Unit 14

Electrical Circuits

This unit online, with its interactives:
science-playground.com/s/12f14

A street light, a phone charger and a car's wiring are all just loops of wire, cells and resistors, yet some of them can't be untangled into "this pair in series, that pair in parallel". Two batteries side by side, a bridge of four resistors with a meter across its middle: Ohm's law alone gets stuck. Two simple rules from a 21-year-old student in 1845 solve every one of them.

This unit gives you those rules, **Kirchhoff's laws**, and then the instruments built on them: the **Wheatstone bridge** and **meter bridge**, which measure a resistance by finding where a meter reads nothing; the **potentiometer**, which measures an e.m.f. without taking any current from the cell; and the way a delicate galvanometer becomes an **ammeter** or a **voltmeter**. It ends with **superconductors**, which have no resistance at all, and **Joule's laws** of the heat a current makes.

Gustav Kirchhoff was still a student at Königsberg when, in 1845, he wrote down the two circuit laws that carry his name, to work out the currents in any network of wires. He went on to work with the chemist **Robert Bunsen** at Heidelberg. Together they found that every element, heated in a flame, gives out its own set of colors, and invented **spectrum analysis**: by passing sunlight through a prism and finding the dark lines of sodium, iron and other elements in it, they could say what the Sun is made of, without going there. He also built on the work of **Georg Simon Ohm**, extending his laws to currents that spread through solid, three-dimensional conductors, and coined the term "black body" for an ideal absorber of radiation, the start of the road to quantum physics.



Figure 14.1 *Gustav Robert Kirchhoff (1824–1887).*
Photo: Universitätsbibliothek Heidelberg, CC BY-SA 4.0

Ohm's law is enough for a simple circuit: if you can see which resistances are in series and which in parallel, you can find the effective resistance, then the current, then every p.d. But a circuit with more than one source of e.m.f., or with resistors joined in a way that is neither series nor parallel, can't be reduced like that. For these, Kirchhoff's two laws give enough equations to find every current.

DID YOU KNOW?

An instrument that takes nothing

There is a very sensitive and accurate instrument that can measure the e.m.f. of a cell, or compare the e.m.f.s of two cells, **without drawing any current from them**. Because of this, scientists use it to calibrate standard cells and to detect potential differences too small for an ordinary voltmeter. It is the **potentiometer** (Section 14.4), and it is made of little more than a long, uniform wire.

14.1 Kirchhoff's laws

Kirchhoff's laws are not limited to direct current (DC). They hold at every instant in alternating current (AC) circuits too, including circuits with capacitors and inductors (whose "resistances" are then the complex impedances of Unit 19). There are two laws: one about currents at a junction, one about voltages round a loop.

Kirchhoff's first law (the junction rule)

A **junction** (or node) is a point where three or more wires meet. Charge can't pile up at a junction, and it can't be made or destroyed there: every coulomb that flows in each second must flow out in the same second. Charge per second is current, so:

Kirchhoff's first law: the algebraic sum of the currents meeting at a junction in a circuit is zero.

$$\sum I = 0$$

Sign convention:

i. a current flowing **towards** the junction is taken as **positive**;
ii. a current flowing **away from** the junction is taken as **negative**.

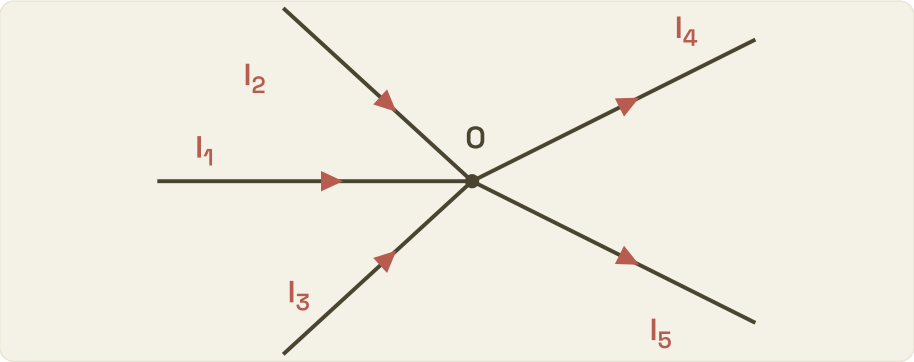


Figure 14.2 *Five currents meet at a junction O. I_1, I_2 and I_3 flow in; I_4 and I_5 flow out.*

At the junction O in Figure 14.2, the currents I_1, I_2 and I_3 flow towards O, so they are positive, and I_4 and I_5 flow away from O, so they are negative. By Kirchhoff's first law,

$$I_1 + I_2 + I_3 + (-I_4) + (-I_5) = 0$$

$$\therefore \quad I_1 + I_2 + I_3 = I_4 + I_5$$

The **total current flowing into a junction equals the total current flowing out of it**. The law follows from the **conservation of charge**, and it is also called the **junction rule** or Kirchhoff's current law (**KCL**).

It is the same as traffic at a chowk: if cars never park or vanish at the crossing, as many must leave each minute as arrive.

Check 1

At a junction, 3 A and 5 A flow in along two wires, and 2 A flows out along a third. What flows in the fourth wire?

a 10 A in
b 4 A in
c 6 A out
d 10 A out

Kirchhoff's second law (the loop rule)

Kirchhoff's second law: in any closed loop of a circuit, the algebraic sum of the e.m.f.s equals the algebraic sum of the potential drops (the products IR) round the loop.

$$\sum E = \sum IR$$

Put another way, if you go once round any closed loop, adding up every rise and fall of potential, the total change in potential is zero:

$$\sum V = 0$$

This is the **loop rule**, also called the mesh rule or Kirchhoff's voltage law (**KVL**). Why must it hold? The potential at a point has one definite value. If you set off from a point, walk round a loop and come back to the same point, you must find the same potential you started with, so the rises and falls on the way must cancel. In energy terms, a coulomb that goes once round a loop gains energy in the cells (their e.m.f.s) and gives up exactly the same energy in the resistors (the IR drops). So the second law follows from the **conservation of energy**, and it holds because the electrostatic force is **conservative**: the work it does round any closed path is zero.

It is like walking round Bhaktapur's Durbar Square: whatever steps you climb and descend on the way, when you are back at the spot you started from, your total change in height is zero.

Sign convention for the loop rule

Choose a direction to go round the loop (clockwise or anticlockwise; either works), and then:

i. Going through a cell from its **negative to its positive** terminal, the potential **rises**: its e.m.f. is taken as **positive**, +E.
ii. Going through a cell from its **positive to its negative** terminal, the potential **falls**: its e.m.f. is taken as **negative**, −E.
iii. Going through a resistor **in the direction of the current**, the potential **falls** (current flows from high potential to low): the product IR is taken as **negative**, −IR.
iv. Going through a resistor **against the current**, the potential **rises**: +IR.

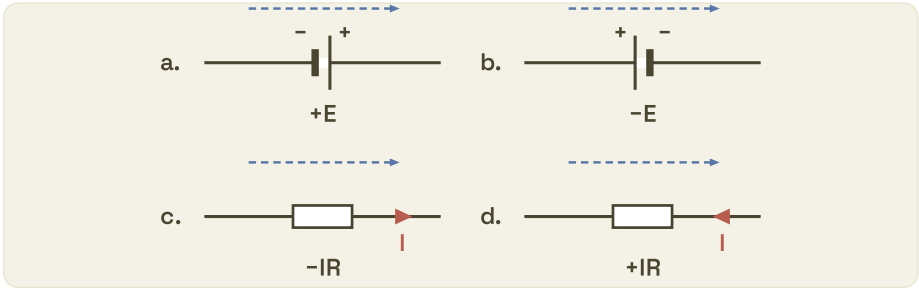


Figure 14.3 *The four signs of the loop rule. a. Going through a cell from − to +: a rise, +E. b. From + to −: a fall, −E. c. Through a resistor in the direction of its current: a fall, −IR. d. Against its current: a rise, +IR.*

The direction you choose for each unknown current doesn't have to be right. Guess it, and mark it on the diagram. If the answer comes out negative, the current flows the other way, with the same size.

CAREFUL

Don't mix up the two choices

There are two separate choices in a Kirchhoff problem: the **direction of each current** (marked once, on the diagram, and used for both laws) and the **direction you travel round each loop** (chosen afresh for each loop). The sign of IR depends on whether you travel *with* or *against* the current you marked, not on which way the current "really" flows. Keep both on the diagram and the signs look after themselves.

DID YOU KNOW?

Cells joined the wrong way round

If n identical cells, each of e.m.f. E and internal resistance r, are joined in series and x of them are connected the wrong way round, each wrongly placed cell cancels one right one: the effective e.m.f. is (n − 2x)E. The internal resistance doesn't cancel, though: it is still nr, since every cell still has its resistance in the path of the current. A torch with one battery put in backwards is dim (or dark), for exactly this reason.

Check 2

Kirchhoff's first and second laws follow from the conservation of, respectively:

a energy and charge
b charge and energy
c momentum and energy
d charge and momentum

Using the laws

To solve any circuit:

1. Mark a current, with a direction, in each branch (a branch is a path between two junctions). Use the junction rule at once to save unknowns: if I_1 and I_2 meet, the next branch carries $I_1 + I_2$.
2. Choose enough loops to give as many equations as there are unknown currents left, each loop going through at least one branch not used before.
3. Go round each loop with the sign convention, and set the sum to zero.
4. Solve the equations together. A negative answer means that current flows opposite to the arrow.

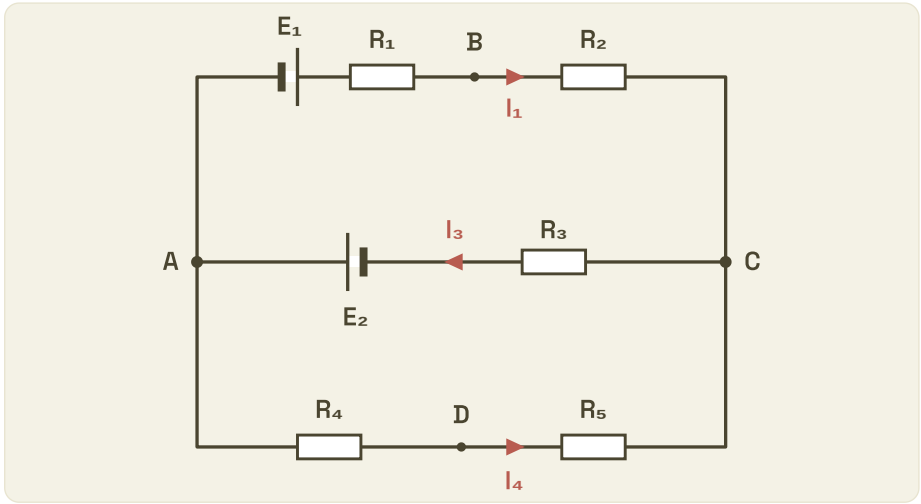


Figure 14.4 *A network ABCDA. Branch AB has the cell E_1 (its + end towards B) and R_1, branch BC has R_2, the middle branch CA has R_3 and the cell E_2 (its + end towards A), branch AD has R_4 and branch DC has R_5. The currents are I_1 along ABC, I_4 along ADC, and I_3 back along CA.*

WORKED EXAMPLE

Example 14.1: a network with two cells

In the network of Figure 14.4, a. write Kirchhoff's laws for the junction A and for the loops ABCA and ADCA. b. Find the currents if $E_1 = 4$ V, $E_2 = 8$ V, $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 2\ \Omega$, $R_4 = 1\ \Omega$ and $R_5 = 1\ \Omega$ (the cells' internal resistances are included in R_1 and R_3).

Answer

a. **The currents.** B joins only R_1 and R_2, so the same current I_1 flows through both, along A → B → C; likewise I_4 flows through R_4 and R_5, along A → D → C. The current I_3 in the middle branch flows from C back to A.

