), and the changing field makes an e.m.f.

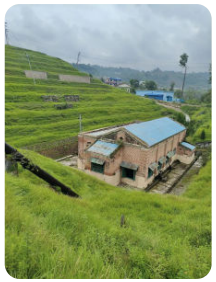


Figure 18.2 *The powerhouse of Chandra Jyoti, Nepal's first hydropower station, at Pharping.*
Photo: Ganesh Paudel, CC BY-SA 4.0

Electromagnetic induction

In 1820 Hans Christian Ørsted found that an electric current makes a magnetic field. Scientists then asked the opposite question: can a magnet make a current? Faraday answered it in 1831. A magnet lying still beside a coil does nothing; but **moving** the magnet towards or away from a closed coil, or moving the coil near the magnet, makes a current flow in the coil.

Electromagnetic induction is the production of an e.m.f. in a conductor, and so of a current in a closed circuit, when the magnetic flux linked with it changes, for example by relative motion between the conductor and a magnetic field. The e.m.f. is called the **induced e.m.f.** and the current it drives the **induced current**.

In a power station, magnets move relative to coils of wire, so that the magnetic flux through the coils keeps changing and an e.m.f. is induced in them. Other parts of a power system, such as transformers, also work by induced e.m.f.

Magnetic flux

Everything in this unit depends on one quantity, met in Unit 16, so here it is again before we start. Imagine field lines passing through a flat loop. The **magnetic flux** Φ through the loop measures how much field passes through it:

$$\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where B is the magnetic flux density, A the area of the loop and θ the angle between the field and the **normal** (the line at right angles) to the loop. The flux is greatest, BA, when the field passes straight through the loop ($\theta = 0$), and zero when the field lies along the loop's plane ($\theta = 90°$). Its SI unit is the **weber** (Wb): 1 Wb = 1 T m². That is why B is also called the flux density, in Wb/m², the same as tesla.

A coil of N turns has the same flux Φ through each turn. The total, $N\Phi$, is the **flux linkage** of the coil.

Check 1
A loop of area 0.02 m² lies in a field of 0.5 T, with its plane **parallel** to the field. The flux through it is:

a 0.01 Wb
b 0.005 Wb
c 0
d 0.25 Wb

Observing induction

18.1 Faraday's experiment

In 1831 Faraday connected a coil of a few turns to a sensitive galvanometer and moved a bar magnet near it. He saw that:

1. When the magnet is moved towards the coil, the galvanometer shows a sudden deflection.
2. The deflection lasts only while the magnet is moving.
3. If the magnet is moved quickly, the deflection is larger.
4. The deflection reverses if the magnet is moved away instead, or if its other pole is used.

He got the same results when he kept the magnet still and moved the coil. So what matters is the **relative motion** of magnet and coil. A current through the galvanometer means an e.m.f. in the coil: the **induced e.m.f.**, driving the **induced current**. Faraday called the effect **electromagnetic induction**: electricity produced by magnetism.

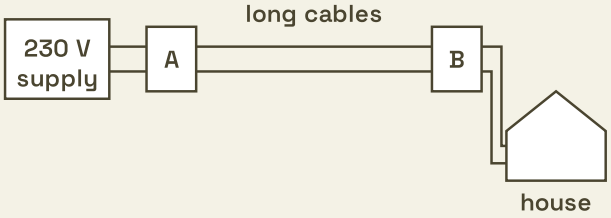


Figure 18.36 *For question 21b: transformer A next to the supply, transformer B next to the house.*

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

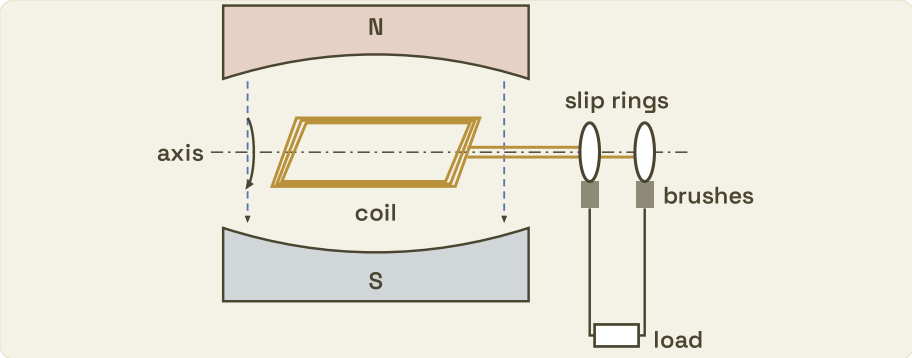


Figure 18.35 *For question 20: a simple a.c. generator.*

21. A farmer connects a house to the mains. The house is a long way from the nearest 230 V supply, and is joined to it by long cables.

    a. The farmer uses 230 V lamps in the house, but they do not light up at full brightness. Explain why the lamps are dim.

    b. The farmer then adds a transformer A next to the supply and a transformer B next to the house. Now the lamps light up at full brightness. Explain why.

    c. The 230 V supply provides 690 W of power to transformer A.

    i. Calculate the current supplied to the transformer.

    ii. Calculate the energy supplied to the transformer in 10 minutes, in joules.

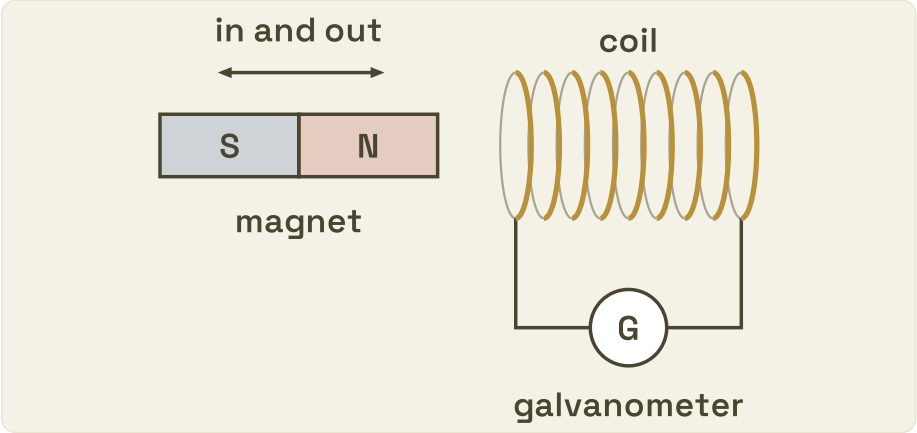


Figure 18.3 *Faraday's experiment. Moving the magnet into or out of the coil makes the galvanometer kick; holding it still does not.*

Faraday's laws of electromagnetic induction

From his experiments, Faraday gave two laws.

First law (qualitative): whenever the magnetic flux linked with a closed circuit changes, an e.m.f. is induced in the circuit. The e.m.f. lasts only as long as the flux keeps changing.

Second law (quantitative): the magnitude of the induced e.m.f. is directly proportional to the rate of change of the magnetic flux linkage.

Let the flux through each turn of a coil of N turns change from Φ_1 to Φ_2 in a time t. The flux linkage changes from $N\Phi_1$ to $N\Phi_2$, so

$$\text{rate of change of flux linkage} = \frac{N\Phi_2 - N\Phi_1}{t}$$

By the second law, the induced e.m.f. is

$$\varepsilon \propto \frac{N\Phi_2 - N\Phi_1}{t}$$

and for a change that isn't steady, taking a very short time dt,

$$\varepsilon \propto \frac{d(N\Phi)}{dt}$$

In SI units the weber and the volt are defined so that the constant of proportionality is 1 (a flux linkage changing at 1 Wb/s gives 1 V). With Lenz's minus sign (Section 18.2):

$$\boxed{\varepsilon = -N\frac{d\Phi}{dt}}$$

For a single loop, N = 1 and $\varepsilon = -d\Phi/dt$: the induced e.m.f. in a closed loop equals the negative of the rate of change of the magnetic flux through it. With N turns, the same flux changes through every turn, so the e.m.f.s of the turns add up: N times as much.

The **minus sign** shows the opposing nature of the induced e.m.f.: the induced current flows in the direction that opposes the change in flux. If the flux is increasing ($d\Phi/dt$ positive), ε comes out negative; if the flux is decreasing ($d\Phi/dt$ negative), ε is positive. Section 18.2 explains what "opposing" means.

If the circuit has a total resistance R, the induced current is

$$I = \frac{\varepsilon}{R} = -\frac{N}{R}\frac{d\Phi}{dt}$$

and the **charge** that flows round it while the flux changes by $\Delta\Phi$ is

$$q = \int I\,dt = -\frac{N}{R}\int d\Phi \quad \Rightarrow \quad |q| = \frac{N\,\Delta\Phi}{R}$$

The charge depends only on the change of flux, not on how fast it happens. Push a magnet in slowly and a small current flows for a long time; push it in quickly and a large current flows for a short time; the same charge flows either way. (A ballistic gal-

vanometer measures this charge, and so measures a change of flux.)

In SI units, ε is in volts, Φ in webers and t in seconds. In the old CGS system, flux was measured in **maxwells** (1 Mx = 10^{-8} Wb) and e.m.f. in electromagnetic units (abvolts, 1 abvolt = 10^{-8} V).

Check 2

A magnet is pushed into a coil, once slowly and once quickly, to the same place. Compared with the slow push, the quick push gives:

a the same e.m.f. and a larger charge
b a larger e.m.f. and the same charge
c a larger e.m.f. and a larger charge
d the same e.m.f. and the same charge

WORKED EXAMPLE

Example 18.1: a field that grows

Find the magnitude of the induced e.m.f. in a 200-turn coil of cross-sectional area 0.16 m², if the magnetic field through the coil changes from 0.10 Wb/m² to 0.50 Wb/m² at a uniform rate over 0.02 s.

Answer

N = 200, A = 0.16 m², B_1 = 0.10 T, B_2 = 0.50 T, t = 0.02 s.
The flux linkage changes from NB_1A to NB_2A, so

$$|\varepsilon| = \frac{NB_2A - NB_1A}{t} = \frac{NA(B_2 - B_1)}{t} = \frac{200 \times 0.16 \times (0.50 - 0.10)}{0.02} = \frac{12.8}{0.02} = \mathbf{640\ V}$$

WORKED EXAMPLE

Example 18.2: a flux given as a formula

The magnetic flux through a coil, at right angles to its plane, is $\Phi = 4t^2 + 5t + 2$, where Φ is in webers and t in seconds. Find the magnitude of the induced e.m.f. at t = 3 s.

Answer

$$|\varepsilon| = \frac{d\Phi}{dt} = \frac{d}{dt}(4t^2 + 5t + 2) = 8t + 5$$

At t = 3 s: $|\varepsilon| = 8 \times 3 + 5 = \mathbf{29\ V}$.

WORKED EXAMPLE

Example 18.3: the charge from a half turn

A coil of 100 turns, each of area 2.0×10^{-3} m², has a resistance of 12 Ω. It lies in a horizontal plane in a vertical magnetic field of flux density 3.0×10^{-3} Wb/m². Its ends are joined (short-circuited), and the coil is turned through 180° about a diameter. What charge flows round it?

Answer

At the start the field passes straight through the coil: $\Phi_1 = BA \cos 0° = +BA$. After half a turn, the same field passes through it the other way: $\Phi_2 = BA \cos 180° = -BA$. So the flux through each turn changes by

$$\Delta\Phi = \Phi_1 - \Phi_2 = BA - (-BA) = 2BA$$

The charge that flows is

$$q = \frac{N\Delta\Phi}{R} = \frac{2NBA}{R} = \frac{2 \times 100 \times 3.0 \times 10^{-3} \times 2.0 \times 10^{-3}}{12} = \frac{1.2 \times 10^{-3}}{12} = \mathbf{1.0 \times 10^{-4}\ C}$$

(In short: charge = change in flux linkage ÷ resistance. Note that the time taken doesn't matter.)

YOUR TURN

Test yourself 18.1: pulled out of the field

A magnetic field of flux density 10 T acts at right angles to a 50-turn coil of area 100 cm². Find the e.m.f. induced in the coil if it is removed from the field in 1/20 s.

Answer

A = 100 cm² = 0.01 m². The flux through each turn falls from BA = 10 × 0.01 = 0.1 Wb to 0.

$$|\varepsilon| = \frac{N\Delta\Phi}{t} = \frac{50 \times 0.1}{1/20} = 50 \times 0.1 \times 20 = \mathbf{100\ V}$$

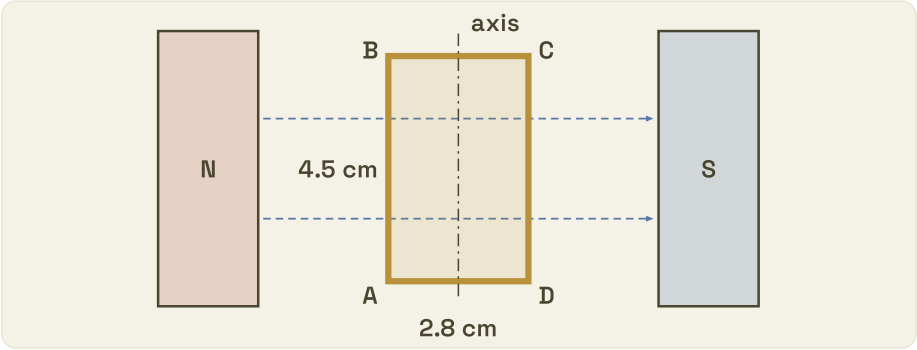


Figure 18.34 *For question 19: the coil ABCD seen from above, between the poles of the magnet.*

17. A straight electrical conductor is placed between the poles of a U-shaped magnet (Figure 18.31). The field in the gap is horizontal, from one pole to the other, and the conductor lies across it. The ends of the conductor are connected to a voltmeter.
    a. Name a material an electrical conductor can be made from.
    b. i. The voltmeter measures the e.m.f. across the conductor. State the unit of e.m.f.
    ii. State what, if anything, the meter shows when the conductor is moved: horizontally, from side to side, between the poles (towards one pole and then the other); vertically, up and down, between the poles.
    c. State two factors that affect the size of the e.m.f. across the conductor.

