Answers: Magnetic Fields

Quick checks

1 b A line of force is drawn along the field's direction. If two crossed, there would be two tangents, two field directions, at the crossing point. The field at a point is a single vector, so this can't happen. (Where two fields cancel, at a neutral point, there is no field and no line passes through it.)

2 a A surface in the y–z plane has its normal along the x-axis, $\vec{A} = 20\hat{i}$. Only the x-component of the field passes through it: $\phi = \vec{B} \cdot \vec{A} = 5 \times 20 = 100$ units. The y- and z-components run along the surface and add nothing.

3 b The angle in $N\phi = NBA\cos\theta$ is between the field and the **normal**. The plane is at 30° to the field, so the normal is at 60°: $N\phi = 50 \times 0.2 \times 40 \times 10^{-4} \times \cos 60^\circ = 0.04 \times 0.5 = 0.020$ Wb. (Using cos 30° by mistake gives 0.035 Wb.)

4 c At a neutral point the magnet's field cancels the Earth's horizontal field: there is no horizontal field to turn the needle, so it rests in any direction. (With the N pole pointing south, these points lie on the magnet's axis, beyond its ends.)

5 a Point your right thumb down along the current: seen from above, your fingers curl **clockwise**. (Maxwell's rule agrees: a screw going down, away from you as you look from above, turns clockwise.)

6 b $F = Bqv\sin\theta$ depends on the charge, not the mass. The two charges are equal in size (1.6×10^{-19} C) and opposite in sign, so the forces are equal and opposite. (The electron, with about 1/1836 of the mass, is then turned far more sharply.)

7 a With the wire along the field, θ = 0 and $F = BIl\sin 0^\circ = 0$. The drifting electrons move along the field, and charges moving along a field feel no force.

8 c $dB \propto \sin\theta / r^2$. Doubling r divides by 4; sin 30° = 0.5 against sin 90° = 1 halves it again: dB × ½ × ¼ = **dB/8**.

9 b $B = \dfrac{\mu_0 NI}{2\pi r} = \dfrac{2 \times 10^{-7} \times 500 \times 2}{0.10} = 2 \times 10^{-3}$ T = **2 mT**. (Between 2.2 mT at the inner edge and 1.8 mT at the outer.)

10 a Turning a moment M from $\theta_1 = 0°$ to $\theta_2 = 180°$ takes $W = MB(\cos 0^\circ - \cos 180^\circ) = 2MB = 2 \times 10 \times 0.4 \times 10^{-4} = 8 \times 10^{-4}$ J.

11 d Far from a magnet (or a current loop), the field on its axis falls as $1/d^3$ (Section 16.9: $B \approx \mu_0 NIa^2/2x^3$; for a bar magnet, $B \approx \dfrac{\mu_0}{4\pi}\dfrac{2M}{d^3}$). Doubling the distance divides the field by $2^3 = 8$. It is only **approximately** 8 : 1, because the distances are measured from the pole, not the magnet's center (A and B are x + 1 and 2x + 1 cm from the center), and the $1/d^3$ law holds only far away.

12 c $B = \mu_r \dfrac{\mu_0 N I}{2\pi r} = 2000 \times \dfrac{2 \times 10^{-7} \times 300 \times 1}{0.10} = 2000 \times 6 \times 10^{-4} = 1.2$ T.

13 b Each charge goes round a circle of radius l, f = ω/2π times a second, so it is a current qω/2π round an area πl²: its moment is $\dfrac{q\omega}{2\pi}\pi l^2 = \dfrac{q\omega l^2}{2}$. For both, $M = q\omega l^2$. The angular momentum is $L = 2ml^2\omega$. So $M/L = q/2m$, whatever ω and l.

14 b $F/l = \dfrac{\mu_0 I_1 I_2}{2\pi r}$ with $I_1 = I_2 = I$ and r = b: $\dfrac{\mu_0 I^2}{2\pi b}$.

15 d M = NIA, in A m²: the torque on the coil in a field is $\tau = MB\sin\theta$.