Junction A: I_3 flows in; I_1 and I_4 flow out:

$$I_3 - I_1 - I_4 = 0 \quad \Rightarrow \quad I_3 = I_1 + I_4$$

Loop ABCA, traveling A → B → C → A:

- through E_1 from − to +: $+E_1$;
- through R_1 and R_2 with I_1: $-I_1R_1 - I_1R_2$;
- through R_3 from C to A, with I_3: $-I_3R_3$;
- through E_2 from − to + (its + end is towards A): $+E_2$.

$$E_1 - I_1R_1 - I_1R_2 - I_3R_3 + E_2 = 0 \quad \Rightarrow \quad E_1 + E_2 = I_1(R_1 + R_2) + I_3R_3$$

Loop ADCA, traveling A → D → C → A:

$$-I_4R_4 - I_4R_5 - I_3R_3 + E_2 = 0 \quad \Rightarrow \quad E_2 = I_4(R_4 + R_5) + I_3R_3$$

In each loop, the e.m.f.s on one side equal the IR drops on the other, as the second law says.

b. Putting in the numbers:

$$12 = 3I_1 + 2I_3 \qquad 8 = 2I_4 + 2I_3 \qquad I_3 = I_1 + I_4$$

Substituting for I_3: $12 = 5I_1 + 2I_4$ … (i) and $8 = 2I_1 + 4I_4$, that is $4 = I_1 + 2I_4$ … (ii).
(i) − (ii): $8 = 4I_1$, so $\mathbf{I_1 = 2\ A}$; then from (ii), $2I_4 = 2$, $\mathbf{I_4 = 1\ A}$; and $\mathbf{I_3 = 2 + 1 = 3\ A}$.
Check with the loop ABCA: $3 \times 2 + 2 \times 3 = 12$ V = $E_1 + E_2$. ✓

28.

a. A metal wire of constant resistance is used in an electric heater. So as not to overload the circuit, the supply voltage to the heater is reduced from 230 V to 220 V. Find the percentage reduction in the heater's power.

b. A uniform wire AB of length 100 cm is connected across a cell of e.m.f. 1.5 V and negligible internal resistance, as in Figure 14.36. An ammeter of internal resistance 5.0 Ω is connected to the end A of the wire and to a contact C that can be moved along it. Find the ammeter reading with C i. at A; ii. at B.

c. Using this circuit, the ammeter reading I is recorded for different distances L of C from A; some of the points are plotted in Figure 14.37.

i. Plot the points for C at A and at B (your answers to b.).

ii. Draw a line of best fit through all the points, and find the ammeter reading with C at the midpoint of the wire.

iii. Use your answer to ii. to calculate the p.d. between A and C with C at the midpoint.

d. Explain why, although C is at the midpoint of AB, your answer to c.iii is not half the e.m.f. of the cell.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

length XJ of the wire.

iii. Use your answer to ii. to state the value of E.

iv. State why the internal resistance of cell B is not needed to find E.

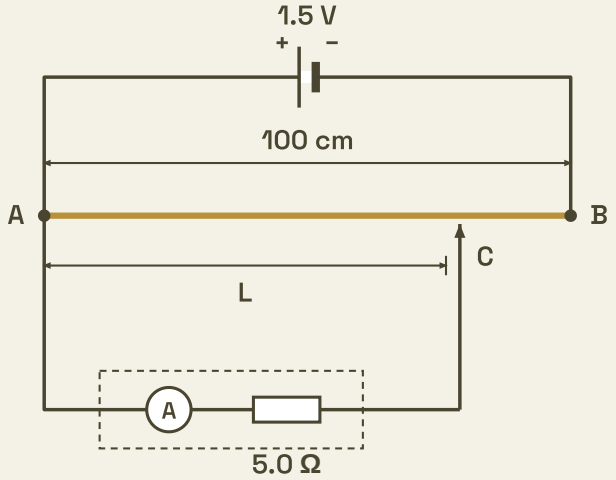


Figure 14.36 For question 28. A 1.5 V cell across a uniform wire AB, 100 cm long. An ammeter of internal resistance 5.0 Ω is connected from A to a contact C, a distance L from A.

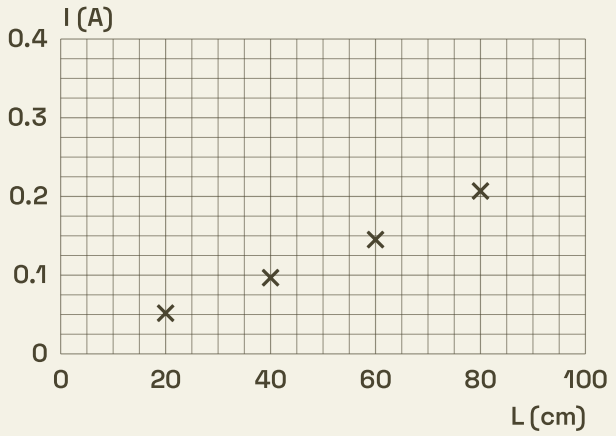


Figure 14.37 The ammeter reading I against the distance L of the contact C from A: points at L = 20, 40, 60 and 80 cm.

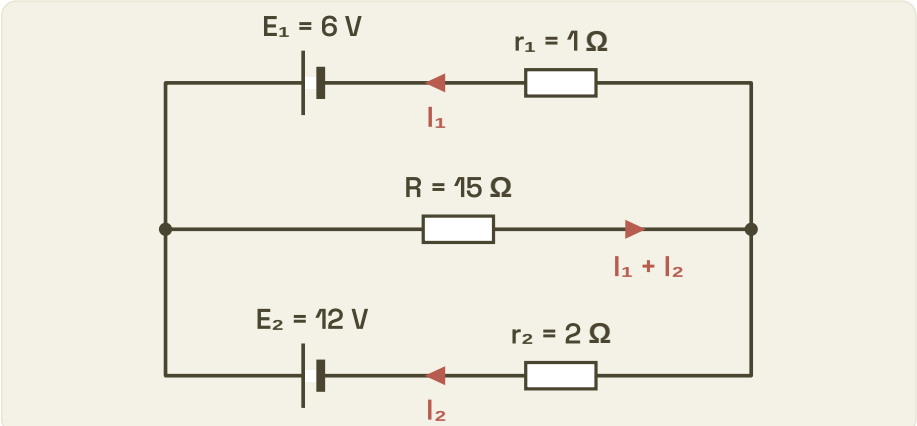


Figure 14.5 Two cells in parallel across a 15 Ω resistor: $E_1 = 6$ V with $r_1 = 1\ \Omega$ in the top branch, R in the middle and $E_2 = 12$ V with $r_2 = 2\ \Omega$ in the bottom branch, both with their + ends to the left. The currents I_1 and I_2 are marked leaving the + ends, and $I_1 + I_2$ returns through R.

WORKED EXAMPLE

Example 14.2: two cells in parallel

Two batteries of e.m.f. 6 V and 12 V and internal resistances 1 Ω and 2 Ω are connected in parallel with a resistance of 15 Ω (Figure 14.5). Find the current through each branch of the circuit and the potential difference across the 15 Ω resistance.

Answer

Let the currents from the cells be I_1 and I_2, as marked. By the junction rule, the current through R = 15 Ω is $I_1 + I_2$.

Loop with E_1 and R: going round in the direction of I_1,

$$E_1 = I_1 r_1 + (I_1 + I_2)R$$

$$6 = I_1 \times 1 + (I_1 + I_2) \times 15 \quad \Rightarrow \quad 16I_1 + 15I_2 = 6 \quad \text{... (i)}$$

Loop with E_2 and R:

$$E_2 = I_2 r_2 + (I_1 + I_2)R$$

$$12 = I_2 \times 2 + (I_1 + I_2) \times 15 \quad \Rightarrow \quad 15I_1 + 17I_2 = 12 \quad \text{... (ii)}$$

Multiply (i) by 15 and (ii) by 16: $240I_1 + 225I_2 = 90$ and $240I_1 + 272I_2 = 192$. Subtracting, $47I_2 = 102$, so

$$I_2 = \frac{102}{47} = \mathbf{2.17\ A}$$

and from (i), $16I_1 = 6 - 15 \times 2.17 = -26.6$, so

$$I_1 = \mathbf{-1.66\ A}$$

I_1 is negative: the current in the 6 V cell flows **opposite** to the arrow, into its + terminal. The stronger 12 V cell is pushing 1.66 A backwards through the weaker one (charging it, if it is rechargeable).

Current through R: $I_1 + I_2 = -1.66 + 2.17 = 0.51$ A, and the p.d. across it is

$$V = (I_1 + I_2)R = \frac{24}{47} \times 15 = \mathbf{7.66\ V}$$

WORKED EXAMPLE

Example 14.3: a weaker and a stronger battery

A battery of e.m.f. 6 V and internal resistance 0.5 Ω is joined in parallel with another battery of e.m.f. 10 V and internal resistance 1 Ω. The combination sends a current through an external resistance of 12 Ω. Find the current through each battery and through the resistance.

Answer

The circuit is the same as Figure 14.5, with $E_1 = 6$ V, $r_1 = 0.5\ \Omega$, $E_2 = 10$ V, $r_2 = 1\ \Omega$ and $R = 12\ \Omega$. Let I_1 and I_2 leave the + ends of the two batteries; $I_1 + I_2$ flows through R.

Loop with the 6 V battery and R:

$$6 = 0.5I_1 + 12(I_1 + I_2) \quad \Rightarrow \quad 12.5I_1 + 12I_2 = 6 \quad \text{... (i)}$$

Loop with the 10 V battery and R:

$$10 = 1 \times I_2 + 12(I_1 + I_2) \quad \Rightarrow \quad 12I_1 + 13I_2 = 10 \quad \text{... (ii)}$$

Multiply (i) by 13 and (ii) by 12: $162.5I_1 + 156I_2 = 78$ and $144I_1 + 156I_2 = 120$. Subtracting,

$$18.5I_1 = -42 \quad \Rightarrow \quad I_1 = -2.27\ \text{A}$$

From (i): $12I_2 = 6 - 12.5 \times (-2.27) = 6 + 28.38 = 34.38$, so $I_2 = 2.86$ A.

So **2.27 A** flows through the 6 V battery, backwards (into its + terminal), **2.86 A** through the 10 V battery, and $I_1 + I_2 = 0.59$ A through the **12 Ω resistance**.

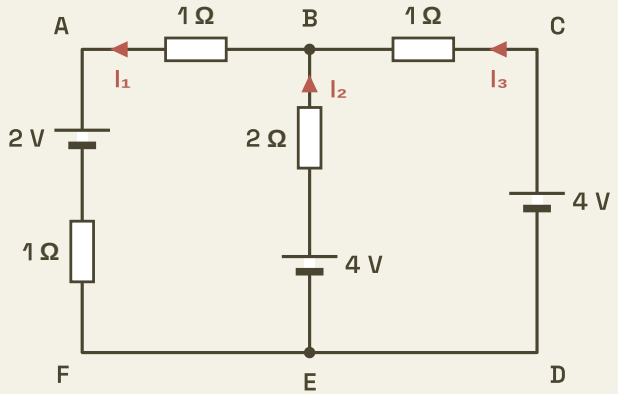


Figure 14.6 For Test yourself 14.2. Along the top, A to B through 1 Ω and B to C through 1 Ω. Down the left, A to F through a 2 V cell (+ end up) and 1 Ω. The middle branch from E up to B has a 4 V cell (+ end up) and 2 Ω. Down the right, C to D through a 4 V cell (+ end up). F, E and D are joined along the bottom. I_1 flows from B to A, I_2 from E up to B and I_3 from C to B.

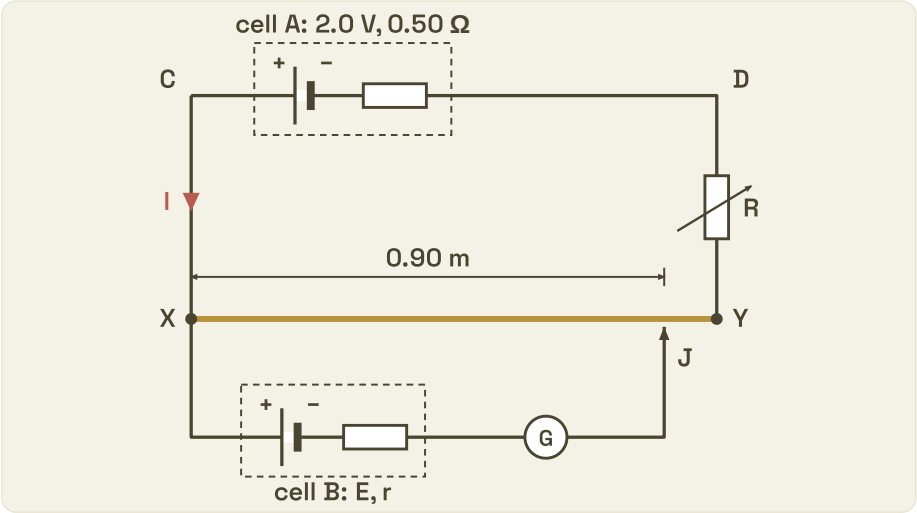


Figure 14.35 For question 27. Cell A (2.0 V, 0.50 Ω) and a variable resistor R (2.5 Ω) in series with the uniform resistance wire XY (1.00 m, 4.0 Ω): C–X down the left, D–Y down the right. Cell B (e.m.f. E, internal resistance r), its + terminal at X, is connected through a galvanometer to the contact J, 0.90 m from X.