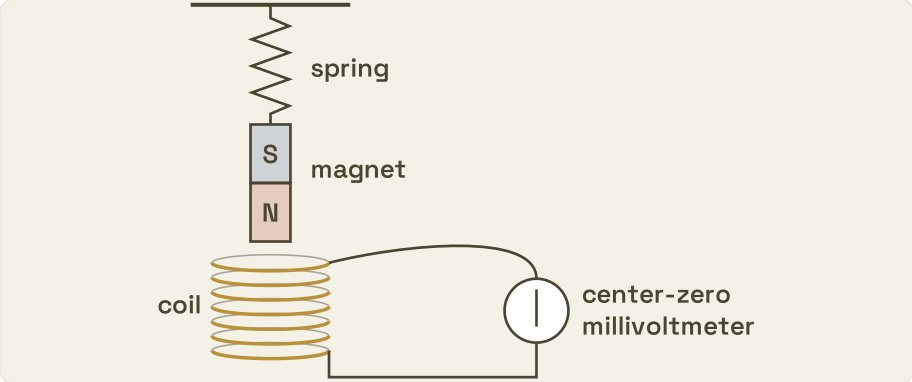


Figure 18.32 *For question 18a: a magnet on a spring above a coil.*

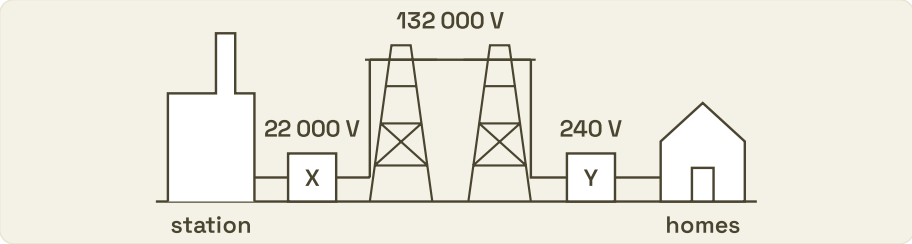


Figure 18.33 *For question 18b: from the power station to homes, through X and Y.*

18.2 Lenz's law

Faraday's law gives the size of the induced e.m.f. Its direction is given by **Lenz's law** (Heinrich Lenz, 1834):

Lenz's law: the induced current always flows in such a direction as to oppose the change that produces it.

Read it carefully: the induced current opposes the **change** in flux, not the flux itself.

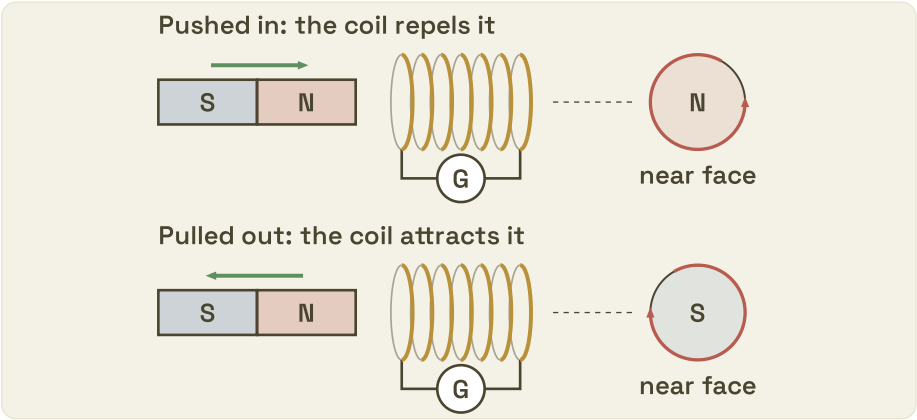


Figure 18.4 *Lenz's law. Pushed in, the magnet makes the coil's near face a north pole (anticlockwise current, seen from the magnet), which repels it. Pulled out, the near face becomes a south pole (clockwise current), which attracts it.*

A magnet approaching a coil. Push the north pole of a magnet towards one end of a coil. The flux through the coil, pointing away from the magnet, is growing. The induced current flows round the coil so that its own field points the other way, back towards the magnet: the near end of the coil becomes a **north pole**, and repels the approaching north pole. Look at that end of the coil: the current flows **anticlockwise** (an end with anticlockwise current, seen face-on, is a north pole).

A magnet moving away. Pull the north pole away. Now the flux through the coil is shrinking. The induced current flows the other way, making the near end a **south pole**, which attracts the retreating north pole: the coil tries to hold on to the flux it is losing. Seen from the magnet, the current is **clockwise**.

Either way, the coil **opposes the motion** of the magnet.

> CAREFUL
> **Opposes the change, not the field**
>
> A common slip is to say "the induced field is always opposite to the magnet's field". That is true only while the flux is **growing**. When it is **shrinking**, the induced field points the **same** way as the magnet's field, to keep the flux up. Always ask first: is the flux increasing or decreasing?

Lenz's law and the conservation of energy

Energy can't be created or destroyed, only changed from one form into another. Lenz's law is just what this demands.

When you push a magnet towards a coil, the induced current makes the near end of the coil repel the magnet, so you have to push against a force: you **do work**. That mechanical work is exactly the electrical energy that appears in the coil (and ends up as heat in its resistance). Pulling the magnet away, you work against the attraction.

Suppose the rule were the other way round, and the induced current **helped** the change. Then a magnet nudged towards a coil would be pulled in, faster and faster, making a bigger and bigger current: electrical energy and kinetic energy from nothing. That would break the conservation of energy. So **Lenz's law is a consequence of the law of conservation of energy**: the induced current always makes you pay for the electrical energy you get, in mechanical work.

Fleming's right-hand rule

For a straight conductor moving through a field, there is a quick way to find the direction of the induced current, which always agrees with Lenz's law. **Fleming's right-hand rule:** hold the thumb, first finger and middle finger of your **right** hand at right angles to one another. Then:

Check 3

A bar magnet is pulled away from a coil connected to a galvanometer. The end of the coil nearest the magnet's north pole becomes:

a a north pole, repelling the magnet
b a south pole, attracting the magnet
c no pole, since the magnet is moving away
d a north pole, attracting the magnet

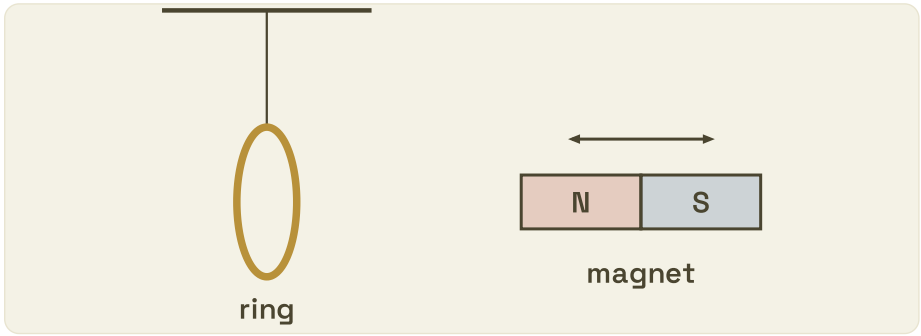


Figure 18.5 *A metal ring hangs from a thread; a bar magnet is moved towards it and away along its axis.*

WORKED EXAMPLE

Example 18.4: a ring that follows the magnet

A metal ring hangs freely from a thread in a vertical plane. A bar magnet, its axis along the ring's axis, is moved towards the ring and then away from it. How does the ring move?

Answer

When the magnet moves **towards** the ring, the flux through the ring increases. The induced current in the ring opposes this: the face of the ring nearest the magnet becomes the **same** pole as the magnet's near pole, so the ring is **repelled** and swings away from the magnet.

When the magnet moves **away**, the flux through the ring decreases. The induced current now makes the ring's near face the **opposite** pole, so the ring is **attracted** and swings after the magnet.

So the ring follows the magnet's motion, always moving so as to reduce the relative motion between them. (A ring with a cut in it carries no current and doesn't move at all.)

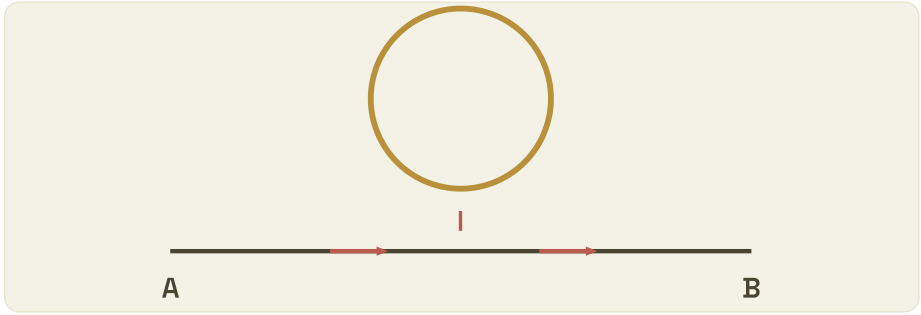


Figure 18.6 *A current I from A to B in a straight wire, with a circular loop above it in the same plane (Example 18.5 and Test yourself 18.2).*

1. the **first finger** (forefinger) points in the direction of the magnetic **field**;
2. the **thumb** points in the direction of **motion** of the conductor;
3. the **middle finger** points in the direction of the **induced current** (conventional current).

A way to remember it: thuMb for Motion, First finger for Field, seCond finger for Current. Behind the rule is the force on the charges in the moving wire, $\vec{F} = q\,\vec{v} \times \vec{B}$: the current flows along $\vec{v} \times \vec{B}$.

16.

a. An ideal transformer has a primary coil of 2700 turns and a secondary coil of 450 turns on a soft iron core, with a load resistor connected to the secondary. Explain why, when there is an alternating current in the primary coil, there is a current in the load resistor.

b. A step-down transformer is used on a 220 V line to supply a current of 0.5 A to a 15 W bulb. If the secondary has 20 turns, find the number of turns in the primary and the current in the primary.

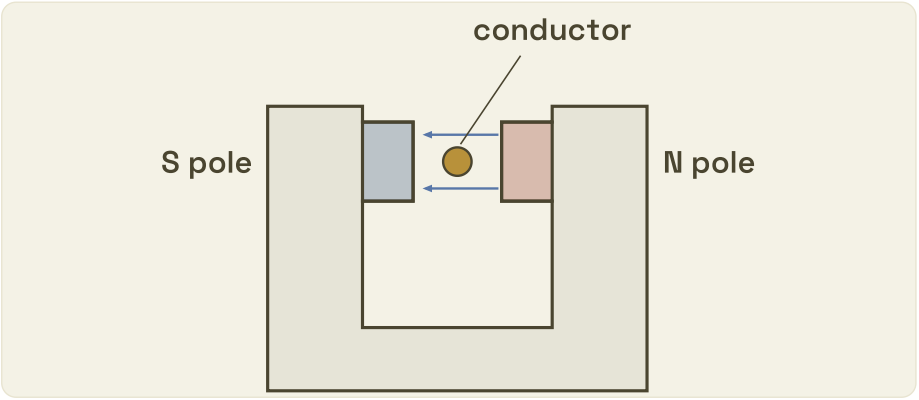


Figure 18.31 *For question 17: a U-shaped magnet seen from the front, and a straight conductor in the gap between its poles, running into the page (the voltmeter joined to its ends is behind).*

and 75 turns. A constant current in the solenoid makes a uniform field of flux density 1.4 mT throughout the small coil. The direction of the current in the solenoid is reversed in 0.12 s. Calculate the average e.m.f. induced in the small coil.

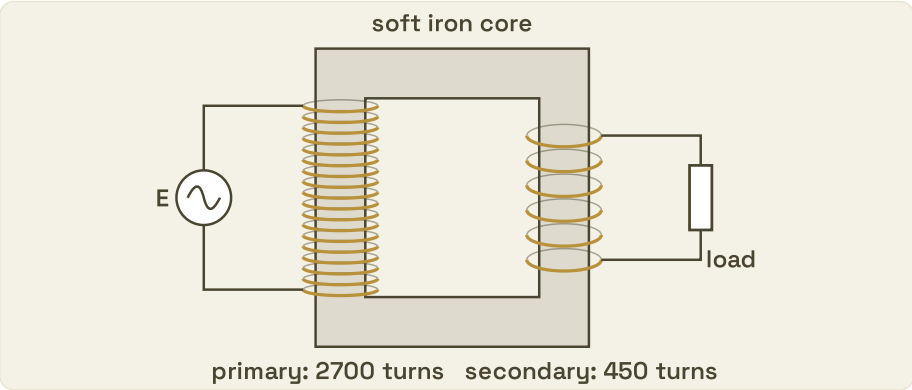


Figure 18.30 *For question 16a: an ideal transformer with 2700 turns on the primary and 450 on the secondary.*

WORKED EXAMPLE

Example 18.5: a growing current beside a loop

A straight wire carries a current from A to B that is increasing. A circular loop lies above the wire, in the same plane. What is the direction of the induced current in the loop?

Answer

By the right-hand grip rule, the field of a current from left to right (A to B) points **out of the page** above the wire. So the flux through the loop is out of the page, and since the current is increasing, it is **increasing**.

By Lenz's law the induced current must oppose this increase: its own field inside the loop must point **into** the page. A current makes a field into the page inside it when it flows **clockwise**. So the induced current in the loop is **clockwise**.