16 c Each arm is a straight wire from the corner out to infinity. The point is a distance a from the arm, and the foot of the perpendicular is a distance a from the corner, so the corner is seen at 45° and the far end at 90°: $B_1 = \dfrac{\mu_0 I}{4\pi a}(\sin 45^\circ + \sin 90^\circ)$. The current flows towards the corner along one arm and away along the other, and both fields point the same way at a point inside the angle, so

$$B = 2B_1 = \frac{\mu_0 I}{2\pi a}\left(1 + \frac{1}{\sqrt{2}}\right) = \frac{\mu_0 I(\sqrt{2}+1)}{2\sqrt{2}\,\pi a}$$

Some books print option c as $\mu_0 I/2\sqrt{2}\pi a$, without the factor (√2 + 1). That is not the field at this point, and neither is any other option they give; option c here is the field worked out above.

17 a The flux linkage is $NBA\cos\omega t$, and the e.m.f. is its rate of change (Unit 18): $e = NBA\omega\sin\omega t$, largest when sin ωt = 1: $e_0 = NAB\omega$.

18 c The e.m.f. is the rate of change of flux (Unit 18): $e = \dfrac{\Delta\phi}{t} = \dfrac{(4B_0 - B_0)A_0}{t} = \dfrac{3B_0A_0}{t}$ (for one turn, with the field at right angles to the coil).

19 a The field at any point has one direction, the tangent to the line through it, so two lines can never cross. (At a neutral point the field is zero, and no line passes through it.)

20 b The two fields are equal and at right angles: $B = \sqrt{B_1^2 + B_1^2} = \sqrt{2}\,B_1$ (Example 16.23).

21 c The straight leads lie along radii, pointing at O: every element of them is along the line to O (θ = 0), so they give no field there. The arc is three quarters of a circle:

$$B = \frac{3}{4} \times \frac{\mu_0 I}{2r} = \frac{3}{4} \times \frac{4\pi \times 10^{-7} \times 4}{2 \times 0.03}$$
$$= \frac{3}{4} \times \frac{8\pi}{3} \times 10^{-5} = 2\pi \times 10^{-5}$$

22 c At each corner where the current enters, it divides between the two ways round the frame in inverse proportion to their resistances (their lengths). In a and c it splits ¾ along the top side and ¼ round the other three; the top side's field at the center (¾ × one side's) and the other three sides' (3 × ¼ of one side's, the other way) cancel. In b and d the two ways round are equal, and cancel too. So the **frame** never gives a field at its center; only the **leads** can. In a and b, the leads lie along the diagonals, pointing at the center: no field. In d, the leads lie along the lines of the bottom and top sides, carrying the current the same way at the same distance on opposite sides of the center, and their fields cancel. In **c**, both leads lie along the line of the top side, carrying the current the same way, on the same side of the center: their fields add, so the field at the center is **not zero**.

23 c The deflecting torque NBIA balances the restoring torque kθ: $\theta = \dfrac{NAB}{k} I$.

24 b The magnetic forces between two moving charges, or two current elements that aren't parallel, need not be equal and opposite (Test yourself 16.24): the field itself carries momentum. Between two complete circuits with steady currents they are equal and opposite. Neither the size of the current nor the strength of the field changes this.

25 c The near side of the loop carries its current the same way as the wire (like currents) and is attracted; the far side carries it the opposite way and is repelled. The near side is closer, where the wire's field ($\propto 1/r$) is stronger, so the attraction wins. The forces on the two short sides are equal and opposite. The net force is towards the wire.