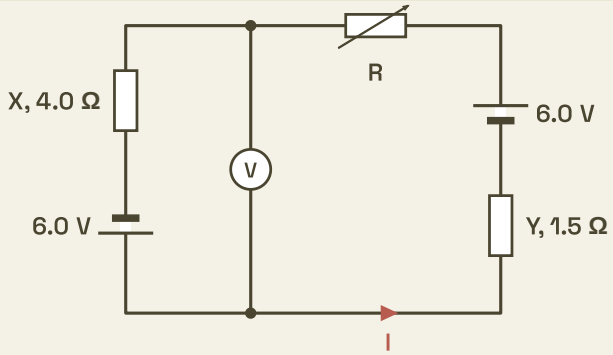


Figure 14.33 *For question 25. A loop: down the left, resistor X (4.0 Ω) above a 6.0 V battery; along the top, the variable resistor R; down the right, a second 6.0 V battery above resistor Y (1.5 Ω); both batteries drive the current I the same way round. A voltmeter is connected across X and the left battery.*

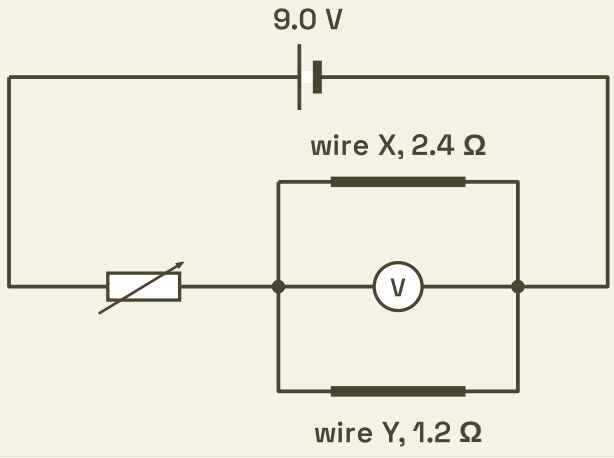


Figure 14.34 *For question 26. A 9.0 V supply, a variable resistor, and wires X (2.4 Ω) and Y (1.2 Ω) in parallel, with a voltmeter across the two wires.*

YOUR TURN

Test yourself 14.2: three branches

In the circuit of Figure 14.6, find the currents I_1, I_2 and I_3.

Hint

Junction B gives $I_1 = I_2 + I_3$. Then use two loops: the left one (ABEFA) and the right one (BCDEB).

Answer

Junction B: I_2 and I_3 flow in, I_1 flows out: $I_1 = I_2 + I_3$.

Loop ABEFA, traveling A → B → E → F → A (against I_1 in AB, against I_2 in BE):

- A to B through 1 Ω against I_1: $+I_1 \times 1$;
- B down to E: through 2 Ω against I_2, $+2I_2$; then the 4 V cell from + to −, -4;
- F up to A: through 1 Ω against I_1 (I_1 flows down this branch), $+I_1 \times 1$; then the 2 V cell from − to +, $+2$.

$$I_1 + 2I_2 - 4 + I_1 + 2 = 0 \quad \Rightarrow \quad 2I_1 + 2I_2 = 2 \quad \Rightarrow \quad I_1 + I_2 = 1 \quad \text{... (i)}$$

Loop BCDEB, traveling B → C → D → E → B:

- B to C through 1 Ω against I_3: $+I_3$;
- C down to D through the 4 V cell from + to −: -4;
- E up to B: the 4 V cell from − to +, $+4$; then 2 Ω with I_2, $-2I_2$.

$$I_3 - 4 + 4 - 2I_2 = 0 \quad \Rightarrow \quad I_3 = 2I_2 \quad \text{... (ii)}$$

So $I_1 = I_2 + 2I_2 = 3I_2$, and (i) gives $4I_2 = 1$:

$$I_2 = \mathbf{0.25\ A}, \qquad I_3 = \mathbf{0.50\ A}, \qquad I_1 = \mathbf{0.75\ A}$$

All three are positive, so they flow as marked. Check at B: 0.25 + 0.50 = 0.75 A in, 0.75 A out. ✓

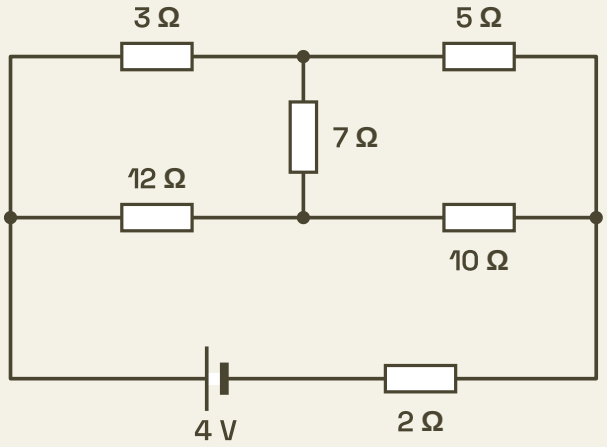


Figure 14.7 *For Test yourself 14.3. A 3 Ω and a 5 Ω resistor in series along the top, and a 12 Ω and a 10 Ω in series below them, joined at their ends; a 7 Ω resistor joins the point between 3 Ω and 5 Ω to the point between 12 Ω and 10 Ω. A 4 V cell and a 2 Ω resistor complete the circuit along the bottom.*

YOUR TURN

Test yourself 14.3: a bridge that isn't balanced

Find the current drawn from the battery in the circuit of Figure 14.7.

Hint

3/5 is not equal to 12/10, so current flows through the 7 Ω resistor and it can't be left out. Call the battery current I, the current in the 3 Ω resistor I_1 (so $I - I_1$ in the 12 Ω), and the current down through the 7 Ω I_g. Write three loop equations.

Answer

Let L and R be the left and right ends of the network, T the point between 3 Ω and 5 Ω, and M the point between 12 Ω and 10 Ω. Currents: I from the battery into L; I_1 through 3 Ω (L to T), $I - I_1$ through 12 Ω (L to M); I_g through 7 Ω (T to M); then $I_1 - I_g$ through 5 Ω and $I - I_1 + I_g$ through 10 Ω.

Loop L–T–M–L (no cell): $-3I_1 - 7I_g + 12(I - I_1) = 0$, that is $15I_1 + 7I_g - 12I = 0$ … (i)

Loop T–R–M–T: $-5(I_1 - I_g) + 10(I - I_1 + I_g) + 7I_g = 0$, that is $15I_1 - 22I_g - 10I = 0$ … (ii)

Outer loop through the battery, 3 Ω and 5 Ω: $4 = 2I + 3I_1 + 5(I_1 - I_g)$, that is $2I + 8I_1 - 5I_g = 4$ … (iii)

(i) – (ii): $29I_g - 2I = 0$, so $I_g = \dfrac{2I}{29}$.

Then (i): $15I_1 = 12I - \dfrac{14I}{29} = \dfrac{334I}{29}$, so $I_1 = \dfrac{334I}{435}$.

Into (iii), multiplied by 435: $870I + 2672I - 150I = 1740$, so $3392I = 1740$ and

$$I = \frac{1740}{3392} = \mathbf{0.51\ A}\ \text{(about 0.5 A)}$$

(The network of five resistors is equivalent to 5.8 Ω: 4 V ÷ (5.8 + 2) Ω = 0.51 A.)

14.2 Wheatstone bridge principle from Kirchhoff's laws

A **Wheatstone bridge** is a circuit for measuring an unknown resistance X very accurately. It has four resistances joined in a ring ABCD (Figure 14.8): two fixed, known resistances P and Q (the **ratio arms**, usually of manganin or constantan, whose resistance hardly changes with temperature), a variable known resistance R, and the unknown X. A cell is connected between A and C, and a sensitive **galvanometer** G between B and D, "bridging" the two paths.

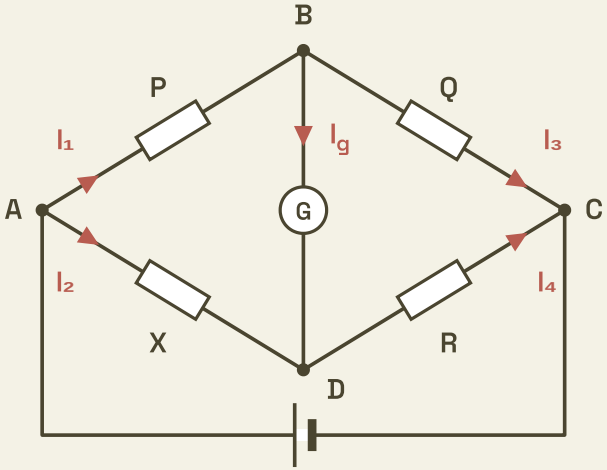


Figure 14.8 *A Wheatstone bridge. P is in arm AB, Q in BC, R in CD and X in AD; a cell is connected across A and C, and a galvanometer G across B and D. The currents I_1 (in P), I_3 (in Q), I_2 (in X), I_4 (in R) and I_g (in G, from B to D) are marked.*

R is adjusted until the galvanometer shows **no deflection**: the **null point**. Then B and D are at the same potential, and

$$\frac{P}{Q} = \frac{X}{R}$$

This is the **principle of the Wheatstone bridge**. Here is why, from Kirchhoff's laws.

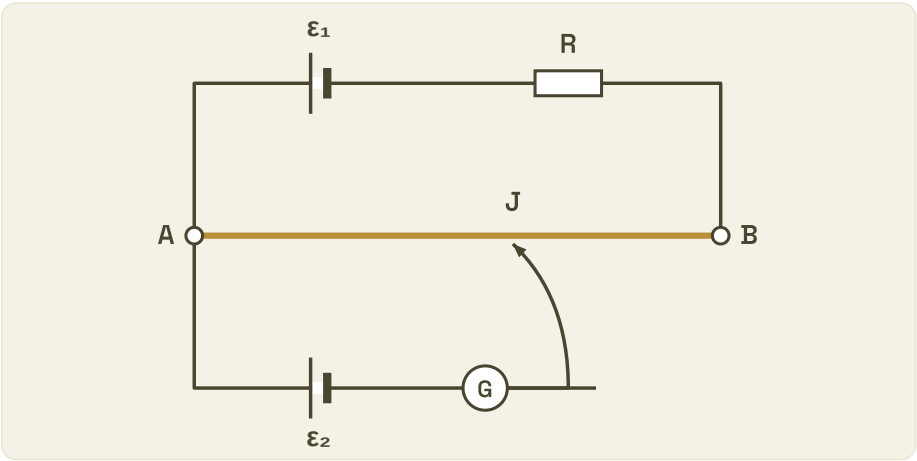


Figure 14.31 *For question 23. A cell ε_1 and a resistance R in series with the uniform wire AB. A cell ε_2, its + terminal at A, is connected through a galvanometer G to the jockey J.*

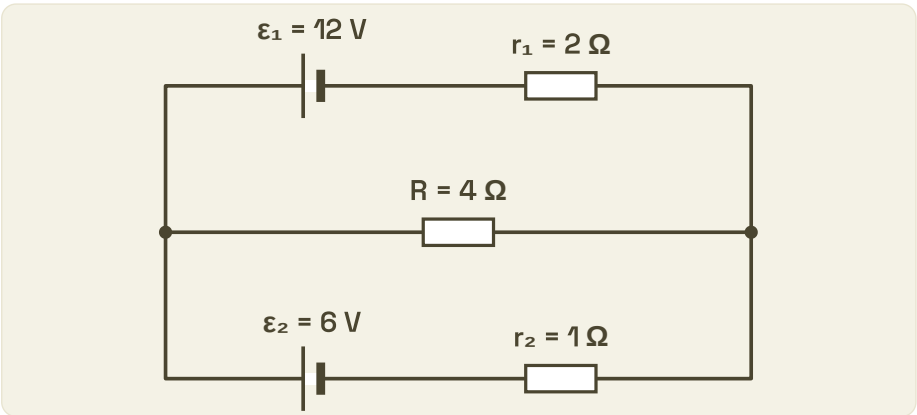


Figure 14.32 *For question 24. Three branches in parallel between two junctions: $\varepsilon_1 = 12$ V with $r_1 = 2\ \Omega$ along the top, $R = 4\ \Omega$ in the middle, and $\varepsilon_2 = 6$ V with $r_2 = 1\ \Omega$ along the bottom; both cells have their + ends at the left junction.*

25. a. State Kirchhoff's second law.

b. Two batteries, each of e.m.f. 6.0 V and negligible internal resistance, are connected in series with three resistors as in Figure 14.33. Resistor X has resistance 4.0 Ω and resistor Y 1.5 Ω. The resistance R of the variable resistor is changed until the voltmeter reads zero. Calculate i. the current I in the circuit; ii. the resistance R.