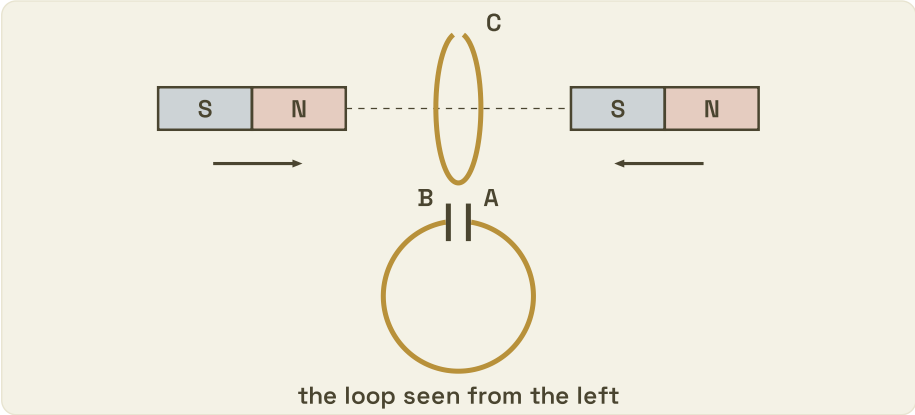


Figure 18.7 *Two magnets moving towards a loop with a capacitor C in a gap at its top (Examples 18.6 and 18.7). Below: the loop as seen from the left.*

WORKED EXAMPLE

Example 18.6: two magnets and a capacitor

Two bar magnets are moved quickly towards a metal loop, one from each side, along its axis. The magnet on the left has its north pole towards the loop; the one on the right has its south pole towards the loop. A capacitor is connected in a gap at the top of the loop (Figure 18.7): seen from the left, plate A is on the right side of the gap and plate B on the left. Find the polarity of the capacitor.

Answer

Field lines leave a north pole and enter a south pole. So **both** magnets make a field through the loop pointing from left to right, and as they approach, the flux to the right through the loop **increases**.

The induced current opposes the increase: its field inside the loop must point to the left, towards the left magnet. Seen from the left, a current whose field points at you flows **anticlockwise**. (The loop's left face becomes a north pole, repelling the approaching north pole; its right face becomes a south pole, repelling the approaching south pole.)

Seen from the left, anticlockwise means that along the top of the loop the current flows from right to left. The wire on the right of the gap brings charge to plate A, and the wire on the left carries charge away from plate B. So **A is positive and B negative**. (Charge doesn't cross the gap: it piles up on the plates, which is why the current lasts only while the magnets move.)

WORKED EXAMPLE

Example 18.7: the same, from the other side

Now the loop is seen from the right, as an observer standing behind the right-hand magnet sees it. The magnets still move towards the loop as in Example 18.6 (the left one with its N pole leading, the right one with its S pole leading). Which plate is positive, and which way does the current flow as seen from the right?

Answer

The current is the same one: anticlockwise seen from the left is **clockwise seen from the right** (turn a clock round and it runs backwards). Nothing about the capacitor changes: **A is still positive**, B negative.

A quick check with poles: the face of the loop towards the right-hand magnet must repel that magnet's approaching **south** pole, so it is a south pole; a face with clockwise current, seen face-on, is a south pole. ✓

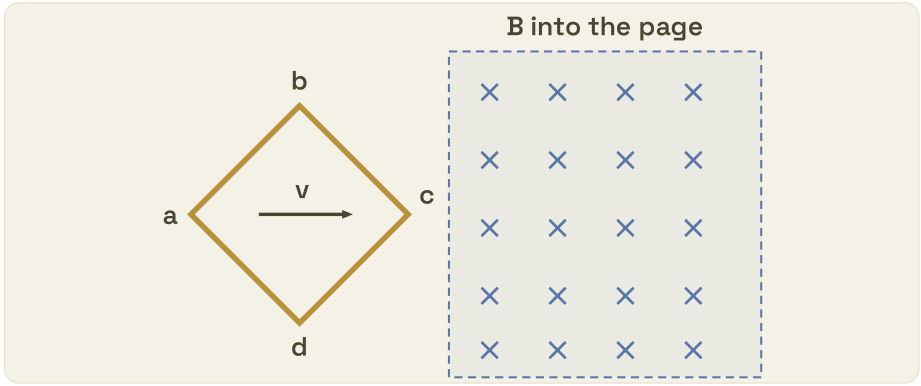


Figure 18.8 *A square loop abcd moving into a field into the page (Test yourself 18.5).*

Check 4

Test yourself 18.6. A metal ring is held horizontal, and a bar magnet is dropped through it along the ring's axis. The acceleration of the falling magnet is:

a more than g
b equal to g
c less than g
d zero

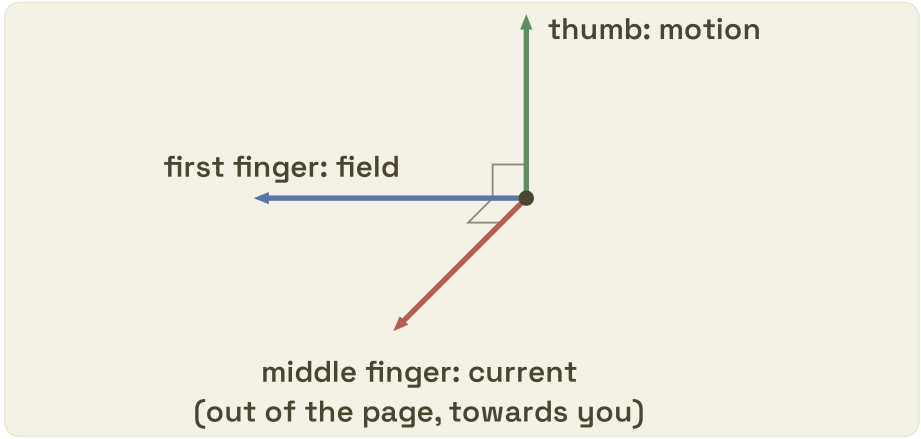


Figure 18.9 *Fleming's right-hand rule: thumb, first finger and middle finger at right angles. Here the conductor moves up, the field points left, and the induced current comes out of the page.*

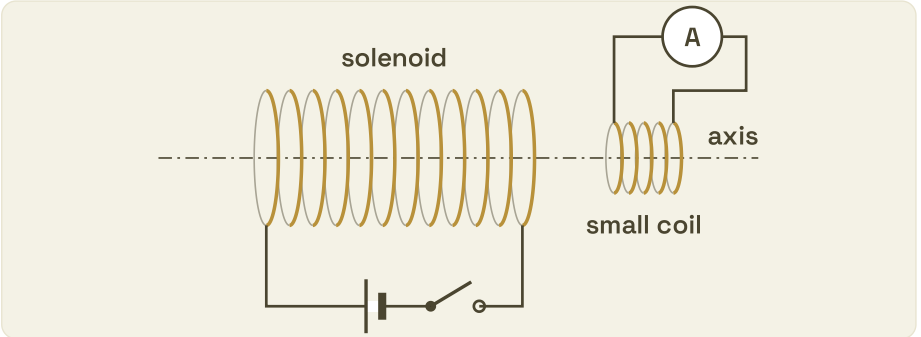


Figure 18.29 *For question 15: a solenoid with a battery and a switch, and a small coil joined to an ammeter on its axis.*

13. Lenz's law gives the direction of an induced e.m.f.
    a. Explain how Lenz's law agrees with the conservation of energy.
    b. A straight vertical conductor carries an increasing current from P (below) to Q (above), with a circular loop to its right in the same plane. Find the direction of the current induced in the loop, and give your reasoning.
    c. What is called the electrical inertia of a circuit? Find its expression for a solenoid.
    d. What is the electrical inertia of a straight conductor?
    e. A wheel with metal spokes 1.2 m long turns in a magnetic field of flux density 5×10^{-5} T at right angles to its plane. An e.m.f. of 10^{-2} V is induced between the rim and the axle. Find the rate of rotation of the wheel.

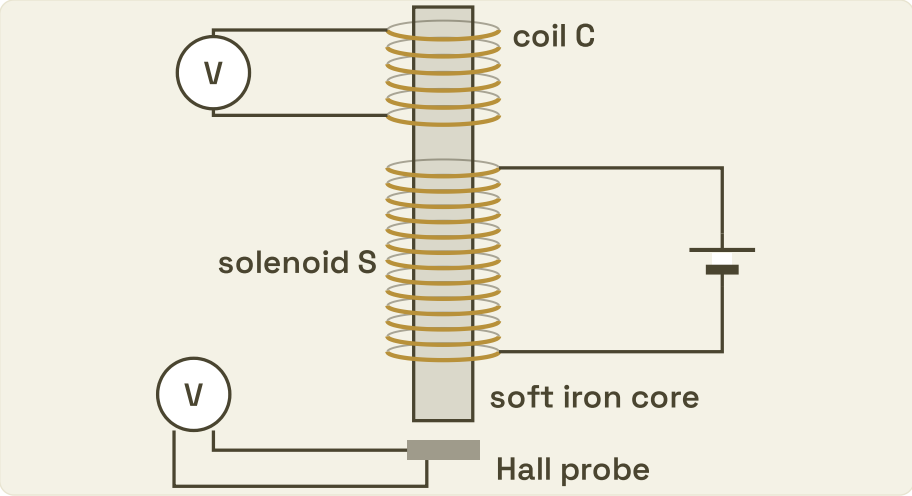


Figure 18.28 *For question 14: a solenoid S and a coil C on a soft iron core, with a Hall probe at its other end.*

CAREFUL

Right hand for generators, left hand for motors

Fleming's **right**-hand rule gives the current **induced** in a conductor that is **moved** (a generator). Fleming's **left**-hand rule (Grade 12) gives the **force** on a conductor that **carries** a current (a motor). Mixing them up reverses your answer.

WORKED EXAMPLE

Example 18.8: two reversals in one turn

A horizontal magnetic field is at right angles to the plane of a vertical loop. The loop is turned about a horizontal axis lying in its plane, at right angles to the field. The current produced in the loop reverses its direction twice in each rotation. Explain why.

Answer

The flux through the loop is $\Phi = BA\cos\theta$, where θ is the angle the loop's normal makes with the field. In one rotation:

- from $\theta = 0°$ to $180°$ the flux **decreases**, from $+BA$ through 0 to $-BA$;
- from $\theta = 180°$ to $360°$ it **increases**, from $-BA$ back to $+BA$.

The direction of the induced current depends on whether the flux is increasing or decreasing (the sign of $d\Phi/dt$). This sign changes twice per rotation, at $\theta = 0°$ and at $\theta = 180°$, where the flux passes its greatest and least values and for an instant stops changing. So the current reverses twice in every rotation, at the moments when the loop is at right angles to the field.

Ways to induce an e.m.f.

From Faraday's experiments, an e.m.f. is induced in a coil whenever the magnetic flux linked with it changes. The flux $\Phi = BA\cos\theta$ can change in several ways:

1. **Relative motion between a coil and a magnet:** moving either of them changes B at the coil.
2. **A changing current in the same coil:** the coil's own field, and so its own flux, changes, and an e.m.f. is induced in the coil itself (**self-induction**, Section 18.5).
3. **A changing current in a nearby coil:** the neighbor's field changes, and an e.m.f. is induced in this coil (**mutual induction**, Section 18.5).
4. **Moving a rod across a magnetic field:** the area the circuit encloses changes, and an e.m.f. is induced across the rod (**motional e.m.f.**, Section 18.3).
5. **Rotating a coil in a magnetic field:** the angle θ changes, the basic principle of the electric **generator**.

A few points worth remembering:

- The induced **e.m.f.** does not depend on the resistance of the circuit; the induced **current** does ($I = \varepsilon/R$).
- The induced e.m.f. in a rotating coil depends on the number of turns, the area of the coil, its angular velocity and the strength of the field (Section 18.3).
- Lenz's law gives the direction of the current round the closed circuit, and follows from the conservation of energy: mechanical energy is converted into electrical energy.

18.3 Motional e.m.f.

The e.m.f. induced in a conductor because it **moves** through a magnetic field is called **motional e.m.f.** The flux through the circuit changes not because the field changes but because the conductor moves.

Motional e.m.f. in a straight rod

A straight conductor PQ of length l slides along two parallel rails PR and QS, joined at RS, in a uniform field B directed into the page (at right angles to the plane of the rails). The rod moves at a steady speed v, at right angles both to itself and to the field. Let x be the rod's distance from RS at some instant.

1. The circuit PQRS encloses an area $A = lx$, and the field is at right angles to it, so the flux through it is
$$\Phi = BA = Blx$$
2. As the rod moves, x changes at the rate $\dfrac{dx}{dt} = v$.
3. By Faraday's law, the magnitude of the induced e.m.f. is

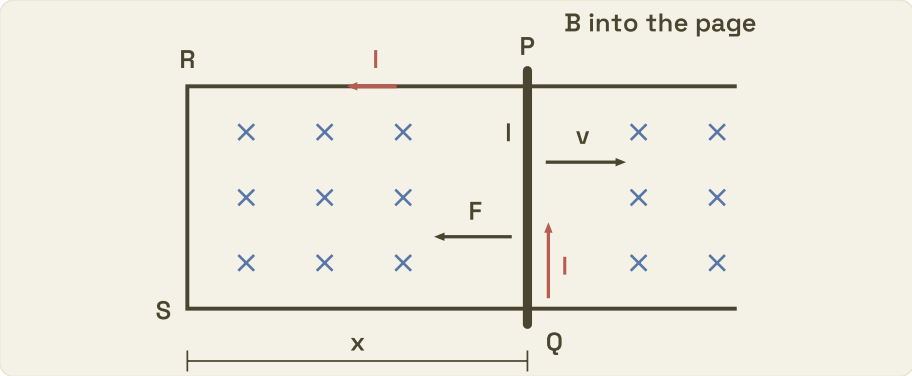


Figure 18.10 *A rod PQ sliding to the right on rails in a field into the page. The induced current I flows up the rod from Q to P, and the force F = BIl on it points back, against the motion.*

4. so

$$\boxed{\varepsilon = Blv}$$

If the rod moves at an angle θ to the field, only the part of its velocity at right angles to the field, v sin θ, cuts the field lines, and

$$\varepsilon = Blv\sin\theta$$

which is greatest, Blv, when the rod moves at right angles to the field (θ = 90°), and zero when it moves along the field (θ = 0).