17.

a. From the **N pole to the S pole**, across the gap at right angles to the wire. Check with Fleming's left-hand rule: first finger from N to S, second finger along the current, away from you; the thumb points down, the way the force is shown.

b. The force is **reversed**: it acts vertically upward. (Reversing either the field or the current reverses the force; reversing both leaves it unchanged.)

c. $F = BIl \sin\theta = 0.34 \times 10 \times 2 \times \sin 60^\circ = 6.8 \times 0.866 = \mathbf{5.89\ N}$

18.

a. i. A wire from the bottom of the conductor to one terminal of a cell (long line +, short line −), from its other terminal through a switch and back up to the top of the conductor, as in Figure 16.37.

ii. The filings form **concentric circles** round the conductor, centered on it, in the plane of the card: closest together and clearest near the wire, fading farther out, where the field is weaker.

iii. The **pattern stays the same**: circles. Only the field's **direction** round the circles reverses, which the filings can't show; a compass on the card would turn round.

b. m = 50 g = 0.05 kg, l = 0.5 m, B = 0.2 T.

$$BIl = mg \quad \Rightarrow \quad I = \frac{mg}{Bl} = \frac{0.05 \times 10}{0.2 \times 0.5} = \mathbf{5\ A}$$

19.

a. The magnetic flux density at a point is the magnetic flux per unit area through a small area at right angles to the field there, $B = \phi/A$. Equivalently, it is the force on a charge of 1 C moving at 1 m/s at right angles to the field, from $F = Bqv$. It is a vector, measured in teslas (1 T = 1 Wb/m² = 1 N/(A m)).

b. i. Inside the field, the particle moves along an **arc of a circle**. By Fleming's left-hand rule (field into the page, positive charge moving to the right), the force is **up the page**, so the path curves upward (anticlockwise). On leaving the field it goes on in a **straight line**, along the tangent to the arc where it left, as in Figure 16.38.

ii. The magnetic force is always **at right angles to the velocity**, so it does no work ($W = Fs\cos 90^\circ = 0$): it changes the direction of the velocity, not its size. The kinetic energy, and so the speed, stay the same: v.

c. In the field, the magnetic force provides the centripetal force:

$$Bqv = \frac{mv^2}{r} \quad \Rightarrow \quad r = \frac{mv}{qB}$$

For the same path, r must be the same. With the charge 2q and speed v′, $r = \dfrac{mv'}{2qB}$. Setting this equal to $\dfrac{mv}{qB}$ gives **v′ = 2v**.

20.

a. X: the current in CD flows the **opposite way** to that in AB (it goes round the loop). P: the force on CD is the **opposite** of F, since the current is reversed and the field is the same: with F downward on AB, P is **upward** on CD. The two forces form a couple that turns the coil.

b. Any two of: **increase the current**; use a **stronger magnet** (a stronger field); use a coil with **more turns** (F = NBIl on each side); make the sides AB and CD **longer**.

c. i. **Free electrons.**

ii. A thinner wire of the same length and metal has a smaller cross-section, so a **larger resistance** ($R = \rho l/A$). With the same battery, the **current is smaller** (I = V/R, if the battery's internal resistance is small). In $I = nAev_d$ the area A is smaller and v_d, set by the electric field along the wire (the same p.d. across the same length), is the same, so again I is smaller, in proportion to the area.

21.

a. The magnetic force on the proton (positive charge moving to the right, field into the page) is **up the page** (Fleming's left-hand rule). The electric force must be down the page, so **E points down the page**. Then $qE = qvB$, or $v = E/B$.

b. i. **No deflection.** The α-particle has charge 2e and four times the mass, but the condition $qE = qvB$ gives $v = E/B$, which depends on neither charge nor mass. At the same speed v, both forces double and still balance.

ii. **Deflected down the page.** For the electron both forces are reversed: the magnetic force is down the page, $e(2v)B$, and the electric force up the page, eE = evB. The magnetic force is twice the electric force, so the net force, evB, is down the page, and the electron curves downward.

22. i. Between faces **CFSP** and **DERQ**, the two side faces, at right angles to both the field and the current.

ii. Let the slice be d wide (between CFSP and DERQ) and t thick (along the field), so the current flows through an area A = dt.