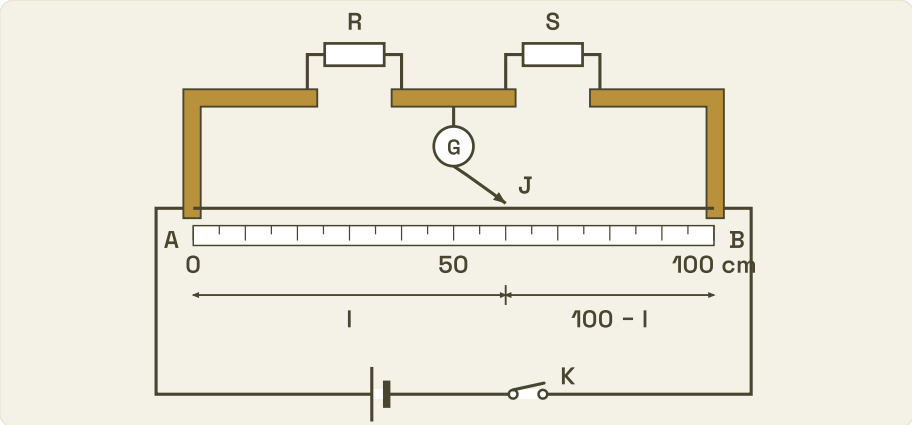


Figure 14.29 *For question 18. A meter bridge with R in the left gap and S in the right gap; the galvanometer joins the middle strip to the jockey.*

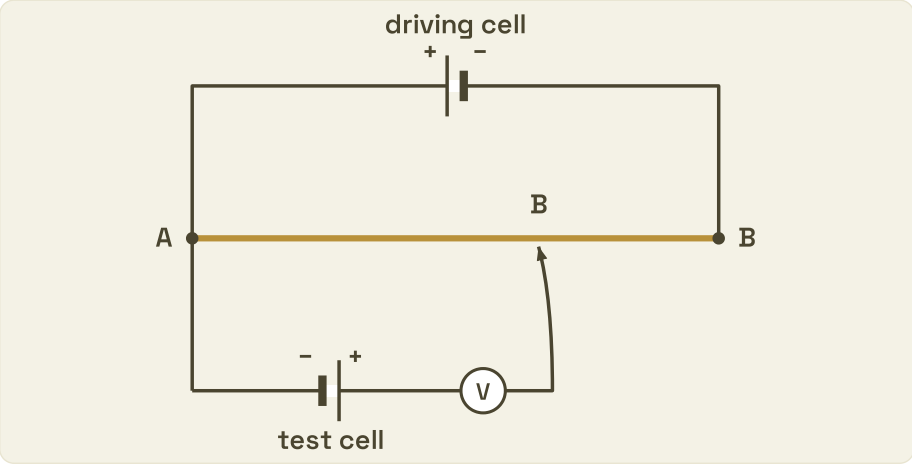


Figure 14.30 *For question 22, as the student set it up. The driver cell has its + terminal at A. The test cell has its − terminal at A, and its + terminal goes through a voltmeter to the jockey at B.*

Let I_1, I_3, I_4 and I_2 be the currents in P, Q, R and X, and I_g the current in the galvanometer (resistance G), from B to D.

Step 1. Loop ABDA (second law), traveling A → B → D → A: with I_1 through P, with I_g through G, then against I_2 through X:

$$-I_1P - I_gG + I_2X = 0 \quad \dots (14.1)$$

Step 2. Loop BCDB, traveling B → C → D → B: with I_3 through Q, against I_4 through R, against I_g through G:

$$-I_3Q + I_4R + I_gG = 0 \quad \dots (14.2)$$

Step 3. Junction B (first law): I_1 in, I_g and I_3 out:

$$I_1 - I_g - I_3 = 0 \quad \dots (14.3)$$

Step 4. Junction D: I_g and I_2 in, I_4 out:

$$I_g + I_2 - I_4 = 0 \quad \dots (14.4)$$

Step 5. Balance. R is adjusted until the galvanometer shows no deflection: $I_g = 0$. Then the four equations become

$$I_1P = I_2X \quad \dots (14.5) \qquad I_3Q = I_4R \quad \dots (14.6) \qquad I_1 = I_3 \qquad I_2 = I_4$$

Step 6. Divide. Using $I_3 = I_1$ and $I_4 = I_2$ in (14.6): $I_1Q = I_2R$. Dividing (14.5) by this,

$$\frac{I_1P}{I_1Q} = \frac{I_2X}{I_2R} \quad \Rightarrow \quad \boxed{\frac{P}{Q} = \frac{X}{R}}$$

So $X = \dfrac{P}{Q}R$: knowing P, Q and R, the unknown X is found.

A memory aid: at balance, the ratio of the two resistances on one side of the galvanometer equals the ratio on the other side, taken in the same order from the cell's end A.

DID YOU KNOW?

Why measuring "nothing" is so accurate

The bridge is a **null method**: you don't read a value off the galvanometer, you only look for zero. A meter that is badly calibrated, or whose scale is cramped, still shows clearly whether its needle moves at all. So the result depends only on how well P, Q and R are known, not on the meter, and a Wheatstone bridge measures resistances (from about an ohm to a megohm) more accurately than an ammeter and a voltmeter can. For very small resistances, a fraction of an ohm, it is less good, because the resistance of the connecting wires and contacts then matters; laboratories use a variant, the Kelvin double bridge, for those. The meter bridge (Section 14.3) and the post office box are Wheatstone bridges in other forms, and the potentiometer (Section 14.4) is a null method too.

A STORY FROM SCIENCE

Not Wheatstone's invention

The bridge was invented by **Samuel Hunter Christie** in 1833. **Charles Wheatstone**, a London professor (and maker of musical instruments), saw how useful it was, described it in 1843, and always gave Christie the credit. It is called after Wheatstone anyway. Wheatstone also built one of the first electric telegraphs and invented the concertina.

Check 3

In a balanced Wheatstone bridge, the galvanometer and the cell are interchanged. The galvanometer now:

a still shows no deflection
b shows a large deflection
c shows a deflection depending on the cell's e.m.f.
d burns out

14.3 Meter bridge

The **meter bridge** (also called the **slide wire bridge**) is the simplest practical Wheatstone bridge. It works on the principle of the Wheatstone bridge: two of the four arms are the two parts of one uniform wire, divided by a sliding contact.

Construction (Figure 14.9). A **constantan** (or manganin) wire exactly **1 m** long and of uniform cross-section is stretched along a wooden board between two thick copper strips at A and C. A meter scale is fixed on the board beside the wire. A third copper strip between them leaves **two gaps**: a resistance box R is connected across one gap and the unknown resistance X across the other. One terminal of a sensitive galvanometer is joined to the middle strip (point B), and the other to a **jockey**, a knife-edge contact that can be slid along the wire and pressed on it at any point J. A cell and key are connected across the ends of the wire.

The circuit is exactly that of a Wheatstone bridge: the part of the wire from A to the jockey is one arm (call its resistance P), and the part from the jockey to C is another (Q); X and R are the other two. The jockey is moved until the galvanometer shows no

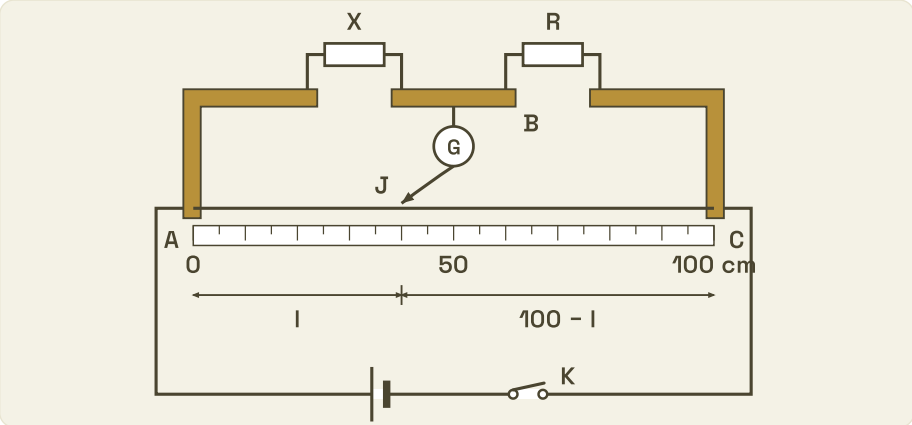


Figure 14.9 *A meter bridge: a 1 m constantan wire AC stretched over a meter scale between thick copper strips. The unknown X is in the left gap and a resistance box R in the right gap; the galvanometer joins the middle strip B to the jockey J. A cell and key are connected across A and C. The balance length is l from A, and 100 − l from C.*

deflection.



Figure 14.10 *A meter bridge in a school laboratory.*

Photo: Stephanie cheks, CC BY-SA 4.0

To measure an unknown resistance

Suppose the balance point J is at a distance l (in cm) from the end A, so that JC = (100 − l) cm. The wire is uniform, so the resistance of each part is proportional to its length:

$$P \propto l \quad \text{and} \quad Q \propto (100 - l)$$

$$\frac{P}{Q} = \frac{l}{100 - l} \quad \ldots (14.7)$$

From the principle of the Wheatstone bridge, $\frac{P}{Q} = \frac{X}{R}$, so

$$\frac{X}{R} = \frac{l}{100 - l} \quad \Rightarrow \quad \boxed{X = \frac{l}{100 - l} R}$$

Knowing R and l, X is found. The same circuit can **compare two resistances**: put one in each gap, and their ratio is $l/(100 - l)$.

The result is most accurate when the balance point is near the **middle** of the wire: a small error in reading l then makes the smallest error in X. So R is chosen close to the expected value of X.

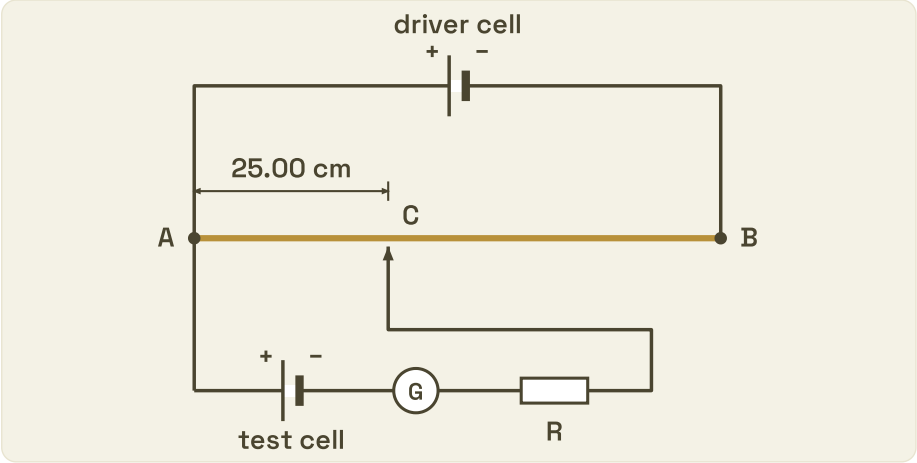


Figure 14.27 *For question 16. A driver cell across the wire AB. A test cell, its + terminal at A, a galvanometer and a protective resistance R lead to the jockey, which balances at C, 25.00 cm from A.*

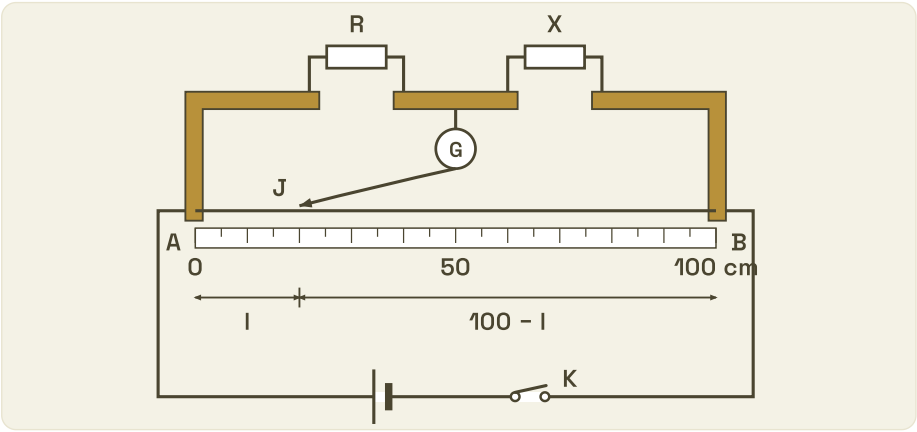


Figure 14.28 *For question 17. A meter bridge with R in the left gap and X in the right gap; the balance point J is a length l from A.*

21. Kirchhoff's laws are very useful in solving complicated circuits.
    a. What is the significance of the first law?
    b. State and explain the second law with a circuit diagram.
    c. Apply these laws to find an unknown resistance.
    d. What is a meter bridge? Name the material used for its wire.