Why the charges move. A free electron in the rod moves with it at velocity v, so it feels the magnetic force $\vec{F} = q\vec{v} \times \vec{B}$, along the rod. The electrons are pushed to one end, which becomes negative, leaving the other end positive: the rod becomes a cell of e.m.f. Blv. The work done on a unit charge by this force over the length l is (qvB × l)/q = Blv, the same e.m.f. as Faraday's law gives. Its direction is given by Fleming's right-hand rule.

Where the energy comes from. With a resistance R round the circuit, a current I = Blv/R flows in the rod. A current in a field feels a force BIl, and by Lenz's law it acts **against** the motion. To keep the rod moving at speed v, you must push it with an equal force, and your power is

$$P = Fv = BIl \times v = \frac{B^2 l^2 v^2}{R}$$

which is exactly the electrical power $\varepsilon^2/R = (Blv)^2/R$ turned into heat in the circuit. Every joule of electrical energy is a joule of your work.

Check 5

A rod of length 0.5 m moves at 4 m/s at right angles to a field of 0.2 T, on rails joined by a 2 Ω resistor. The force needed to keep it moving steadily is:

a 0.4 N
b 0.2 N
c 0.02 N
d 0

WORKED EXAMPLE

Example 18.9: a falling wire

A horizontal straight wire 10 m long, lying east–west, falls at 5.0 m/s at right angles to the horizontal component of the Earth's magnetic field, 0.30×10^{-4} Wb/m². What is the instantaneous e.m.f. induced in the wire?

Answer

l = 10 m, v = 5.0 m/s, $B = 0.30 \times 10^{-4}$ T, all at right angles to one another.

$$\varepsilon = Blv = 0.30 \times 10^{-4} \times 10 \times 5.0 = \mathbf{1.5 \times 10^{-3}\ V}$$

12. Lenz's law gives the direction of an induced e.m.f.
    a. State Lenz's law.
    b. A straight conductor carries an increasing current from A to B, with a circular loop above it in the same plane. Find the direction of the current induced in the loop, and give your reasoning.
    c. A rectangular metal frame PQRS, 4.0 cm wide (QR) and 7.0 cm tall (PQ), stands in a vertical plane in a uniform horizontal magnetic field of 4.5×10^{-3} T, directed along QR. A current of 2.5 A flows round it, up RS, along RQ and down QP. If the frame turns with a steady angular velocity ω about a vertical axis through its middle, obtain an expression for the e.m.f. induced in it.
    d. Calculate the force on the side PQ.
    e. Why does the side QR feel no force?

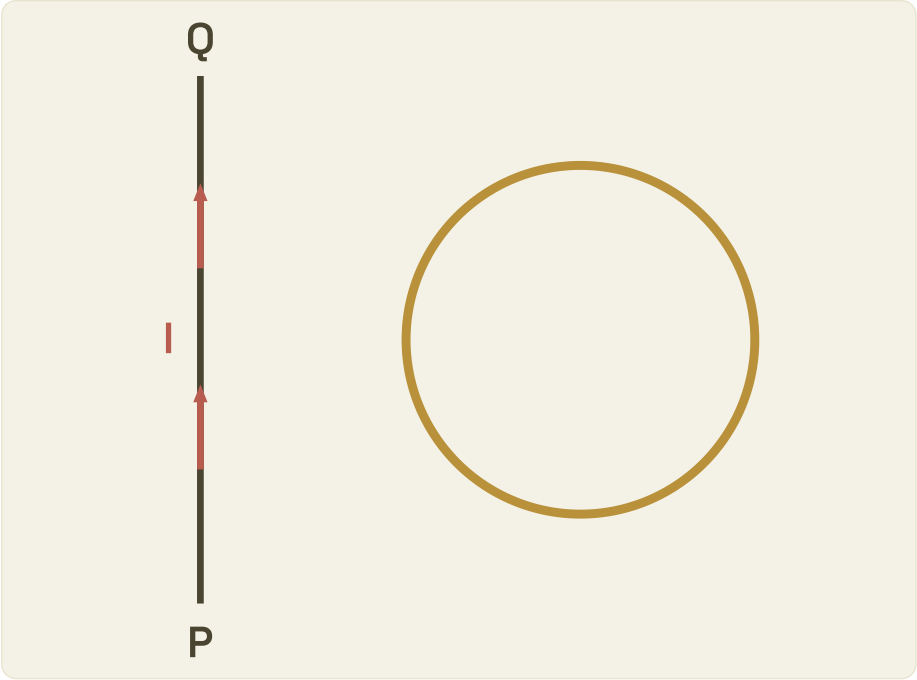


Figure 18.27 *For question 13b.*

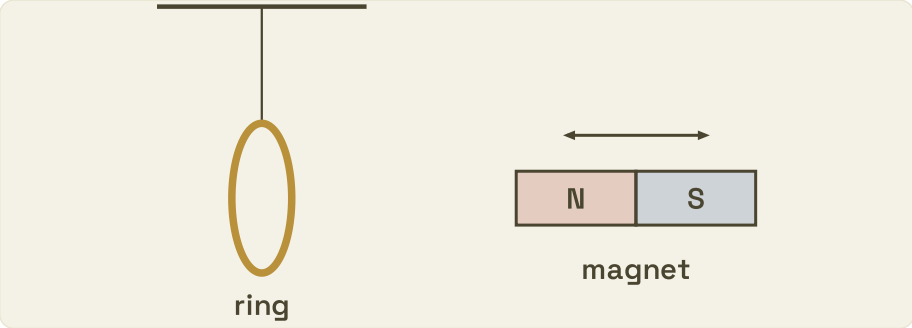


Figure 18.24 For question 11c.

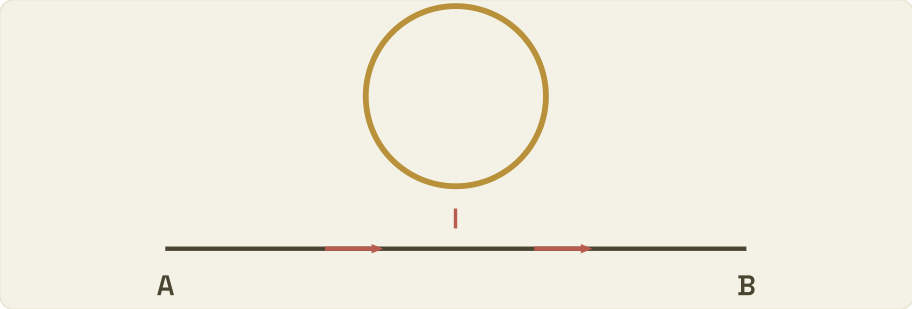


Figure 18.25 For question 12b.

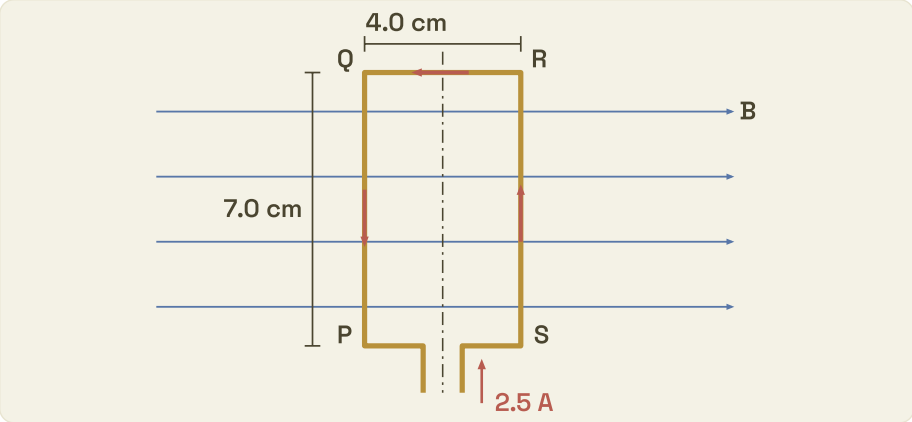


Figure 18.26 For question 12c: the frame PQRS in a horizontal field.

WORKED EXAMPLE

Example 18.10: a conductor on a dynamo's armature

Find the e.m.f. induced in a straight conductor 20 cm long on the armature of a dynamo, 10 cm from the axis, when it moves in a uniform radial field of 0.5 T and the armature turns at 1000 revolutions per minute.

Answer

l = 0.2 m, r = 0.1 m, B = 0.5 T, f = 1000 r.p.m. = 1000/60 rev/s.

The conductor moves in a circle of radius r, so its speed is $v = r\omega = r \times 2\pi f$. In a radial field it always moves at right angles to the field, so

$$\varepsilon = Blv = Bl \times 2\pi fr = 0.5 \times 0.2 \times 2\pi \times \frac{1000}{60} \times 0.1 = \mathbf{1.05\ V}$$

WORKED EXAMPLE

Example 18.11: the wings of an aeroplane

A metal aircraft with a wingspan of 40 m flies east at a ground speed of 1080 km/h, at a constant height, where the vertical component of the Earth's magnetic field is 1.75×10^{-5} T. Find the e.m.f. between the tips of its wings.

Answer

v = 1080 km/h = 1080 × 1000 ÷ 3600 = 300 m/s, l = 40 m.

The wings are horizontal and move horizontally, so only the **vertical** component of the field is cut by them:

$$\varepsilon = B_V lv = 1.75 \times 10^{-5} \times 40 \times 300 = \mathbf{0.21\ V}$$

(Nothing in the plane can use this e.m.f.: any wire joining the wing tips moves with them and has the same e.m.f. induced in it, so no current flows round.)

WORKED EXAMPLE

Example 18.12: a spinning disc

A vertical metal disc of radius 8×10^{-2} m spins about its center at 50 rev/s, with its plane at right angles to a horizontal magnetic field of 0.1 T. Find the e.m.f. between the center and the rim.

Answer

Think of the disc as many spokes, each of length r, spinning about the center. Every point of a spoke moves at right angles to the field, but at different speeds: 0 at the center and rω at the rim. Since the speed grows evenly along the spoke, the average speed is

$$v = \frac{0 + r\omega}{2} = \frac{r\omega}{2}$$

so with l = r,

$$\varepsilon = Blv = Br \times \frac{r\omega}{2} = \frac{1}{2}B\omega r^2 = \frac{1}{2}B \times 2\pi f \times r^2$$

$$\varepsilon = \frac{1}{2} \times 0.1 \times 2\pi \times 50 \times (8 \times 10^{-2})^2 = \mathbf{0.10\ V}$$

This is Faraday's disc dynamo of 1831: a brush on the axle and another on the rim take the current out. All the spokes are in parallel, so the e.m.f. is that of one spoke.

Written another way, $\frac{1}{2}B\omega r^2 = \frac{1}{2}B(2\pi f)r^2 = B(\pi r^2)f = BAf$: the flux density times the disc's area times the turns per second. Each turn, a spoke sweeps the whole area A, cutting a flux BA.

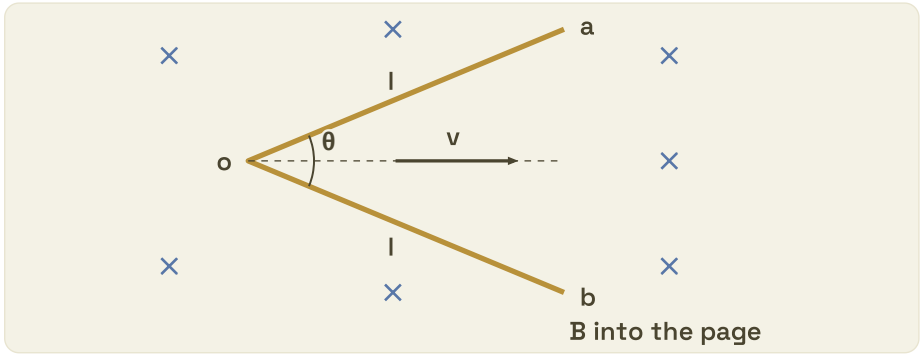


Figure 18.11 *A wire bent into an angle aob, moving along its bisector through a field into the page (Example 18.13).*

WORKED EXAMPLE

Example 18.13: a bent wire

A conducting wire bent into an angle aob, of angle θ, with each arm of length l, moves at speed v along the bisector of the angle (the vertex o behind), through a uniform magnetic field B at right angles to the plane of the angle. Find the e.m.f. between the free ends a and b.

Answer

Only the part of each arm's length at right angles to the velocity cuts field lines. Each arm makes an angle θ/2 with the bisector (the direction of motion), so its effective length is l sin(θ/2), and each arm is like a cell of e.m.f.

$$\varepsilon_1 = Bvl\sin\frac{\theta}{2}$$

By the right-hand rule (or $\vec{v}\times\vec{B}$), the force on positive charges points across the direction of motion, to the same side for both arms: towards a in arm oa, and towards o in arm ob. So the two cells are in **series**, the + end of ob's cell joined to the − end of oa's at o:

$$\varepsilon_{ab} = 2Bvl\sin\frac{\theta}{2}$$

with a at the higher potential (for the field and motion as drawn).

A quicker way: the e.m.f. of any bent wire moving in a uniform field equals that of the straight line joining its ends. The line ab has length 2l sin(θ/2) and moves at right angles to itself at speed v, so ε = B × 2l sin(θ/2) × v. ✓

YOUR TURN

Test yourself 18.7: a wheel with metal spokes

A wheel with metal spokes 1.2 m long is turned in a magnetic field of flux density 5×10^{-5} T, at right angles to the plane of the wheel. An e.m.f. of 10^{-2} V is induced between the rim and the axle. Find the rate of rotation of the wheel.

Hint

Each spoke is a rod turning about one end: ε = ½Bωr².