Each carrier feels the magnetic force $F_m = Bqv$, across the slice; carriers collect on one side face until the electric field they set up, $E = V_H/d$, pushes back on each with $F_e = qV_H/d$. In balance,

$$\frac{qV_H}{d} = Bqv \quad \Rightarrow \quad V_H = Bvd$$

The current is $I = nqvA = nqv\,dt$, so $v = \dfrac{I}{nq\,dt}$, and

$$V_H = Bd\,\frac{I}{nq\,dt} = \frac{\mathbf{BI}}{\mathbf{nqt}}$$

iii. Copper has a very large number of free electrons per unit volume (n ≈ 10^{29} per m^3). For a given current they drift very slowly, so the Hall voltage, BI/nqt, is tiny, well under a microvolt for ordinary currents and fields, and hard to measure.

23. a. When a magnetic field is applied at right angles to a current in a conductor, a voltage is set up across the conductor in the direction at right angles to both the current and the field.

b. t is the thickness along the field: **t = 0.10 mm** = 1.0×10^{-4} m.

c. $$V_H = \frac{BI}{nqt} = \frac{0.13 \times 3.8}{6.0 \times 10^{28} \times 1.6 \times 10^{-19} \times 1.0 \times 10^{-4}} = \frac{0.494}{9.6 \times 10^{5}} = \mathbf{5.1 \times 10^{-7}\ V}$$

about 0.51 μV.

24. a. The magnetic field dB at a point P due to a small element dl of a conductor carrying a current I is in direct proportion to I, to dl and to sin θ (θ the angle between the element and the line from it to P), and in inverse proportion to the square of the distance r from the element to P:

$$dB = \frac{\mu_0}{4\pi}\frac{I\,dl\sin\theta}{r^2}$$

with $\mu_0 = 4\pi \times 10^{-7}$ H/m. Its direction is at right angles to both the element and the line to P, by the right-hand thumb rule. The field of a whole conductor is found by adding up (integrating) the fields of all its elements.

b. For a circular coil of radius r, every element dl is a distance r from the center O, and the line to O is at right angles to it (sin θ = 1), so each gives $dB = \frac{\mu_0}{4\pi}\frac{I\,dl}{r^2}$, all along the axis in the same direction. Adding them up over the circumference 2πr:

$$B = \frac{\mu_0 I}{4\pi r^2} \times 2\pi r = \frac{\mu_0 I}{2r}, \qquad B = \frac{\mu_0 N I}{2r}$$

along the axis, by the right-hand grip rule (Figure 16.20).

c. N = 100, a = 0.10 m, I = 5 A, x = 0.05 m. $a^2 + x^2 = 0.0100 + 0.0025 = 0.0125\ ^2$, and $(0.0125)^{3/2} = 1.40 \times 10^{-3}\ ^3$.

$$B = \frac{\mu_0 N I a^2}{2(a^2+x^2)^{3/2}} = \frac{4\pi \times 10^{-7} \times 100 \times 5 \times 0.0100}{2 \times 1.40 \times 10^{-3}} = \frac{6.28 \times 10^{-6}}{2.80 \times 10^{-3}} = \mathbf{2.25 \times 10^{-3}\ T}$$

(At the center it would be $\mu_0 NI/2a = 3.14 \times 10^{-3}$ T.)

25. a. $\vec{F} = q\left(\vec{v} \times \vec{B}\right)$, of size $Bqv\sin\theta$, at right angles to both $\vec{v}$ and $\vec{B}$.

b. Stretch the thumb, forefinger and middle finger of the left hand at right angles to one another. If the forefinger points along the field and the middle finger along the current (the motion of positive charge), the thumb points along the force.

c. As in Example 16.11: l = 0.05 m, m = 1.2×10^{-3} kg/m × 0.05 m = 6×10^{-5} kg, R = 3.8 Ω/m × 0.05 m = 0.19 Ω.

$$BIl = mg, \quad I = \frac{V}{R} \quad \Rightarrow \quad V = \frac{mgR}{Bl} = \frac{6 \times 10^{-5} \times 9.8 \times 0.19}{0.6 \times 0.05} = \mathbf{3.7 \times 10^{-3}\ V}$$

about 3.7 mV (3.8 mV with g = 10 m/s²).