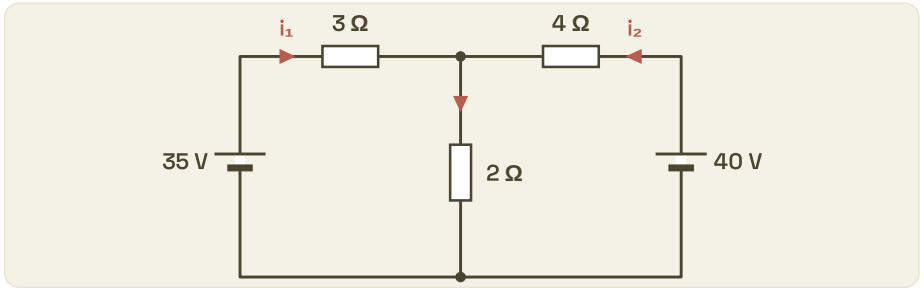


Figure 14.25 *For question 14. A 35 V cell on the left and a 40 V cell on the right, both with their + ends up, joined along the bottom. Along the top, 3 Ω from the left and 4 Ω from the right meet at a junction; a 2 Ω resistor runs from the junction down to the bottom wire. i_1 flows through 3 Ω towards the junction, i_2 through 4 Ω towards the junction.*

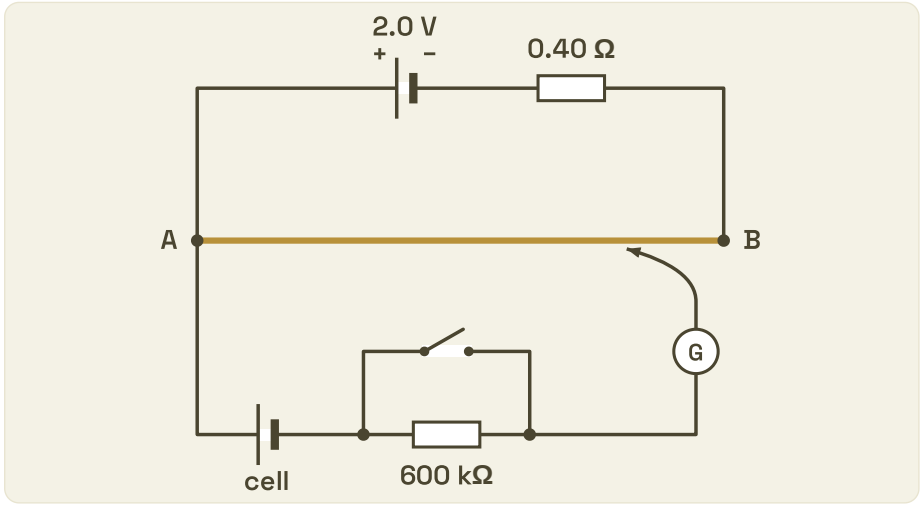


Figure 14.26 *For question 15. A 2.0 V cell with 0.40 Ω internal resistance drives a current through the potentiometer wire AB. A test cell, with a 600 kΩ resistor in series (which a switch can short out), is connected from A through a galvanometer G to the jockey.*

17.

a. State the principle of the potentiometer. Why is a potentiometer also called a voltmeter of infinite resistance?

b. In a meter bridge experiment (Figure 14.28), the balance point is at J with l = 20 cm.

i. The values of R and X are doubled and then interchanged. Where is the new balance point?

ii. If the galvanometer and the battery are interchanged at the balance position, how is the balance point affected?

18.

a. State Kirchhoff's two laws for electrical circuits.

b. In the meter bridge of Figure 14.29, the null point is 60 cm from A. When a resistance of 5 Ω is connected in series with S, the null point is at 50 cm from A. Find R and S.

WORKED EXAMPLE

Example 14.4: a meter bridge

In a meter bridge, a resistance of 10 Ω is in the left gap and the unknown resistance in the right gap. The balance point is 40 cm from the left end. Calculate the unknown resistance.

Answer

The left gap faces the 40 cm part of the wire, and the right gap the 60 cm part:

$$\frac{\text{left}}{\text{right}} = \frac{l}{100-l} \quad \Rightarrow \quad \frac{10}{X} = \frac{40}{60}$$

$$X = 10 \times \frac{60}{40} = \mathbf{15\ \Omega}$$

(Sense check: the balance point is nearer the left end, so the left gap holds the smaller resistance.)

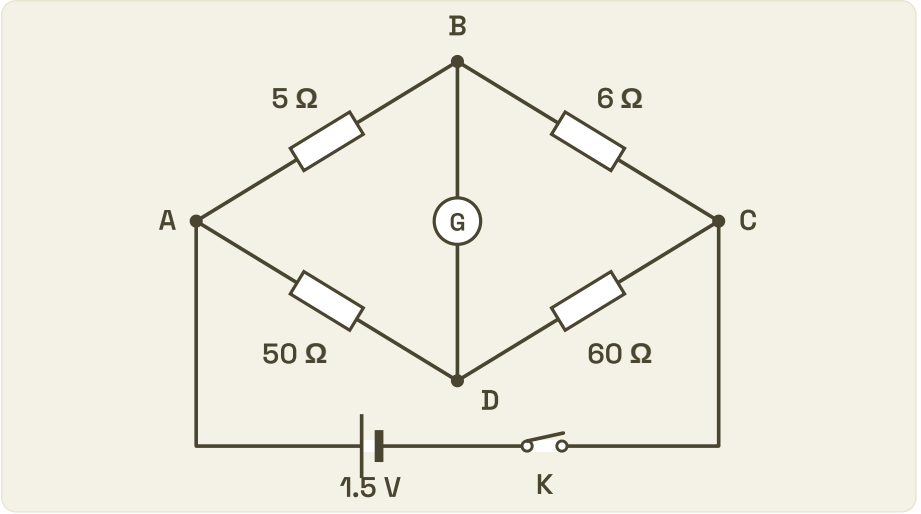


Figure 14.11 *For Example 14.5: P = 5 Ω in AB, Q = 6 Ω in BC, X = 50 Ω in AD and R = 60 Ω in DC; a 1.5 V cell and key across A and C, the galvanometer across B and D.*

WORKED EXAMPLE

Example 14.5: the currents in a balanced bridge

Four resistances are connected in a Wheatstone bridge as in Figure 14.11: 5 Ω in arm AB, 6 Ω in BC, 50 Ω in AD and 60 Ω in DC. A cell of e.m.f. 1.5 V and negligible internal resistance is connected across A and C. Calculate the current in each arm and in the cell.

Answer

Is it balanced? $\frac{P}{Q} = \frac{5}{6}$ and $\frac{X}{R} = \frac{50}{60} = \frac{5}{6}$. Yes, so no current flows through the galvanometer, and the bridge is just two separate paths across the cell.

Path ABC (P and Q in series, the same current I_1 in both): going round the loop from the cell through P and Q,

$$1.5 = I_1(5+6) \quad \Rightarrow \quad I_1 = \frac{1.5}{11} = \mathbf{0.136\ A}$$

Path ADC: $1.5 = I_2(50+60)$, so $I_2 = \frac{1.5}{110} = \mathbf{0.0136\ A}$.

The cell: $I = I_1 + I_2 = 0.136 + 0.0136 = \mathbf{0.15\ A}$.

(Kirchhoff's second law round ABDA gives the same balance: $-5I_1 + 50I_2 = 0$, so $I_2 = I_1/10$.)

WORKED EXAMPLE

Example 14.6: balancing with a shunt

P, Q, R and X are four coils of resistance 2 Ω, 2 Ω, 2 Ω and 3 Ω, arranged to form a Wheatstone bridge. Calculate the resistance with which the coil X must be shunted (connected in parallel) so that the bridge is balanced.

Answer

Balance needs $\frac{P}{Q} = \frac{X'}{R}$, where X′ is the resistance of X with its shunt S. Here P/Q = 1, so X′ must equal R = 2 Ω.

X and S are in parallel:

$$X' = \frac{XS}{X+S} \quad \Rightarrow \quad 2 = \frac{3S}{3+S}$$

$$6 + 2S = 3S \quad \Rightarrow \quad S = \mathbf{6\ \Omega}$$

Check: 3 Ω and 6 Ω in parallel: $\frac{3 \times 6}{9} = 2\ \Omega$. ✓

Check 4

In a meter bridge with 5 Ω in the left gap and 15 Ω in the right gap, the balance point is:

a 25 cm from the left end
b 33 cm from the left end
c 75 cm from the left end
d 50 cm from the left end

14.4 Potentiometer

A **potentiometer** is an instrument for measuring the e.m.f. of a cell, comparing the e.m.f.s of two cells and finding the internal resistance of a cell. A voltmeter draws some current from the cell it measures, and so reads the terminal p.d., which is less than the e.m.f. by Ir. A potentiometer, at its balance point, draws **no current** from the cell, so it measures the e.m.f. itself.

Construction. A long wire of uniform cross-section, usually **10 m** of constantan or manganin, is stretched on a wooden board in ten lengths of 100 cm each, side by side, joined at their ends by thick copper strips, with a meter scale beside them. A battery (the **driver cell**), a key, an ammeter and a **rheostat** are connected across the ends A and B of the whole wire, so that a steady current flows through it; the rheostat keeps the current constant and sets its value. In diagrams the wire is drawn as one straight line AB.

Principle

When a **constant current** flows through a wire of **uniform cross-section**, the potential difference across any length of the wire is **directly proportional to that length**.

Proof. Let V be the p.d. across a length l of the wire, of resistance R, when a current I flows. By Ohm's law, $V = IR$. The resistance of the length is $R = \frac{\rho l}{A}$, where ρ is the resistivity of the material and A the area of cross-section. So

$$V = I\frac{\rho l}{A} = \left(\frac{I\rho}{A}\right) l$$

With a constant current I and a uniform wire (ρ and A the same everywhere), $\frac{I\rho}{A}$ is a constant, k:

$$V = kl \qquad \text{or} \qquad V \propto l \quad \ldots (14.8)$$

The constant $k = \frac{V}{l} = \frac{I\rho}{A}$ is the **potential gradient**: the fall of potential per unit length of the wire, in V/m or V/cm.

How the balance point measures an e.m.f.

The test cell is connected with its **+ terminal to A**, the same end as the driver cell's + terminal, and its other terminal through the galvanometer to the jockey (Figure 14.12). Then the cell's e.m.f. E and the p.d. along the wire from A to J push against each other in the galvanometer's loop:

- if J is too near A, the p.d. AJ is less than E, and the cell pushes a current one way through G;
- if J is too far, the p.d. AJ is more than E, and current flows the other way;
- at one point, the **balance point** (or null point), the p.d. AJ exactly equals E, and **no current** flows through G, or through the cell.