Answer

$$\omega = \frac{2\varepsilon}{Br^2} = \frac{2\times10^{-2}}{5\times10^{-5}\times1.2^2} = \frac{2\times10^{-2}}{7.2\times10^{-5}} = 278\ \text{rad/s}$$

$$f = \frac{\omega}{2\pi} = \frac{278}{2\pi} = \mathbf{44.2\ rev/s}$$

Check 16

3. Which of these circuit elements opposes a change in the current in a circuit?

a a capacitor
b an inductor
c a resistor
d a resistor and a capacitor

Check 17

4. A coil of N turns and area A turns with angular velocity ω in a uniform magnetic field B. The greatest e.m.f. induced in it is:

a NABω
b ½NABω
c NAω/B
d Bω/NA

Check 18

5. A rectangular loop of sides 8 cm and 2 cm, with a resistance of 1.6 Ω, lies in a magnetic field of 0.3 T at right angles to the loop. The field is steadily reduced at 0.02 T/s. How much power is dissipated in the loop as heat?

a 1.6×10^{-4} W
b 3.2×10^{-8} W
c 6.4×10^{-10} W
d 12.8×10^{-8} W

Check 19

6. A current of 3 A in the primary coil is reduced to zero in 0.001 s, and the e.m.f. induced in the secondary is 15 000 V. The mutual inductance of the two coils is:

a 0.5 H
b 5 H
c 1.5 H
d 10 H

Check 20

7. The magnetic flux linked with a coil is $\Phi = 5t^2 + 3t + 6$ (in webers, with t in seconds). The magnitude of the e.m.f. induced in the coil at t = 4 s is:

a 20 V
b 43 V
c 10 V
d 80 V

Check 21

8. A current changing at 3 A/s in one coil induces an e.m.f. of 8 mV in a nearby coil. The mutual inductance of the two coils is:

a 2.67 H
b 2.67×10^{-3} mH
c 0.267 H
d 2.67 mH

Check 22

9. A coil of wire of radius r has 600 turns and a self-inductance of 108 mH. The self-inductance of a similar coil of 500 turns is:

a 108 mH
b 90 mH
c 75 mH
d 100 mH

Check 23

10. What is the coefficient of mutual induction when the magnetic flux linked with one coil changes by 2×10^{-2} Wb for a change of 0.01 A in the current in the other?

a 3 H
b 0.5 H
c 2 H
d zero

Answer the following

11.

a. State Faraday's laws of electromagnetic induction.

b. A coil has 5000 turns. What e.m.f. is induced in it when the magnetic flux through it changes by 8×10^{-4} Wb in 0.1 s?

c. A light aluminium ring hangs from a long thread, with a bar magnet near it along its axis. When the magnet is moved away from the ring, the ring follows the magnet. Why? What would happen if the magnet were moved

YOUR TURN

Test yourself 18.17: a step-down transformer

A step-down transformer changes a 220 V supply into 100 V. The primary coil has 500 turns. The efficiency of the transformer is 80% and it delivers 80 kW. Find a. the number of turns in the secondary, b. the power supplied to it.

Answer

a. $N_S = N_P \times \dfrac{E_S}{E_P} = 500 \times \dfrac{100}{220} = 227$ turns (to the nearest whole turn).

b. $\eta = \dfrac{\text{output}}{\text{input}} \Rightarrow \text{input} = \dfrac{80\ \text{kW}}{0.80} = 100\ \text{kW} = \mathbf{10^5\ W}$

YOUR TURN

Test yourself 18.18: 4000 V in

A transformer has 500 turns on its primary and 100 turns on its secondary. What is the output voltage if the input voltage is 4000 V? If the transformer is 100% efficient, what primary current is needed to draw 2000 W from the secondary?

Answer

$E_S = E_P \times \dfrac{N_S}{N_P} = 4000 \times \dfrac{100}{500} = \mathbf{800\ V}$

At 100% efficiency the input power is also 2000 W, so

$I_P = \dfrac{2000}{4000} = \mathbf{0.5\ A}$

(The secondary current is 2000/800 = 2.5 A, five times larger, as the turns ratio says.)

IN SHORT

- **Magnetic flux** Φ = BA cos θ (weber, Wb); flux linkage NΦ.
- **Faraday's laws:** an e.m.f. is induced whenever the flux linked with a circuit changes, and $\varepsilon = -N\,d\Phi/dt$. Induced charge $q = N\Delta\Phi/R$, independent of time.
- **Lenz's law:** the induced current opposes the change that causes it: the conservation of energy.
- **Motional e.m.f.:** ε = Blv sin θ; a rod turning about one end, ε = ½Bωr² (= BAf for a spinning disc or wheel). Fleming's right-hand rule gives the direction.
- **Rotating coil (a.c. generator):** ε = NBAω sin ωt, peak ε_0 = NBAω when the coil's plane is parallel to the field.
- **Eddy currents:** induced in solid metal; laminate cores to reduce them; use them for damping, brakes, induction heating.
- **Self-induction:** NΦ = LI, ε = −L dI/dt; a long solenoid has L = μ_0N²A/l. 1 H = 1 V s/A.
- **Mutual induction:** $N_S\Phi_S$ = MI, ε = −M dI/dt.
- **Energy in an inductor:** E = ½LI²; energy density B²/2μ_0.
- **Transformer:** $E_S/E_P = N_S/N_P = I_P/I_S$ (ideal); η = output/input; losses: flux, copper, hysteresis, eddy currents. High-voltage transmission cuts the I²R loss.

Exercise

Choose the correct option

Check 14

1. A conducting wire 10 cm long moves at 10 m/s at right angles to a magnetic field of flux density 0.01 T. The e.m.f. induced in it is:

a 0 V
b 0.01 V
c 0.1 V
d 1.0 V

Check 15

2. Lenz's law is a statement of the law of conservation of:

a charge
b energy
c mass
d linear momentum

YOUR TURN

Test yourself 18.8: a jet's wings

A jet plane travels due west at 1800 km/h. What p.d. is set up between the ends of its wings, 25 m apart, if the Earth's magnetic field there is 5×10^{-4} T and the angle of dip is 30°?

Hint

The horizontal wings, moving horizontally, cut only the vertical component of the field, B sin δ.

Answer

v = 1800 km/h = 500 m/s. The vertical component of the field is

$$B_V = B\sin\delta = 5 \times 10^{-4} \times \sin 30^\circ = 2.5 \times 10^{-4}\ \text{T}$$

$$\varepsilon = B_V l v = 2.5 \times 10^{-4} \times 25 \times 500 = \mathbf{3.1\ V}$$

(The Earth's real field is about ten times weaker, near 5×10^{-5} T, which would give about 0.3 V.)

YOUR TURN

Test yourself 18.9: a wire in a field given in gauss

A straight wire 2 m long moves at 500 cm/s at right angles to a uniform field of 100 gauss. Find the e.m.f. induced, in volts.

Answer

1 gauss = 10^{-4} T, so B = 100 G = 0.01 T; v = 500 cm/s = 5 m/s.

$$\varepsilon = Blv = 0.01 \times 2 \times 5 = \mathbf{0.1\ V}$$

YOUR TURN

Test yourself 18.10: a loop with a sliding arm

A rectangular loop PQMN has a movable arm PQ of length 10 cm and resistance 2 Ω. It lies in a uniform magnetic field of 0.1 T at right angles to its plane. The resistances of the arms MN, NP and MQ are negligible. When the arm PQ moves at 20 m/s, find a. the e.m.f. induced in PQ, b. the current induced in the loop.

Answer

a. $\varepsilon = Blv = 0.1 \times 0.10 \times 20 = \mathbf{0.2\ V}$

b. The only resistance in the loop is the arm's 2 Ω: $I = \dfrac{\varepsilon}{R} = \dfrac{0.2}{2} = \mathbf{0.1\ A}$

Motional e.m.f. in a rotating coil

A rectangular coil of N turns and area A turns at a steady angular velocity ω in a uniform magnetic field B, about an axis in its plane that is at right angles to the field. Let θ be the angle between the **normal** to the coil and the field at time t, starting from θ = 0 (field straight through the coil) at t = 0, so that θ = ωt.

1. The flux linkage of the coil is

$$N\Phi = NBA\cos\theta = NBA\cos\omega t$$

2. By Faraday's law,

$$\varepsilon = -\frac{d(N\Phi)}{dt} = -\frac{d}{dt}(NBA\cos\omega t) = -NBA(-\omega\sin\omega t)$$

3. so

$$\varepsilon = NBA\omega\sin\omega t \qquad \ldots (18.1)$$

As time goes on, sin ωt rises from 0 to 1, falls back to 0, goes to −1 and back to 0, again and again: the e.m.f. keeps reversing. An e.m.f. like this is an **alternating e.m.f.** It is greatest when sin ωt = ±1, that is when θ = 90° or 270°: when the **plane of the coil is parallel to the field**. Then the flux through the coil is zero, but it is changing fastest, because the coil's sides are moving straight across the field lines. The greatest value,

$$\varepsilon_0 = NBA\omega \qquad \ldots (18.2)$$

is the **peak value** of the alternating e.m.f. It depends on the number of turns, the strength of the field, the area of the coil and its angular velocity. In terms of it,

$$\varepsilon = \varepsilon_0\sin\omega t \qquad \ldots (18.3)$$

As the coil turns:

i. at $\theta = 0°$ (coil at right angles to the field, greatest flux), $\varepsilon = \varepsilon_0 \sin 0° = 0$;
ii. at $\theta = 90°$ (coil parallel to the field, no flux), $\varepsilon = \varepsilon_0 \sin 90° = \varepsilon_0$;
iii. at $\theta = 180°$, $\varepsilon = \varepsilon_0 \sin 180° = 0$;
iv. at $\theta = 270°$, $\varepsilon = \varepsilon_0 \sin 270° = -\varepsilon_0$;
v. at $\theta = 360°$, $\varepsilon = \varepsilon_0 \sin 360° = 0$, and the cycle repeats.

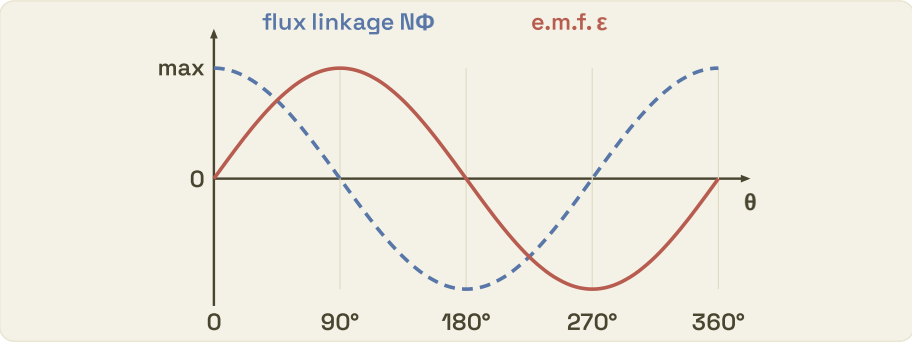


Figure 18.12 *Flux linkage (dashed) and e.m.f. of a coil turning in a uniform field, against the angle θ between its normal and the field.*

Notice that the e.m.f. is zero where the flux is greatest, and greatest where the flux is zero: the e.m.f. follows the **slope** of the flux graph, not its height.

Check 6
A coil turns in a uniform field. Its induced e.m.f. is greatest when:
a the plane of the coil is at right angles to the field
b the plane of the coil is parallel to the field
c the flux through the coil is greatest
d the coil is at 45° to the field

WORKED EXAMPLE
Example 18.14: a 2000-turn coil

A rectangular coil of 0.10 m × 0.5 m, with 2000 turns, turns about an axis parallel to its long side at 2100 revolutions per minute, in a field of 0.1 T at right angles to the axis. What is the greatest e.m.f. induced in the coil? Also find the e.m.f. at the instant when the plane of the coil makes an angle of 30° with the field.

Answer
N = 2000, A = 0.10 × 0.5 = 0.05 m², B = 0.1 T, f = 2100/60 = 35 rev/s.

$$\omega = 2\pi f = 2\pi \times 35 = 220 \text{ rad/s}$$

$$\varepsilon_0 = NBA\omega = 2000 \times 0.1 \times 0.05 \times 220 = \mathbf{2200\ V}$$

In $\varepsilon = \varepsilon_0 \sin\theta$, θ is the angle between the **normal** to the coil and the field. With the plane of the coil at 30° to the field, the normal is at $90° - 30° = 60°$ to it:

$$\varepsilon = \varepsilon_0 \sin 60° = 2200 \times 0.866 = \mathbf{1.9 \times 10^3\ V}$$

(If instead the normal were at 30° to the field, the plane at 60°, the e.m.f. would be 2200 sin 30° = 1100 V. Always check which angle a question gives.)

The a.c. generator

An **a.c. generator** (alternator) converts mechanical energy into electrical energy.

Principle: electromagnetic induction. When a coil turns about an axis at right angles to a uniform magnetic field, the flux through it keeps changing, and an alternating e.m.f. is induced in it.

Construction:

1. **Armature:** a rectangular coil of many turns of insulated copper wire, wound on a soft iron core, which increases the magnetic flux through it.
2. **Field magnet:** a strong magnet with concave (curved) pole pieces. The armature turns between them, its axis at right angles to the field.

WORKED EXAMPLE
Example 18.24: a transformer on d.c.

Can a transformer work on d.c.? Justify your answer.

Answer
No. With d.c. on the primary, the current in it is steady (once it has grown, in the first instant), so the magnetic flux it makes in the core is constant. The flux linked with the secondary then does not change, so no e.m.f. is induced in it. A transformer needs a **changing** flux, which only a changing (alternating) current provides.
Worse, with d.c. the primary has no back e.m.f. to limit its current, only its small resistance: a large current flows and the primary can burn out.

WORKED EXAMPLE
Example 18.25: a laminated core

Why is the core of a transformer laminated?

Answer
The alternating flux in the core induces e.m.f.s in the core itself. If the core were a solid block of iron, these would drive large eddy currents round it, wasting energy as heat and heating the transformer. Making the core of thin sheets insulated from one another (laminations), lying along the flux, breaks up the eddy currents' paths: they are confined to thin sheets of high resistance, and the loss becomes small.

WORKED EXAMPLE
Example 18.26: an ideal transformer

What is an ideal transformer?