26. a. A **parabola**. The electric force, eE, is constant and along the field (opposite to E for an electron), so the electron accelerates steadily across the field while its velocity at right angles to the field stays the same: like a ball thrown horizontally under gravity.

b. **No.** There may be a magnetic field along its direction of motion (sin θ = 0, no force), or electric and magnetic fields crossed so that their forces cancel (eE = evB, as in a velocity selector).

c. **Yes.** $F = Bev\sin\theta = 0$ when the electron moves **along** the field (θ = 0° or 180°), or is **at rest** (v = 0).

27. a. $dB = \dfrac{\mu_0}{4\pi}\dfrac{I\,dl\sin\theta}{r^2}$, or in vector form $d\vec{B} = \dfrac{\mu_0}{4\pi}\dfrac{I\,d\vec{l}\times\hat{r}}{r^2}$.

b. As in question 24 b: each element dl of a coil of radius r is at distance r from the center, at right angles to the line to it, so $dB = \dfrac{\mu_0 I\,dl}{4\pi r^2}$, all in the same direction. Adding over one turn (Σdl = 2πr) gives $\mu_0 I/2r$, and for N turns

$$B = \frac{\mu_0 N I}{2r}$$

Direction (Figure 16.20): along the coil's axis, by the right-hand grip rule: out of the page at the center for a current going anticlockwise as you look at it, into the page for a clockwise one.

28. a. Each wire lies in the magnetic field of the other, and a current in a magnetic field feels a force. Wire 1 makes a field $B_1 = \dfrac{\mu_0 I_1}{2\pi r}$ at wire 2, at right angles to it (Figure 16.31). A length l of wire 2 then feels

$$F = B_1 I_2 l = \frac{\mu_0 I_1 I_2 l}{2\pi r}, \qquad \frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}$$

By Fleming's left-hand rule this force points towards wire 1. In the same way, wire 1 feels an equal force per unit length towards wire 2. So the wires attract.

b. One ampere is the steady current which, flowing in each of two infinitely long, straight, parallel conductors of negligible cross-section 1 m apart in a vacuum, produces a force of 2×10^{-7} N per meter of length between them. (This was the definition until 2019; the ampere is now defined from the charge of the electron, which gives the same force.)

29. a. i. The field of XY circles it anticlockwise (current out of the page, right-hand thumb rule); at Q, to the left of Y, that is **down the page**.

ii. Fleming's left-hand rule (field down the page, current out of the page) gives a force on Q **towards Y**: like currents attract.

b. i. If body A exerts a force on body B, then B exerts an equal and opposite force on A.

ii. The force on XY is equal and opposite to the force on PQ: **towards PQ** (towards Q).

c. Take a cable with two wires about d = 1 cm apart, carrying I = 10 A (a heater):

$$B = 2.0 \times 10^{-7} \times \frac{10}{0.01} = 2 \times 10^{-4}$$

Force on 1 m of the other wire: $F = BIl = 2 \times 10^{-4} \times 10 \times 1 = 2 \times 10^{-3}$ N. A meter of such copper wire (1.5 mm^2) has a mass of about 13 g, a weight of about 0.13 N, sixty times larger; and the wire is held by its insulation and fixings. The alternating force, reversing 100 times a second, makes the wires vibrate by far too little to be seen (though in a transformer such forces cause its hum).

30.

a. A **circle centered on X**, passing through A, with its direction **clockwise** (seen from above, the current goes away from you, into the card).

b. At B, X's field and Y's field are **equal in size** (equal currents, equal distances) and **opposite in direction**: X's clockwise circle points one way at B, which is to its right, and Y's clockwise circle points the other way at B, which is to its left. They cancel.

c. i. **Towards X** (to the left): like currents attract.

ii. Wire Y carries a current and lies in the magnetic field made by the current in X. A current-carrying conductor in a magnetic field feels a force $F = BIl$, here towards X by Fleming's left-hand rule.