At balance the cell gives no current, so there is no drop across its internal resistance: the p.d. it balances is its full e.m.f.

$$E = kl$$

Check 13

7. A resistance of 5 Ω is connected in the left gap of a meter bridge and 15 Ω in the right gap. The balance point is at:

a 25 cm
b 15 cm
c 60 cm
d 75 cm

Check 14

8. The resistivity of a potentiometer wire is 40×10^{-8} Ω m and its area of cross-section is 8×10^{-6} m². If a current of 0.2 A flows through the wire, the potential gradient is:

a 10^{-2} V/m
b 10^{-1} V/m
c 3.2×10^{-2} V/m
d 1 V/m

Check 15

9. A potentiometer has a wire 400 cm long, of resistance 10 Ω, connected to a cell of e.m.f. 2 V with negligible internal resistance. The potential difference per unit length of the wire is:

a 5 V/m
b 0.5 V/m
c 0.05 V/m
d none of these

Check 16

10. Two cells of e.m.f. 1.25 V and 0.75 V are connected in parallel (+ to +). The effective voltage will be:

The textbook gives the options 0.75, 1.25, 2.00 and 0.50 V, none of which is right, so the book's options don't work; the fourth is replaced here.

a 0.75 V
b 1.25 V
c 2.00 V
d between 0.75 V and 1.25 V

Check 17

11. The instrument for the accurate measurement of the e.m.f. of a cell is:

a a voltmeter
b an ammeter
c a potentiometer
d a slide wire bridge

Check 18

12. The internal resistance of an ideal voltmeter is:

a zero
b infinite
c small
d large but finite

Check 19

13. A galvanometer of resistance 99 Ω should carry 10 % of the main current. The shunt to be connected in parallel with it is:

a 9 Ω
b 9.9 Ω
c 10 Ω
d 11 Ω

Answer these questions

14. a. State Kirchhoff's current law and voltage law.
    b. Using Kirchhoff's laws, find the current in the 2 Ω resistor in Figure 14.25.
    c. Can Kirchhoff's laws be used for both AC and DC circuits?

15. Figure 14.26 shows a potentiometer with a cell of 2.0 V and internal resistance 0.40 Ω maintaining a potential drop across the resistance wire AB. A standard cell of constant e.m.f. 1.02 V gives the balance point at 67.3 cm of the wire. To make sure very little current is drawn from the standard cell, a very high resistance of 600 kΩ is put in series with it. The standard cell is then replaced by a cell of unknown e.m.f., and the balance point, found the same way, is at 82.3 cm.
    a. What is the e.m.f. of the unknown cell?
    b. What is the purpose of the 600 kΩ resistance?
    c. Is the balance point affected by this high resistance?
    d. Would the method work if the driver cell had an e.m.f. of 1.0 V instead of 2.0 V?

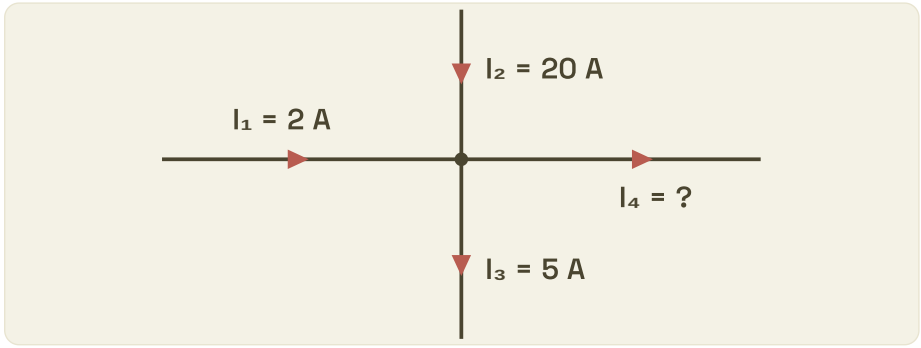


Figure 14.24 *For question 1: $I_1 = 2$ A flows in from the left and $I_2 = 20$ A in from above; $I_3 = 5$ A flows out downwards, and I_4 out to the right.*

Check 7

1. What is the value of I_4 flowing in the circuit shown?

a 13 A b 17 A
c 23 A d 27 A

Check 8

2. The statement "In a closed loop of an electric circuit, the algebraic sum of the e.m.f.s is equal to the algebraic sum of the potential drops" is:

a Kirchhoff's voltage law (KVL) b Ohm's law
c Lenz's law d Kirchhoff's current law (KCL)

Check 9

3. Consider two statements: (i) Kirchhoff's junction law follows from the principle of conservation of charge. (ii) Kirchhoff's junction law follows from the principle of conservation of the electric field.

a Both (i) and (ii) are correct. b (i) is correct but (ii) is wrong.
c (ii) is correct but (i) is wrong. d Both are wrong.

Check 10

4. If the null point of a potentiometer with a cell of e.m.f. E is at a balance length l on the wire, then:

a $E \propto l$ b $E \propto l^2$
c $E \propto 1/l$ d $E \propto 1/l^2$

Check 11

5. A length of 650 cm of a potentiometer wire is needed to balance the e.m.f. of a cell. When a resistor of 8 Ω is connected across the cell's terminals, the balance length is 520 cm. The internal resistance of the cell is:

a 3 Ω b 1.5 Ω
c 2 Ω d 5 Ω

Check 12

6. A galvanometer is connected to the secondary coil of a solenoid. It shows an instantaneous maximum deflection of 7 divisions when the current is started in the primary coil. If the primary coil is now turned through 180° and the current started again, the instantaneous maximum deflection will be:

a 7 divisions (to the other side) b 14 divisions
c 0 divisions d 21 divisions

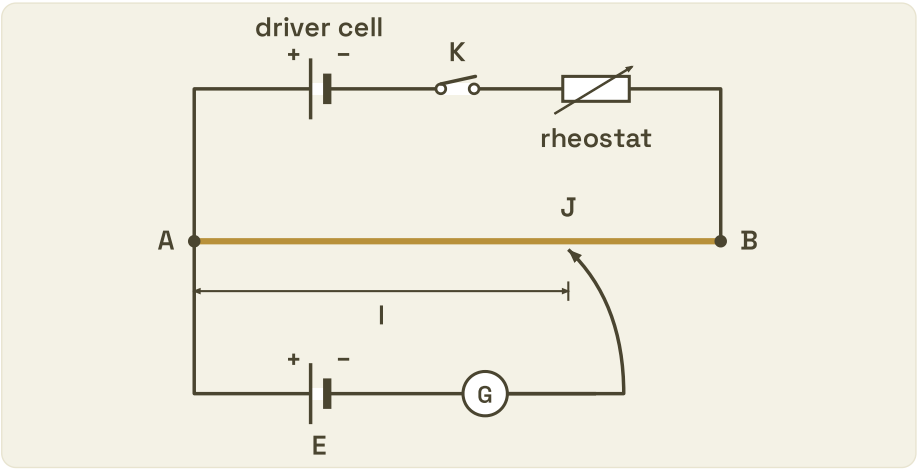


Figure 14.12 *The potentiometer's circuit. The driver cell, key and rheostat send a steady current through the uniform wire AB. A test cell of e.m.f. E has its + terminal joined to A, the same end as the driver cell's + terminal, and its − terminal to a galvanometer G and the jockey J. At the balance point J, a length l from A, the galvanometer reads zero.*

CAREFUL

Two conditions for a balance point

1. The **+ terminals** of the driver cell and the test cell must both go to **the same end A** of the wire. If one is reversed, its e.m.f. adds to the p.d. along the wire instead of opposing it, and the galvanometer deflects the same way at every point.
2. The p.d. across the **whole wire** must be **greater than** the e.m.f. being measured, so the driver cell's e.m.f. must be greater than that of each test cell (and the rheostat must not cut the current too far). Otherwise the balance point would lie beyond B.

Either fault shows the same way: a deflection in one direction only, wherever the jockey touches.

Applications of the potentiometer

1. To compare the e.m.f.s of two cells

The two cells E_1 and E_2 have their + terminals joined to A, and their − terminals to the two outer terminals of a **two-way key**, whose middle terminal is connected through the galvanometer G to the jockey (Figure 14.13). A steady current is kept in the wire by the driver cell (of e.m.f. greater than either), with an ammeter, a rheostat and a one-way key.

First the key is set to connect E_1 alone. The jockey is slid along the wire until the galvanometer shows no deflection, at a balance length l_1 from A. By the principle of the potentiometer,

$$E_1 = kl_1 \quad \ldots (14.9)$$

where k is the potential gradient. Then the key is set to connect E_2 alone, and the balance length is l_2:

$$E_2 = kl_2 \quad \ldots (14.10)$$

The current, and so k, is the same both times (the rheostat is not touched). Dividing,

$$\boxed{\frac{E_1}{E_2} = \frac{l_1}{l_2}}$$

If one cell is a **standard cell** of known e.m.f., the other e.m.f. is found.

2. To measure the internal resistance of a cell

The cell, of e.m.f. E and internal resistance r, is connected as a test cell (+ terminal to A, − terminal through G to the jockey), and a **resistance box** R with a key K_2 is connected across its terminals (Figure 14.14).

K_2 open. The cell gives no current (at balance), so the p.d. balanced is its e.m.f. The balance length is l_1:

$$E = kl_1 \quad \ldots (14.11)$$

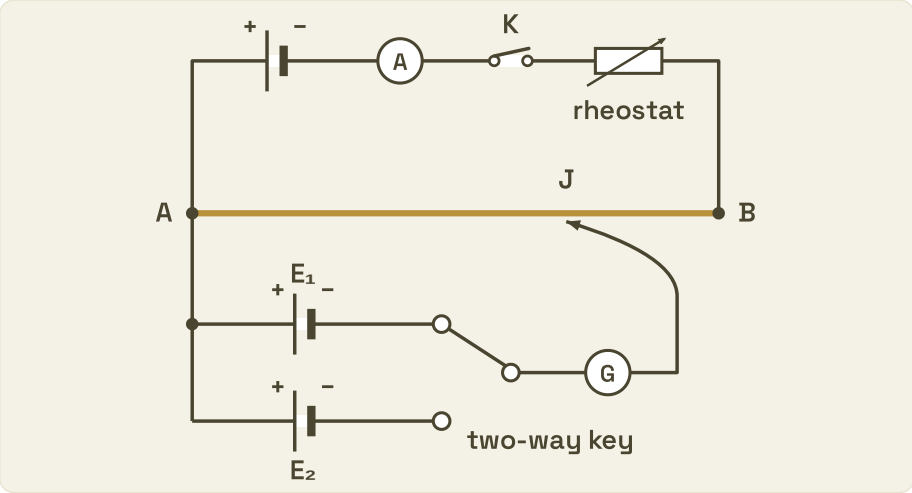


Figure 14.13 Comparing two e.m.f.s. The + terminals of both cells, E_1 and E_2, are joined to A; their − terminals go to the two sides of a two-way key, whose common terminal leads through the galvanometer to the jockey. The driver circuit, with an ammeter, a one-way key K and a rheostat, keeps a steady current in the wire.

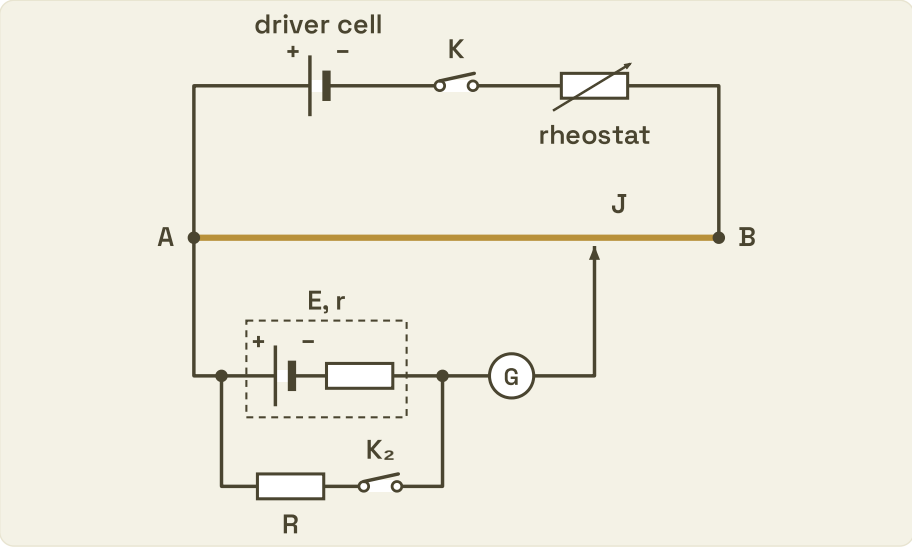


Figure 14.14 Measuring internal resistance. The cell (e.m.f. E, internal resistance r) has its + terminal at A and its − terminal through the galvanometer to the jockey. A resistance box R and a key K_2 are connected across the cell's terminals.