Answer
An ideal transformer is one with no energy losses: no flux leakage, no copper losses and no iron losses. All the power given to the primary is delivered by the secondary, so its efficiency is 100% and

$$E_S I_S = E_P I_P$$

where I_P and I_S are the currents in the primary and secondary, and E_P and E_S the voltages across them.

WORKED EXAMPLE
Example 18.27: a step-up transformer

In a step-up transformer the transformation ratio is 100. The primary voltage is 220 V and the input power is 1100 W. The primary has 100 turns. Calculate a. the number of turns in the secondary, b. the current in the primary, c. the voltage across the secondary, d. the current in the secondary (assume the transformer is ideal).

Answer
$k = N_S/N_P = 100$, $N_P = 100$, $E_P = 220$ V, input power $E_P I_P = 1100$ W.

a. $N_S = kN_P = 100 \times 100 = \mathbf{10\,000}$ turns.
b. $I_P = \dfrac{1100}{E_P} = \dfrac{1100}{220} = \mathbf{5\ A}$
c. $E_S = kE_P = 100 \times 220 = \mathbf{22\,000\ V}$
d. $\dfrac{E_S}{E_P} = \dfrac{I_P}{I_S} \Rightarrow I_S = \dfrac{E_P}{E_S} I_P = \dfrac{220}{22\,000} \times 5 = \mathbf{0.05\ A}$

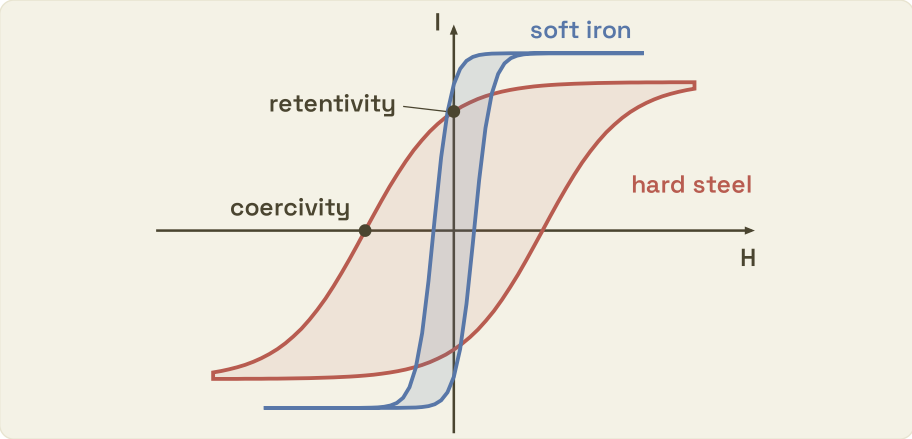


Figure 18.22 *Hysteresis loops of hard steel (wide) and soft iron (narrow): magnetization I against magnetizing field H. The area of a loop is the energy lost per unit volume in each cycle.*

Notice in the figure that soft iron's retentivity is not small: in a closed core it can be as large as steel's, or larger. What makes a magnet "permanent" is a high **coercivity**, so that knocks, heat and stray fields can't undo it; a small reverse field, or a knock, wipes out soft iron's magnetism.

Transformers and long-distance transmission

A power station may be far from the towns that use its electricity. The power lost as heat in the transmission lines, of resistance R, is I^2R, so for a given power $P = VI$, the lower the current, the less is lost. Stepping the voltage up **n** times cuts the current to $1/n$ of its value, and the heat loss to $\mathbf{1/n^2}$.

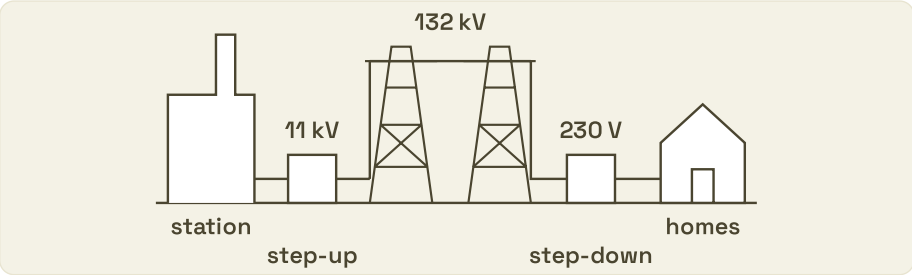


Figure 18.23 *From the power station to homes: a step-up transformer for the long line, a step-down transformer near the town.*

So at the power station a **step-up** transformer raises the generator's voltage (in Nepal, usually 11 kV) to 132 kV or 220 kV for the long lines, or 400 kV on the Dhalkebar–Muzaffarpur line to India. Near the towns, **step-down** transformers lower it in stages: to 33 kV or 11 kV for local lines, and finally to 230 V on the pole outside your home. High voltage is dangerous, so it is stepped down before it reaches homes, and appliances are made for 230 V; as the voltage comes down, the current in the local wires goes up.

In short, about transformers and transmission:

1. A **soft iron core** carries the flux from the primary to the secondary, so that nearly all of it links both coils.
2. **Laminated cores** reduce eddy currents and overheating in transformers, and the laminated armatures of motors and generators do the same for them.
3. **A.c. is used** for transmission because transformers can change its voltage easily and with little loss.
4. **High voltage, low current** reduces the heat lost in the lines ($H = I^2Rt$).

WORKED EXAMPLE

Example 18.15: a 50-turn square coil

A coil of 50 turns, 10 cm × 10 cm, turns at 5 rev/s in a uniform field of flux density 0.7 T. What is the greatest e.m.f. induced in it?

Answer

N = 50, A = 10 × 10 cm² = 100 × 10⁻⁴ m² = 0.01 m², B = 0.7 T, f = 5 rev/s.

$$\omega = 2\pi f = 2 \times \frac{22}{7} \times 5 = \frac{220}{7} \text{ rad/s}$$

$$\varepsilon_0 = NBA\omega = 50 \times 0.7 \times 0.01 \times \frac{220}{7} = \mathbf{11\ V}$$

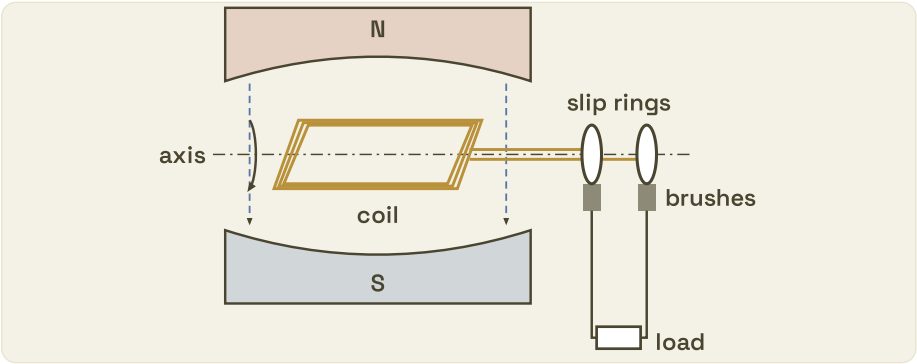


Figure 18.13 *A simple a.c. generator: a coil turning between the poles of a magnet, its ends joined to two slip rings on the axle, with brushes taking the current to the load.*

3. **Slip rings:** two brass rings, each joined to one end of the coil, fixed on the axle so that they turn with the armature.
4. **Brushes:** two strips of carbon (or flexible metal) that press against the slip rings and stay still, carrying the current from the turning coil to the external circuit (the load).

Working: an engine, a water turbine or a steam turbine turns the armature at a steady angular velocity ω. With N turns of area A in a field B, and the normal to the coil at θ = ωt to the field, the flux linkage is

$$N\Phi = NBA\cos\omega t \qquad \text{... (18.4)}$$

and by Faraday's law the e.m.f. is

$$\varepsilon = -\frac{d}{dt}(NBA\cos\omega t) = NBA\omega\sin\omega t \qquad \text{... (18.5)}$$

Connected to an external circuit of total resistance R, it drives a current

$$I = \frac{\varepsilon}{R} = \frac{NBA\omega}{R}\sin\omega t \qquad \text{... (18.6)}$$

which is also sinusoidal: it rises, falls, reverses and repeats, once every turn of the coil. Each slip ring stays joined to the same end of the coil, so the current in the external circuit reverses whenever the current in the coil does: this is **alternating current**.

DID YOU KNOW?

50 times a second

Nepal's mains supply is 230 V at **50 Hz**. In a hydropower station, the turbine turns the generator's magnets (the rotor) inside fixed coils (the stator): it is the same induction, with the magnet turning instead of the coil, which spares the brushes the full current. A large generator has many pairs of poles, so it can turn slower than 50 turns a second: with 10 pairs of poles, it gives 50 Hz turning at only 300 revolutions per minute.

Check 7

Test yourself 18.11. What change is needed to turn an a.c. generator into a d.c. generator?

a the armature with another coil
b the concave magnets with a horseshoe magnet
c the slip rings with a split ring (commutator)
d all of the above

Check 8

Test yourself 18.12. What decides the frequency of the a.c. produced by a simple generator?

a only the number of turns of the coil per second
b only the speed of rotation of the coil
c the coil's rotation, whether given as turns per second or as its speed of rotation
d the strength of the magnetic field

18.4 Eddy currents

Induced currents don't need a coil. When a solid piece of metal, a sheet or a block, is in a **changing** magnetic field, or moves through a field, the changing flux induces e.m.f.s in it, and currents flow round closed paths inside the metal itself, swirling like eddies in a river. These are **eddy currents** (also called Foucault currents, after Léon Foucault, who studied them in 1855).

Bring the north pole of a magnet towards a metal block: the flux through the block's face increases, and eddy currents flow round in its face, anticlockwise as the magnet sees them, making the face a north pole that opposes the approach. Like any induced current, **eddy currents oppose the motion that causes them** (Lenz's law). A metal block has very little resistance, so the currents can be large: they **heat** the metal and **brake** its motion, strongly if it is a disc spinning between the poles of a magnet.

Undesirable effects: in the iron cores of transformers, motors and generators, eddy currents waste energy as heat and can overheat the machine. To reduce them, the cores are made of thin iron sheets (**laminations**), each coated with varnish or an oxide layer to insulate it from the next. The sheets lie along the field, so the flux is not hindered, but an eddy current would have to cross from sheet to sheet, through the insulation, to go round: it is confined to a thin sheet, where the induced e.m.f. is small and the path's resistance is large. The heat loss falls roughly as the square of the sheet's thickness.

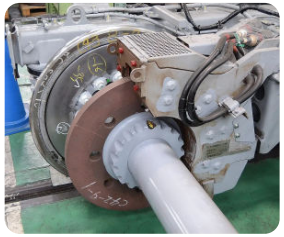


Figure 18.14 *An eddy current brake on a train's axle: electromagnets sit beside a steel disc that turns with the wheel. The eddy currents induced in the disc oppose its motion and slow it down, with no parts rubbing.*
Photo: Toshinori baba, CC BY-SA 4.0

Useful effects:

1. **Electromagnetic damping:** the coil of a moving-coil galvanometer is wound on a metal (aluminium) frame. When the coil swings, eddy currents in the frame oppose the motion, so the pointer comes to rest at its reading quickly instead of swinging to and fro (a **dead-beat** galvanometer).
2. **Induction furnace:** a high-frequency alternating current in a coil induces large eddy currents in a metal placed inside, heating it up to its melting point. It is used to melt and alloy metals without flames or fuel, even in a vacuum.

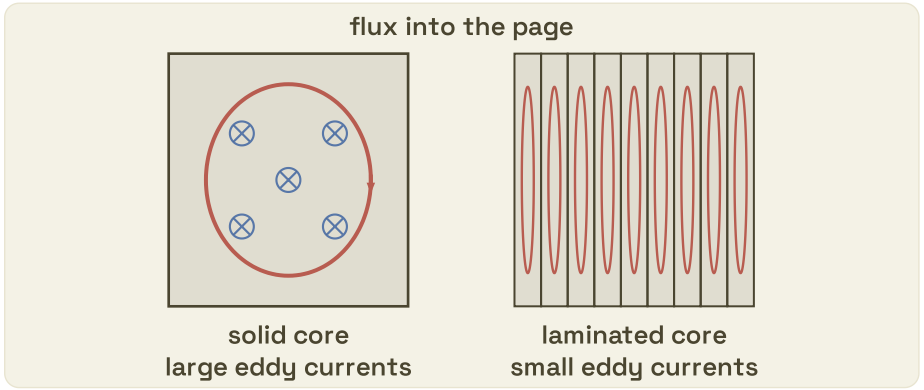


Figure 18.15 *Eddy currents in a solid core (one large loop) and in a laminated core (a small loop in each thin, insulated sheet), with the flux into the page.*

$$E_S = -N_S \frac{d\Phi}{dt} \qquad \dots (18.8)$$

3. Dividing (18.8) by (18.7):

$$\boxed{\frac{E_S}{E_P} = \frac{N_S}{N_P}}$$

The ratio N_S/N_P (= k) is the **transformation ratio** (turns ratio). For a step-up transformer it is greater than 1; for a step-down transformer, less than 1. Since the self-inductance of each coil on the same core is proportional to the square of its turns ($L \propto N^2$), it also equals $\sqrt{L_S/L_P}$.

Currents. In an **ideal transformer** there are no energy losses, so at every instant the output power equals the input power:

$$E_S I_S = E_P I_P \quad \Rightarrow \quad \frac{E_S}{E_P} = \frac{I_P}{I_S} \qquad \dots (18.9)$$

Putting it all together:

$$\frac{E_S}{E_P} = \frac{N_S}{N_P} = \sqrt{\frac{L_S}{L_P}} = \frac{I_P}{I_S}$$

A step-up transformer raises the voltage by **lowering** the current in the same ratio, and a step-down transformer lowers the voltage by **raising** the current, as the conservation of energy requires. A transformer only transforms voltages: it is **not a generator** of electrical energy. And it works only on **a.c.**: a steady current makes a steady flux, which induces nothing.