K_2 closed. Now the cell drives a current I through R, and the p.d. across its terminals, $V = IR$, is less than E. The balance length is shorter, l_2:

$$V = kl_2 \quad \dots (14.12)$$

The internal resistance. The cell's current flows through r and R in series: $E = I(R + r)$, while $V = IR$. Dividing,

$$\frac{E}{V} = \frac{R + r}{R} = 1 + \frac{r}{R} \qquad \Rightarrow \qquad r = \left(\frac{E}{V} - 1\right) R$$

Using (14.11) and (14.12), $\frac{E}{V} = \frac{kl_1}{kl_2} = \frac{l_1}{l_2}$, so

$$H = \frac{I^2 Rt}{J}$$

where J is **Joule's mechanical equivalent of heat**, 4.18 J/cal (4.186 J/cal, more exactly). It is simply a conversion factor: I^2Rt is the energy in joules, and dividing by J gives it in calories.

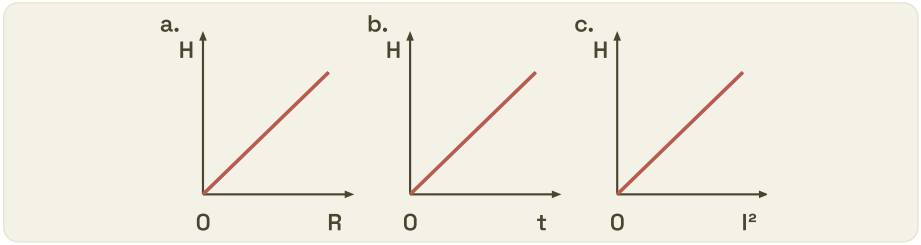


Figure 14.23 Joule's laws as graphs, each with the other two quantities kept constant: heat produced against a. resistance, b. time, c. the square of the current. Each is a straight line through the origin.

Variation with resistance. Keeping the current and the time constant, and changing the resistance, the heat produced is found to be proportional to R: a straight line through the origin (Figure 14.23a).

Variation with time. Keeping the current and the resistance constant, the heat produced is proportional to t: a straight line through the origin (b).

Variation with current. Keeping the resistance and the time constant, the heat produced is proportional to I^2: a graph of H against I^2 is a straight line through the origin (c). (Against I itself, it is a parabola: double the current, four times the heat.)

From all three, the heat developed in the resistor is $H \propto I^2Rt$.

WORKED EXAMPLE

Joule heating in a kettle

An electric kettle element of resistance 46 Ω takes 5 A for 3 minutes. How much heat does it produce, in joules and in calories?

Answer

$$H = I^2Rt = 5^2 \times 46 \times (3 \times 60) = 25 \times 46 \times 180 = \mathbf{207\,000\ J} = 207\ \text{kJ}$$

$$H = \frac{207\,000}{4.18} = \mathbf{4.95 \times 10^4\ cal}\ (\text{about } 49.5\ \text{kcal})$$

That is enough to heat about 0.6 kg of water from 20 °C to 100 °C.

IN SHORT

- **Kirchhoff's first law** (junction rule): $\sum I = 0$ at a junction; current in = current out. Conservation of charge.
- **Kirchhoff's second law** (loop rule): round any closed loop, $\sum E = \sum IR$, or $\sum V = 0$. Conservation of energy. Signs: − to + through a cell, +E; with the current through a resistor, −IR.
- **Wheatstone bridge:** balanced (no current in G) when $P/Q = X/R$. A null method.
- **Meter bridge:** $X = \frac{l}{100 - l}R$; most accurate with the balance point near the middle.
- **Potentiometer:** $V = kl$ for a steady current in a uniform wire; it takes no current at balance. $E_1/E_2 = l_1/l_2$; $r = \left(\frac{l_1}{l_2} - 1\right) R$. More sensitive with a smaller potential gradient.
- **Ammeter:** a shunt $S = \frac{I_g}{I - I_g}G$ in parallel. **Voltmeter:** $R = \frac{V}{I_g} - G$ in series.
- **Superconductivity:** zero resistance below T_c (mercury 4.2 K); the **Meissner effect** expels magnetic fields.
- **Joule's law:** $H = I^2Rt$ joules, or I^2Rt/J calories, J = 4.18 J/cal.

Exercise

Choose the correct option

Discovered by **Walther Meissner** and **Robert Ochsenfeld** in 1933, it is a key property of superconductors. A material in a magnetic field, above T_c, lets the field pass through it almost unchanged. Cool it below T_c, and it **expels the magnetic field from its interior**: currents set up in a thin layer at its surface make a field that exactly cancels the applied field inside. Inside, $B = 0$. A superconductor is a **perfect diamagnet** (its magnetic susceptibility $\chi = -1$; Unit 17).

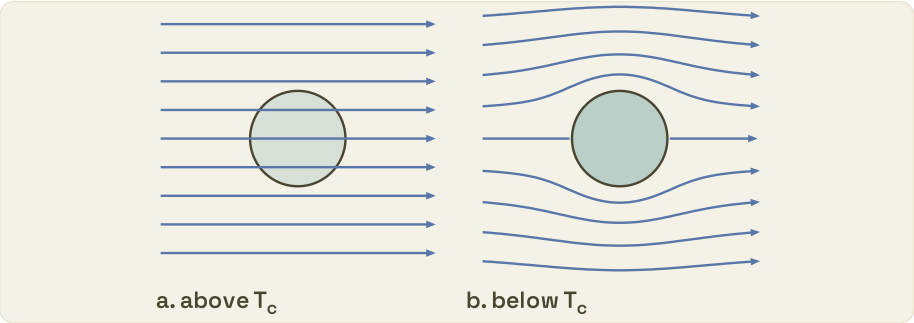


Figure 14.21 *Field lines of a uniform magnetic field and a sphere of superconducting material. a. Above T_c, the lines pass straight through the sphere. b. Below T_c, the field is expelled: the lines bend round the sphere and none passes through it.*

Because the field is pushed out, a superconductor **repels** a magnet. A small magnet placed over a superconductor cooled in liquid nitrogen floats in the air, the most famous demonstration in physics.

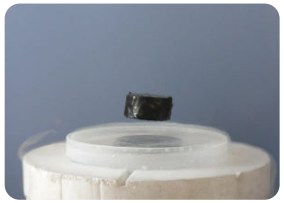


Figure 14.22 *A magnet floating above a superconductor cooled by liquid nitrogen. The superconductor expels the magnet's field, and the magnet is held up.*
Photo: Mai-Linh Doan, CC BY-SA 3.0

Check 6
Which of these does a superconductor do, and a merely perfect conductor not?

a carry current with no resistance
b have no electric field inside
c expel a magnetic field from its interior when it is cooled in that field
d lose no energy as heat

14.7 Joule's laws of heating

In 1841 **James Prescott Joule**, a brewer's son from Manchester, measured how the heat a current produces in a wire depends on the current, the wire's resistance and the time. His results, **Joule's laws of heating**, say that the heat H produced in a conductor is:

i. directly proportional to the **square of the current**: $H \propto I^2$ (R and t kept constant);
ii. directly proportional to the **resistance** of the conductor: $H \propto R$ (I and t kept constant);
iii. directly proportional to the **time** for which the current flows: $H \propto t$ (I and R kept constant).

Combining these, $H \propto I^2Rt$, and in SI units the constant is 1:

$$\boxed{H = I^2Rt}$$

with H in joules. This is the electrical energy turned into heat: a charge $Q = It$ falls through a p.d. $V = IR$, losing energy $QV = It \times IR = I^2Rt$.

To give the heat in **calories**,

$$\boxed{r = \left(\frac{l_1}{l_2} - 1\right) R = \frac{l_1 - l_2}{l_2} R}$$

Knowing l_1, l_2 and R, the internal resistance is found.

Sensitivity, and other uses

A potentiometer is more **sensitive** when a small change in p.d. moves the balance point a long way, that is, when its **potential gradient is small**. It can be made smaller by **using a longer wire** (that is why the wire is 10 m long), or by **reducing the current** in the wire (more resistance from the rheostat). The potentiometer is also used to **calibrate voltmeters and ammeters**, and to measure very small e.m.f.s such as those of thermocouples (Unit 15). Because it takes no current at balance, it behaves like an **ideal voltmeter**, one of infinite resistance.

WORKED EXAMPLE
Example 14.7: an unknown e.m.f.

In a potentiometer experiment, the balance point with a cell of unknown e.m.f. is 55 cm from one end, and with a cell of e.m.f. 1.5 V it is 60 cm from the same end. Calculate the unknown e.m.f.

Answer

$$\frac{E_1}{E_2} = \frac{l_1}{l_2} \quad \Rightarrow \quad E_1 = E_2 \frac{l_1}{l_2} = 1.5 \times \frac{55}{60} = \mathbf{1.375\ V}$$

WORKED EXAMPLE
Example 14.8: the potential gradient

A potentiometer wire of length 10 m and resistance 1.5 Ω/m is connected in series with a cell of e.m.f. 4 V and internal resistance 1 Ω. What is the potential gradient along the wire?

Answer
Resistance of the wire: $1.5 \times 10 = 15\ \Omega$.
Current: $I = \dfrac{E}{R+r} = \dfrac{4}{15+1} = 0.25$ A.
P.d. across the wire: $0.25 \times 15 = 3.75$ V, over 10 m, so

$$k = \frac{3.75\ \text{V}}{10\ \text{m}} = \mathbf{0.375\ V/m}$$

(Or directly: k = I × resistance per meter = 0.25 × 1.5 = 0.375 V/m.)

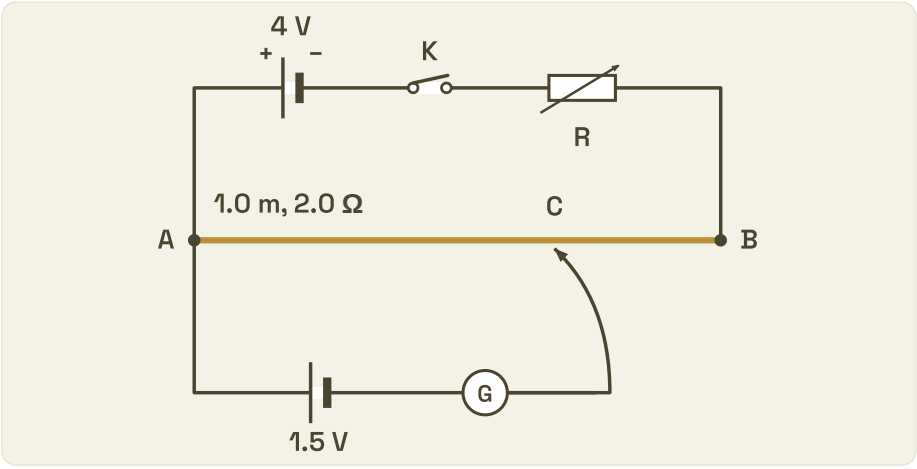


Figure 14.15 *For Example 14.9. A 4 V battery, a key and a variable resistor R in series with the uniform wire AB (1.0 m, 2.0 Ω). A 1.5 V cell, with its + terminal at A, is connected through a galvanometer to the contact C on the wire.*

WORKED EXAMPLE

Example 14.9: setting the gradient

A simple potentiometer circuit is set up as in Figure 14.15, using a uniform wire AB, 1.0 m long, of resistance 2.0 Ω. The resistance of the 4 V battery is negligible.

a. If the variable resistor R is set to 2.4 Ω, what is the length AC for zero galvanometer deflection?

b. If R is made 1.0 Ω, and the 1.5 V cell and galvanometer are replaced by a voltmeter of resistance 20 Ω, what does the voltmeter read when C is at the midpoint of AB?

Answer

a. The current in the wire: $I = \dfrac{4}{R + R_{AB}} = \dfrac{4}{2.4 + 2.0} = \dfrac{4}{4.4} = 0.909\ \text{A}$.

The resistance per meter is 2.0 Ω/m, so a length l_{AC} has resistance $2.0\,l_{AC}$. For zero deflection the p.d. across AC equals 1.5 V:

$$1.5 = \frac{4}{4.4} \times 2.0\,l_{AC} \quad \Rightarrow \quad l_{AC} = \frac{1.5 \times 4.4}{4 \times 2.0} = \mathbf{0.825\ m}$$

b. Now the voltmeter **takes current**, so it is in parallel with AC, which has $2.0 \times 0.5 = 1.0\ \Omega$.

AC and the voltmeter together: $\dfrac{1}{R'_{AC}} = \dfrac{1}{1.0} + \dfrac{1}{20} = \dfrac{21}{20}$, so $R'_{AC} = \dfrac{20}{21}\ \Omega$.