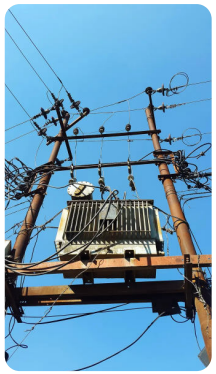


Figure 18.21 *A distribution transformer on a pole in Nepal. It steps the 11 kV of the local line down to 400 V, shared out along the street as 230 V to each home.*
Photo: Bijay Chaurasia, CC BY-SA 4.0

Efficiency. A real transformer loses a little energy, so its output power is a little less than its input. Its **efficiency** is the output power as a fraction (usually a percentage) of the input power:

$$\eta = \frac{\text{output power}}{\text{input power}} \times 100\% = \frac{E_S I_S}{E_P I_P} \times 100\% \qquad \dots (18.10)$$

Large power transformers reach 98–99%: among the most efficient machines ever built.

Check 13

An ideal transformer has 200 turns on its primary and 1000 on its secondary. With 230 V a.c. on the primary and a current of 0.5 A drawn from the secondary, the primary current is:

a 0.1 A
b 0.5 A
c 2.5 A
d 5 A

Energy losses in a transformer

1. **Flux leakage:** the coupling is never perfect, and some of the primary's flux doesn't pass through the secondary. It is reduced by winding the two coils one over the other, on a closed core.
2. **Copper loss:** the resistance of the windings turns some energy into heat (I^2R). It is reduced by using thick copper wire, especially for the coil carrying the larger current.
3. **Iron losses**, in the core:
   i. **Hysteresis loss:** the alternating current magnetizes the core one way, then the other, 50 times a second, and each cycle of magnetization wastes some energy as heat. The energy lost per unit volume in each cycle equals the area of the core's **hysteresis loop**. It is reduced by using a core of **soft iron** (or special silicon steel), whose loop is narrow, with a small area.
   ii. **Eddy-current loss:** eddy currents induced in the core heat it. It is reduced by **laminating** the core.
4. **Humming:** the alternating magnetization makes the core's sheets vibrate a little, 100 times a second, making the transformer's hum; a little energy goes as sound.

Hysteresis. When a ferromagnetic material is taken round a full cycle of magnetizing field H (one way, back to zero, the other way, and back), its magnetization I lags behind the field, and the graph of I against H is a closed loop. (A graph of the flux density B against H has the same shape: in iron, $B = \mu_0(H + I)$ is almost all $\mu_0 I$.) Where the loop cuts the I axis (H = 0) is the **retentivity** (the magnetization left behind); where it cuts the H axis (I = 0) is the **coercivity** (the reverse field needed to remove the magnetization).

- **Soft iron** has a **low coercivity** and a narrow loop of small area. Its **permeability** and **susceptibility** are high, so a small field magnetizes it strongly, and a small reverse field demagnetizes it, with little loss each cycle. It is used for transformer cores and electromagnets.
- **Hard steel** has a **high coercivity** and a wide loop of large area, and a lower permeability and susceptibility. It keeps its magnetism, so it is used for permanent magnets, but it would waste a lot of energy in a transformer.

WORKED EXAMPLE

Example 18.23: the energy in a flat coil

A flat circular coil has 200 turns and a radius of 0.10 m. When it is connected to a battery and switched on, a current of 2 A is set up in it. Calculate the energy stored in the coil.

Answer

N = 200, r = 0.10 m, I = 2 A.

From Example 18.19, a flat coil has $L = \frac{\mu_0 \pi N^2 r}{2}$ (taking $B = \mu_0 NI/2r$ over the whole face). So

$$E = \frac{1}{2}LI^2 = \frac{1}{2} \times \frac{\mu_0 \pi N^2 r}{2} \times I^2 = \frac{\mu_0 \pi N^2 I^2 r}{4}$$

$$E = \frac{4\pi \times 10^{-7} \times \pi \times 200^2 \times 2^2 \times 0.10}{4} = \mathbf{1.58 \times 10^{-2}\ J}$$

18.7 Transformer

A **transformer** is a device that changes an alternating voltage: a low alternating voltage at a high current into a high voltage at a low current, or the other way round.

Principle: mutual induction. A changing current, and so a changing magnetic flux, in one coil induces an e.m.f. in another coil linked with the same flux.

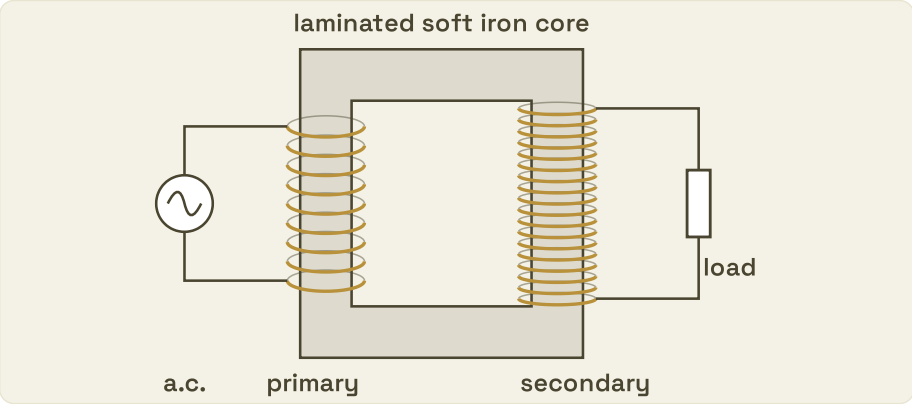


Figure 18.20 A step-up transformer: the primary and secondary coils on one laminated soft iron core, the secondary with more turns.

Construction: two separate coils of insulated copper wire, the **primary** P and the **secondary** S, wound on the same **laminated soft iron core**. The a.c. supply is connected to the primary, and the output is taken from the secondary. The core is made of thin strips of soft iron, coated with varnish to insulate them from one another, to reduce the energy lost to eddy currents.

Types:

- **Step-up transformer:** the secondary has **more** turns than the primary, $N_S > N_P$. It turns a low voltage at a high current into a high voltage at a low current.
- **Step-down transformer:** the secondary has **fewer** turns, $N_S < N_P$. It turns a high voltage at a low current into a low voltage at a high current.

Theory. An alternating e.m.f. E_P is applied to the primary. The alternating current in it makes an alternating flux in the core. Assume the core carries **all** of this flux through the secondary too (no flux leaks out), so the flux Φ through each turn is the same in both coils. If it changes at the rate $d\Phi/dt$:

1. The e.m.f. induced in the primary, which (for a coil of negligible resistance) balances the applied e.m.f., is

$$E_P = -N_P \frac{d\Phi}{dt} \qquad \text{... (18.7)}$$

because the flux changes through each of its N_P turns.

2. The e.m.f. induced in the secondary is

3. **Induction cooker:** the same idea in a kitchen: a coil under the glass makes eddy currents in the steel bottom of the pan, so the pan itself gets hot.
4. **Induction motor:** a rotating magnetic field induces eddy currents in the rotor, and the force on them drags the rotor round after the field. The rotor always turns a little **slower** than the field (if it kept up, the flux through it would stop changing and the currents would stop).
5. **Electromagnetic brakes:** in some trains and roller-coasters, strong magnets near a metal disc or rail, which turns or moves with the wheels, induce eddy currents that brake it smoothly, with no wear.
6. **Diathermy (inductothermy):** eddy currents induced by high-frequency fields are used to heat tissues deep inside the body, for treatment.
7. **Speedometer:** in an old-style vehicle speedometer, a magnet turned by the wheels spins inside an aluminium cup. Eddy currents in the cup drag it round against a hairspring, by an amount that grows with the speed, moving the pointer.
8. **Metal detectors** at airports and in mine-clearing work by sensing the eddy currents a changing field induces in buried or hidden metal.



Figure 18.16 Water boiling on an induction cooktop, with a newspaper under the pot that doesn't even scorch: the heat is made in the pan's own metal, by eddy currents.
Photo: Wtshymanski (talk)., CC BY-SA 3.0

Check 9

A transformer's core is laminated to:

a increase the flux through the core
b reduce eddy-current losses
c reduce copper losses in the windings
d make the core lighter

18.5 Self and mutual induction

Self-induction

A coil is connected to a battery E and a key K. When the key is pressed, a current starts to flow and grows. As it grows, the coil's own magnetic field grows, so the flux through the coil's own turns **increases**, and by Faraday's law an e.m.f. is induced in the coil itself. By Lenz's law this e.m.f. **opposes the growth of the current**.

When the key is released, the current falls, the coil's field and flux decrease, and again an e.m.f. is induced, this time **opposing the decrease**: it tries to keep the current going (which is why you may see a spark at the key).

In both cases the e.m.f. induced in the coil opposes the change of its own current. It is called the **back e.m.f.**

Self-induction is the production of an induced e.m.f. in a coil because of a change in the current, and so in the magnetic flux, linked with the coil itself.

So a coil opposes any **change** in its current, as a heavy body opposes any change in its velocity: self-induction is the **electrical inertia** of a circuit.

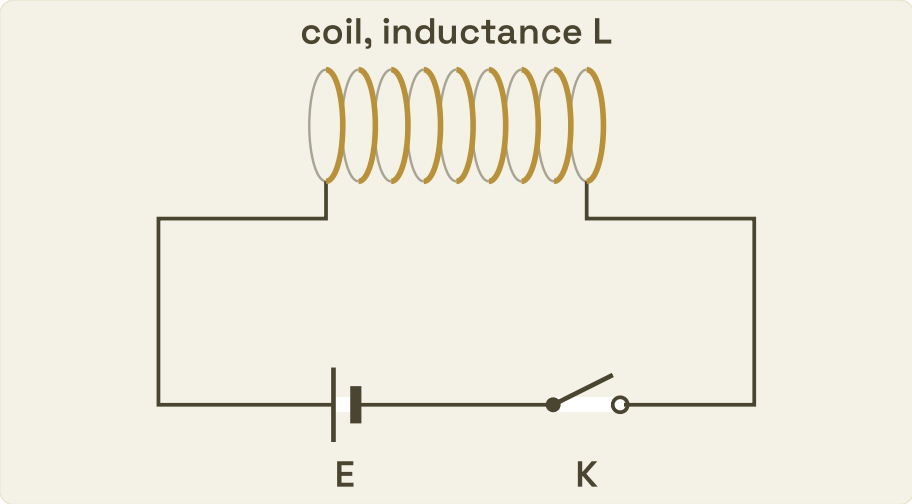


Figure 18.17 *A coil in series with a cell E and a key K: closing or opening the key changes the coil's own flux.*

Coefficient of self-induction

The flux linked with a coil is caused by its own current, so it is directly proportional to the current I:

$$N\Phi \propto I \quad \Rightarrow \quad N\Phi = LI$$

The constant L is the **coefficient of self-induction**, or the **self-inductance**, of the coil. It depends on the number of turns, the area of cross-section and the permeability of the core material. For a single loop (N = 1) this is Φ = LI.

The induced e.m.f. is then

$$\varepsilon = -\frac{d(N\Phi)}{dt} = -\frac{d}{dt}(LI) \quad \Rightarrow \quad \boxed{\varepsilon = -L\frac{dI}{dt}}$$

where dI/dt is the rate of change of the current in the coil. The minus sign shows that the e.m.f. opposes the change in current.

Unit of self-inductance. The SI unit is the **henry** (H), after the American physicist Joseph Henry, who found self-induction in 1832. From ε = L dI/dt, 1 H = 1 V s/A, and from NΦ = LI, 1 H = 1 Wb/A. The self-inductance of a coil is **one henry** if a current changing at **1 ampere per second** induces an e.m.f. of **1 volt** in it.

WORKED EXAMPLE

Example 18.16: the back e.m.f. of a coil

The current in a 10 mH coil increases uniformly from zero to 1 A in 0.01 s. Find the self-induced e.m.f.

Answer

L = 10 mH = 10^{-2} H, dI = 1 − 0 = 1 A, dt = 0.01 s.

$$\varepsilon = -L\frac{dI}{dt} = -10^{-2} \times \frac{1}{0.01} = -\mathbf{1\ V}$$

Its magnitude is 1 V. The minus sign says it acts to oppose the growth of the current.

YOUR TURN

Test yourself 18.16: 12 mV from a changing current

Calculate the mutual inductance of two coils when a current changing from 2.0 A to 5.0 A in 0.25 s in one coil induces an e.m.f. of 12 mV in the other.

Answer

$$\frac{dI}{dt} = \frac{5.0 - 2.0}{0.25} = 12\ \text{A/s}$$

$$M = \frac{\varepsilon}{dI/dt} = \frac{12 \times 10^{-3}}{12} = \mathbf{1.0 \times 10^{-3}\ H}$$

that is, 1 mH.

18.6 Energy stored in an inductor

When a current is switched on in a coil, the back e.m.f. opposes its growth, and the source must do **work** to push charge against it. Once the current is steady, the back e.m.f. is zero and no more work is needed against it. The work done while the current grew is not lost: it is stored as **magnetic potential energy** in the coil's magnetic field. It is the **energy stored in an inductor**, and it comes back out when the current falls (as the spark when the circuit is broken).

Derivation. At some instant the current is i, growing at the rate di/dt.

1. The back e.m.f. is $\varepsilon = L\,\dfrac{di}{dt}$ (in size).
2. To push a charge dq = i dt against it, the source does work
$$dW = \varepsilon\, dq = L\frac{di}{dt} \times i\, dt = Li\, di$$
3. The total work done while the current grows from 0 to I is
$$W = \int_0^I Li\, di = L\left[\frac{i^2}{2}\right]_0^I = \frac{1}{2}LI^2$$
4. This is the energy stored:
$$\boxed{E = \frac{1}{2}LI^2}$$

Compare it with the kinetic energy of a moving body, $\frac{1}{2}mv^2$. Mass resists changes in velocity; inductance resists changes in current. Inductance plays the part of **inertia**, and current the part of velocity. A resistor is different: it turns electrical energy into heat, for good (I^2R, all the time a current flows). An inductor only **stores** energy for a while, in its magnetic field, and can give it back to the circuit, as a spring stores and returns mechanical energy.

The energy depends on:

1. the **inductance** L: for a given current, a larger inductance stores more energy;
2. the **current** I: the energy is proportional to the **square** of the current (double the current, four times the energy).

Where the energy is. For a long solenoid, $L = \mu_0 N^2 A/l$ and $B = \mu_0 NI/l$, so

$$E = \frac{1}{2}\frac{\mu_0 N^2 A}{l} I^2 = \frac{1}{2\mu_0}\left(\frac{\mu_0 NI}{l}\right)^2 Al = \frac{B^2}{2\mu_0} \times Al$$

Al is the volume inside the solenoid, where its field is. So the energy is stored in the magnetic field itself, with an **energy density** (energy per unit volume) of

$$u = \frac{B^2}{2\mu_0}$$

Check 12

The current in a coil is doubled. The energy stored in its magnetic field becomes:

a twice as much
b four times as much
c half as much
d the same

WORKED EXAMPLE

Example 18.20: a loop around a solenoid

A long solenoid is wound with 500 turns per meter, and the current in it is increasing at 100 A/s. Its cross-sectional area is 4.0 cm².

a. Find the magnitude of the e.m.f. induced in a wire loop that surrounds the solenoid, outside it.

b. Find the magnitude of the induced electric field in the loop if its radius is 2.0 cm.

Answer

n = 500 /m, A = 4.0 cm² = 4.0 × 10⁻⁴ m², dI/dt = 100 A/s, r = 2.0 cm = 2.0 × 10⁻² m.

a. The field of a long solenoid is all inside it, B = μ_0nI, so all the flux through the loop is the solenoid's own: Φ = BA = μ_0nIA, where A is the **solenoid's** area, not the loop's.

$$|\varepsilon| = \frac{d\Phi}{dt} = \mu_0 nA\frac{dI}{dt} = 4\pi \times 10^{-7} \times 500 \times 4.0 \times 10^{-4} \times 100 = \mathbf{2.5 \times 10^{-5}\ V}$$

that is, 25 μV.

b. The e.m.f. round the loop is the work done per unit charge by the induced electric field E along the loop's length 2πr, so ε = E × 2πr:

$$E = \frac{\varepsilon}{2\pi r} = \frac{2.5 \times 10^{-5}}{2\pi \times 2.0 \times 10^{-2}} = \mathbf{2.0 \times 10^{-4}\ V/m}$$

A changing magnetic field makes an electric field, even outside the solenoid where there is no magnetic field at all: that electric field is what pushes the charges round the loop.

WORKED EXAMPLE

Example 18.21: switching off the primary

A current of 10 A in the primary coil is reduced to zero at a uniform rate in 10^{-3} s. If the mutual inductance is 3 H, what is the e.m.f. induced in the secondary?

Answer

dI = 10 − 0 = 10 A, dt = 10^{-3} s, M = 3 H.

$$|\varepsilon| = M\frac{dI}{dt} = 3 \times \frac{10}{10^{-3}} = \mathbf{3.0 \times 10^{4}\ V}$$

30 000 V: this is how a car's ignition coil makes the spark from a 12 V battery.

WORKED EXAMPLE

Example 18.22: finding M

When the current in a primary coil rises steadily from 0 to 9.0 A in 0.025 s, an e.m.f. of 288 V appears across the secondary. What is the mutual inductance of the pair?

Answer

dI = 9.0 A, dt = 0.025 s, ε = 288 V.

$$M = \frac{\varepsilon}{dI/dt} = \frac{288 \times 0.025}{9.0} = \mathbf{0.8\ H}$$

YOUR TURN

Test yourself 18.15: flux given in maxwells

What is the mutual inductance of a pair of coils if a current of 10 A in one coil changes the flux through each turn of the second coil, of 1000 turns, by 10 000 maxwells?

Answer

10 000 Mx = 10 000 × 10^{-8} Wb = 10^{-4} Wb per turn, so the flux linkage of the second coil changes by 1000 × 10^{-4} = 0.1 Wb.

$$M = \frac{N_S\Phi_S}{I} = \frac{0.1}{10} = \mathbf{0.01\ H}$$

Check 10

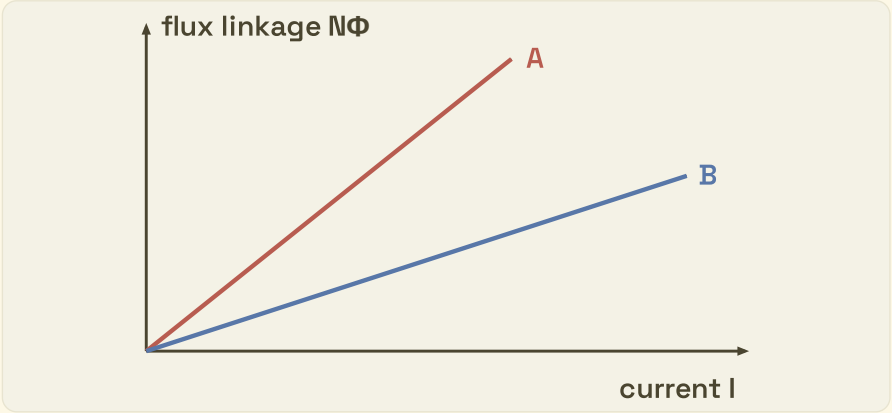


Figure 18.18 *Flux linkage against current for two coils, A and B (Test yourself 18.13).*

Test yourself 18.13. A graph of the magnetic flux linked with a coil against the current in it is drawn for two coils, A and B. Both are straight lines through the origin, A's steeper than B's. Which statement is correct?

a The self-inductance of A is greater than that of B.
b The information is not enough to decide.
c The self-inductance of B is greater than that of A.
d A and B have the same self-inductance.

Self-inductance of a long solenoid

Take a long solenoid (a coil much longer than it is wide) of length l, cross-sectional area A and N turns in all, with air (strictly, a vacuum) inside, carrying a current I.

1. The field inside a long solenoid is uniform: $B = \mu_0 nI$, where n = N/l is the number of turns per unit length. So
$$B = \frac{\mu_0 NI}{l}$$
2. The flux through each turn is
$$\Phi = BA = \frac{\mu_0 NIA}{l}$$
3. The flux linkage of all N turns is
$$N\Phi = \frac{\mu_0 N^2 IA}{l}$$
4. Since NΦ = LI,
$$\boxed{L = \frac{N\Phi}{I} = \frac{\mu_0 N^2 A}{l}}$$

With a core of permeability μ (= $\mu_r\mu_0$) filling it, μ_0 becomes μ. So the self-inductance of a coil is:

1. directly proportional to the **permeability** of the core (an iron core can raise L hundreds of times);
2. directly proportional to the **area of cross-section** A;
3. directly proportional to the **square of the number of turns**, L ∝ N² (doubling the turns doubles the field **and** doubles the turns it links);
4. inversely proportional to the **length** l, for the same number of turns.

The inductance doesn't depend on the current: it is fixed by the coil's shape and core, as a resistor's resistance is by its length, area and material. A **straight wire** has very little self-inductance (in exams it is taken as zero), since its field links hardly any of its own flux.

Uses of self-inductance:

1. **Chokes:** a coil of large inductance (and small resistance) in series with a circuit opposes rapid changes in current. It limits the current in a.c. circuits, as in the ballast of an old tube light, wasting very little energy, and blocks or filters high-

Check 11

A solenoid is stretched to twice its length, keeping the same turns and area. Its self-inductance becomes:

a twice as large
b half as large
c four times as large
d unchanged

frequency signals.

2. **Smoothing:** inductors store energy in their magnetic field and give it back, smoothing out ripples in the current from a power supply.
3. **Tuned circuits:** a coil with a capacitor (an LC circuit) picks out one frequency, as when a radio or television is tuned to a station.
4. **Transformers:** the self-inductance of the primary coil keeps the current in it small when no load is connected.
5. **Induction heating and sparks:** the large back e.m.f. when the current in a coil is suddenly broken makes the spark of a car's ignition coil, and the coils of induction heaters drive the eddy currents that heat the metal.

WORKED EXAMPLE

Example 18.17: doubling the turns

If the number of turns of a solenoid is doubled, keeping everything else the same, how does its self-inductance change?

Answer

$L = \frac{\mu_0 N^2 A}{l}$, so L ∝ N². Doubling N multiplies L by $2^2 = \mathbf{4}$: the self-inductance becomes four times as large.

WORKED EXAMPLE

Example 18.18: a lamp slow to light

A lamp is connected in series with a coil of many turns (on an iron core) and a battery. Why doesn't the lamp light up to full brightness the instant the circuit is switched on?

Answer

The lamp and the coil are in series, so they carry the same current. At the instant of switching on, the current starts to rise from zero, and its rate of change dI/dt is large. This induces a large back e.m.f. in the coil, $L\,dI/dt$, which by Lenz's law opposes the battery's e.m.f. So at first the net e.m.f. driving the current is small, and the current, and the lamp's brightness, grow only gradually.

As the current grows, dI/dt gets smaller, and so does the back e.m.f. In the end the current is steady, dI/dt = 0, the back e.m.f. is zero, and the current reaches its full value E/R: the lamp glows at full brightness. The larger L is, the longer this takes.

WORKED EXAMPLE

Example 18.19: two flat coils

Two flat circular coils have the same number of turns, but one has twice the radius of the other. Find the ratio of their self-inductances.

Answer

At the center of a flat circular coil of N turns and radius r, carrying current I, the field is $B = \frac{\mu_0 NI}{2r}$. Taking this as the field over the whole face of the coil (an approximation), the flux through each turn is

$$\Phi = BA = \frac{\mu_0 NI}{2r} \times \pi r^2 = \frac{\mu_0 NI\pi r}{2}$$

and the flux linkage is

$$N\Phi = LI = \frac{\mu_0 N^2 I\pi r}{2} \quad \Rightarrow \quad L = \frac{\mu_0 \pi N^2 r}{2}$$

So for the same N, L ∝ r:

$$\frac{L_1}{L_2} = \frac{r_1}{r_2} = \frac{r}{2r} = \frac{1}{2}$$

$\mathbf{L_1 : L_2 = 1 : 2}$: the larger coil has twice the self-inductance.

Mutual induction

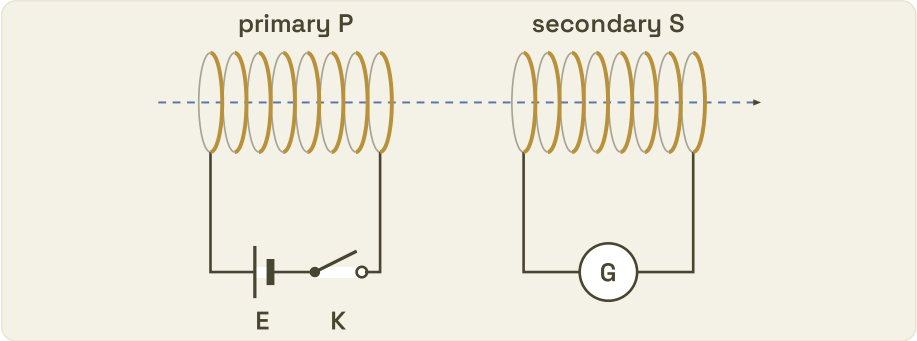


Figure 18.19 *Mutual induction: a change of current in the primary P makes the galvanometer in the secondary S kick.*

Two coils, P (the **primary**) and S (the **secondary**), are placed close together. A battery and a key K are connected to P, and a galvanometer G to S.

When the key is pressed, the current in P starts to grow, and so does its magnetic flux. This growing flux induces an opposing e.m.f. in P itself (self-induction). But because S is close by, much of P's flux passes through S too, and the growing flux through S induces an e.m.f. in S as well. By Lenz's law, the current it drives in S makes a field that opposes the growth of P's flux: the galvanometer kicks.

When the key is released, P's current and flux fall, and an e.m.f. is induced in S again, but of the **opposite** sign: the galvanometer kicks the other way. While the current in P is steady, nothing happens in S.

Mutual induction is the production of an induced e.m.f. in a coil because of a change in the current, and so in the magnetic flux, in a neighboring coil.

Coefficient of mutual induction

The flux linked with S is caused by the current I in P, so it is directly proportional to it:

$$N_S\Phi_S \propto I \quad \Rightarrow \quad N_S\Phi_S = MI$$

The constant M is the **coefficient of mutual induction**, or the **mutual inductance**, of the pair of coils. If I = 1 A, then $N_S\Phi_S$ = M: the mutual inductance is numerically equal to the flux linked with one coil when a current of 1 A flows in the other. It depends on the number of turns of each coil, their sizes and shapes, the distance and angle between them, and the core (an iron core joining them makes M much larger). The same M works both ways: a current I in S links the same flux MI with P.

The e.m.f. induced in S is then

$$\varepsilon_S = -\frac{d(N_S\Phi_S)}{dt} = -\frac{d}{dt}(MI) \quad \Rightarrow \quad \boxed{\varepsilon_S = -M\frac{dI}{dt}}$$

where dI/dt is the rate of change of the current in P. The minus sign shows that the e.m.f. opposes the change.

Unit: the SI unit of mutual inductance is also the **henry**. Two coils have a mutual inductance of 1 H if a current changing at 1 A/s in one induces an e.m.f. of 1 V in the other.

Uses of mutual induction:

1. **Transformers** (Section 18.7) transfer electrical energy from one coil to another, stepping the voltage up or down.
2. **Wireless charging:** a coil in the charging pad and a coil in the phone form a pair; an alternating current in the pad's coil induces a current in the phone's.
3. **Induction motors:** the stator's changing field induces the currents in the rotor.
4. **Signal coupling:** coils pass signals from one stage of a radio circuit, or an antenna, to the next without a wire joining them.
5. **Sensors:** inductive proximity sensors and metal detectors sense metal from the change it makes in a coil's induced e.m.f.