Current from the battery: $I = \dfrac{4}{R + R'_{AC} + R_{CB}} = \dfrac{4}{1 + \frac{20}{21} + 1} = \dfrac{4 \times 21}{62} = 1.355\ \text{A}$.

Voltmeter reading: $V_{AC} = I R'_{AC} = \dfrac{4 \times 21}{62} \times \dfrac{20}{21} = \dfrac{80}{62} = \mathbf{1.29\ V}$

(With an ideal voltmeter it would have read $\dfrac{4}{3} \times 1.0 = 1.33$ V: a real voltmeter disturbs what it measures.)

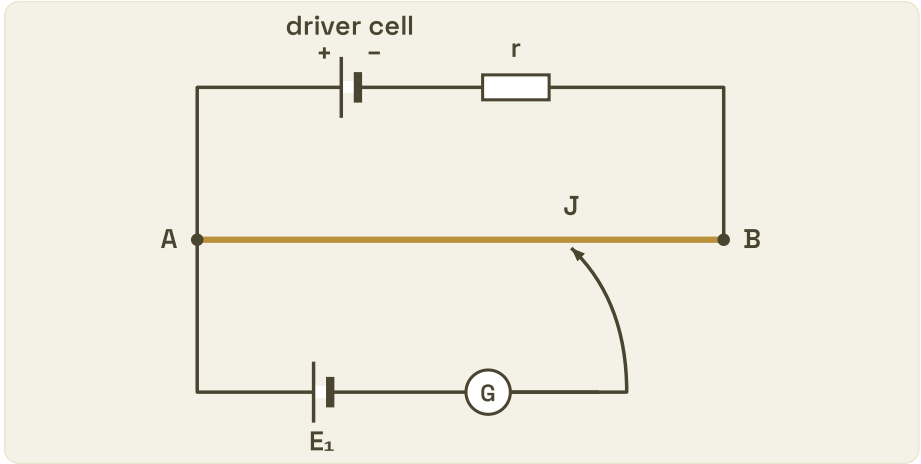


Figure 14.16 *For Test yourself 14.10. A driver cell and its internal resistance send a current through the uniform wire AB. A cell of e.m.f. E_1, with its + terminal at A, is connected through a galvanometer G to the jockey.*

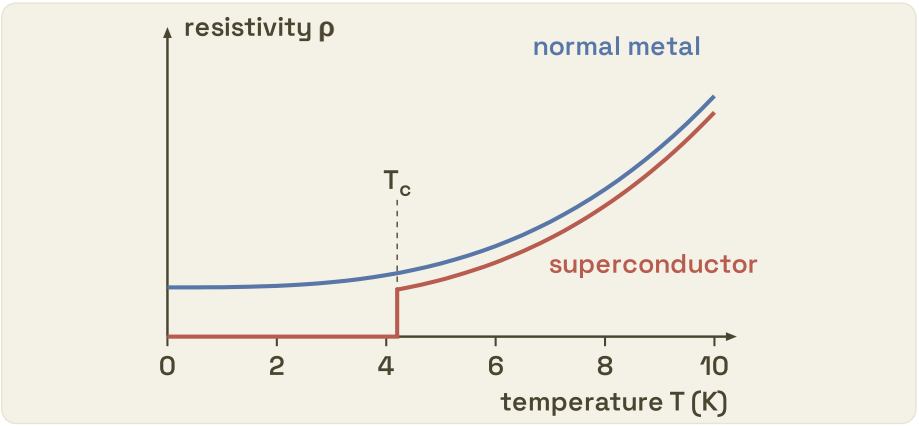


Figure 14.20 *Resistivity against temperature, near absolute zero. A normal metal's resistivity falls as it is cooled but levels off at a small value. A superconductor's falls to exactly zero at its critical temperature T_c.*

flowing for over two years without any measurable decrease. Research goes on to find a superconductor that works at room temperature; so far, none does.

DID YOU KNOW?

Warmer and warmer superconductors

In 1986 Georg Bednorz and Alex Müller found ceramics of copper oxide that superconduct at about 35 K, and within months others reached 93 K (yttrium barium copper oxide), above the 77 K at which nitrogen boils. Liquid nitrogen is cheap, about the price of milk, so these "high-temperature" superconductors can be studied in a school laboratory. The highest critical temperature at ordinary pressure, about 133 K (−140 °C), belongs to a mercury-based copper oxide, found in 1993. Claims of room-temperature superconductors, such as one called LK-99 in 2023, have not survived other laboratories' checks so far.

Applications of superconductors

1. Very **strong electromagnets**: a superconducting coil carries a huge current with no heating. The magnets in **MRI scanners** in hospitals, including those in Kathmandu, are coils of niobium–titanium wire kept at about 4 K in liquid helium.
2. **Research**: the giant magnets that steer particles round the Large Hadron Collider at CERN, and research in materials science and high-energy particle physics.
3. **Very fast electronic devices**: superconducting circuits switch very fast with almost no power, and are used in very sensitive magnetic sensors (SQUIDs) and in many of today's quantum computers.
4. **Transmission of electric power**: superconducting cables could carry electricity with no loss; short ones are already in trial use in a few cities, though the cooling is costly.
5. **Magnetic levitation (maglev) trains**: Japan's superconducting maglev reached a record 603 km/h in 2015.

Perfect conductor

A **perfect conductor** is an idealized material with **zero resistivity** (infinite conductivity). Inside it the electric field is zero, so the potential is the same everywhere in it and there is no potential difference between any two of its points; at its surface the tangential electric field is zero.

A superconductor has zero resistance, but it is **more than** a perfect conductor. A perfect conductor would keep whatever magnetic field was inside it when it lost its resistance (a changing field would drive currents that oppose any change). A superconductor actively **pushes the field out**, whatever was there before: the Meissner effect.

Meissner effect

The **Meissner effect** is the expulsion of a magnetic field from a material when it becomes superconducting.

YOUR TURN

Test yourself 14.14: a two-range ammeter

A galvanometer with a full-scale current of 10 mA and a resistance of 8 Ω is connected as in Figure 14.18. When the terminals A and B are used (C left open), the ammeter's range is I. What is its range when the terminals A and C are used (B left open)?

Answer

Terminals A and B: the shunt is the 2 Ω between A and B. The galvanometer's branch from A to B goes through G and the other 2 Ω: 8 + 2 = 10 Ω. The two branches have the same p.d.:

$$I_g \times 10 = (I - I_g) \times 2 \quad \Rightarrow \quad I = 6I_g = 60 \text{ mA}$$

Terminals A and C: the shunt is both resistors, 4 Ω, and the galvanometer's branch is G alone, 8 Ω:

$$I_g \times 8 = (I' - I_g) \times 4 \quad \Rightarrow \quad I' = 3I_g = 30 \text{ mA}$$

So the range with A and C is $I' = \mathbf{I/2}$ (30 mA, when I is 60 mA).

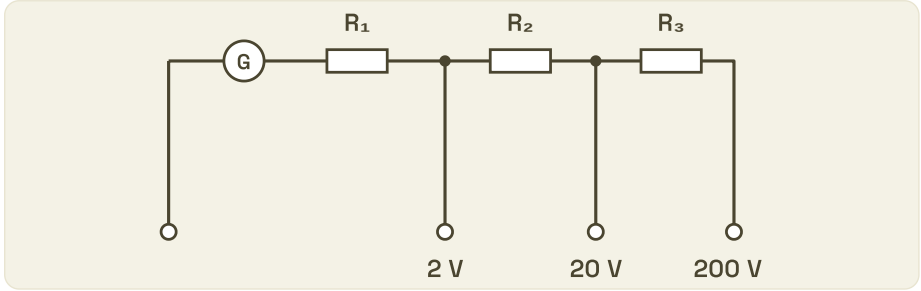


Figure 14.19 *For Test yourself 14.15. A galvanometer G in series with R_1, R_2 and R_3. One terminal is at the free end of G; the others are after R_1 (2 V), after R_2 (20 V) and after R_3 (200 V).*

YOUR TURN

Test yourself 14.15: a multi-range voltmeter

A multi-range voltmeter is made from a galvanometer as in Figure 14.19. It should measure up to 2 V, 20 V and 200 V, using a galvanometer of resistance 10 Ω that gives full-scale deflection for 1 mA. Find R_1, R_2 and R_3.

Answer

At each range the full-scale current, 1 mA = 0.001 A, must flow through all the resistance up to that terminal.

2 V: $G + R_1 = \dfrac{2}{0.001} = 2000\ \Omega$, so $R_1 = 2000 - 10 = \mathbf{1990\ \Omega}$.

20 V: $G + R_1 + R_2 = \dfrac{20}{0.001} = 20\,000\ \Omega$, so $R_2 = 20\,000 - 2000 = \mathbf{18\,000\ \Omega}$.

200 V: $G + R_1 + R_2 + R_3 = \dfrac{200}{0.001} = 200\,000\ \Omega$, so $R_3 = 200\,000 - 20\,000 = \mathbf{180\,000\ \Omega}$.

14.6 Superconductivity

Cool a metal and its resistance falls, because its vibrating ions get in the way of the electrons less. In 1911 the Dutch physicist **Heike Kamerlingh Onnes**, who three years earlier had been the first to liquefy helium (at 4.2 K, −269 °C), cooled mercury with it and saw something no one expected: at 4.2 K its resistance did not just fall, it **vanished**, dropping suddenly to zero.

Superconductivity is the property of certain metals, alloys and compounds of losing **all electrical resistance** below a certain temperature, the **critical temperature** (or transition temperature) T_c. Below T_c, a current can flow without any resistance, and so without losing any energy as heat.

The critical temperature is different for each material: **mercury 4.2 K**, **lead 7.2 K**, **niobium 9.2 K**; an alloy of plutonium, cobalt and gallium becomes superconducting below 18.5 K. Above T_c the material conducts like an ordinary metal; below it, it is a superconductor. In a famous experiment, a current set going in a ring of superconducting wire, with no battery at all, kept

YOUR TURN

Test yourself 14.10: reading the gradient

In Figure 14.16, AB is a resistance wire of uniform cross-section with a potential gradient of 0.01 V/cm along it.

i. If the galvanometer G shows zero deflection at 180 cm, what is the e.m.f. E_1 of the cell?

ii. If the internal resistance of the driver cell increases for some reason, how will the balance point change?

Answer

i. $E_1 = kl = 0.01 \text{ V/cm} \times 180 \text{ cm} = \mathbf{1.8\ V}$

ii. More internal resistance means less current in the wire, so a smaller potential gradient k. A longer length is then needed to balance the same e.m.f. ($l = E_1/k$): the balance point **moves away from A**, towards B. If the gradient falls far enough, it may move off the wire altogether, and no balance point can be found.

YOUR TURN

Test yourself 14.11: a 10 m potentiometer

The total length of a potentiometer wire is 10 m, and a steady current gives a potential gradient of 0.015 V/cm along it. Calculate:

i. the distance of the null point when a standard cell of 1.018 V is connected;

ii. the unknown p.d. if the null point is at 940 cm, and the largest p.d. this instrument can measure.

Answer

i. $l = \dfrac{E}{k} = \dfrac{1.018}{0.015} = \mathbf{67.9\ cm}$

ii. $V = kl = 0.015 \times 940 = \mathbf{14.1\ V}$.

The largest p.d. is the one across the whole wire: $0.015 \text{ V/cm} \times 1000 \text{ cm} = \mathbf{15\ V}$.

14.5 Conversion of galvanometer into ammeter and voltmeter

Galvanometer

A **galvanometer** is an instrument for detecting, and measuring, **small electric currents**. It works by the motor effect: a current through a coil in a magnetic field makes the coil turn (Unit 16). It may be a moving-magnet galvanometer or a moving-coil galvanometer; in the laboratory the usual one is the **Weston-type moving-coil galvanometer**, also called the pointer-type galvanometer, whose pointer's deflection is proportional to the current. A typical one gives full-scale deflection for a current I_g of a few milliamperes or less, and has a coil resistance G of a few ohms to a hundred ohms.

On its own a galvanometer can't carry a large current (its fine coil would overheat) or measure a large p.d. With a suitable resistor added, it becomes an **ammeter** or a **voltmeter**.

Shunt

A **shunt** is a low resistance connected **in parallel** with a galvanometer. It carries most of the current in the circuit, leaving only a small, definite part to flow through the galvanometer.

Uses of a shunt:

i. A moving-coil galvanometer is very sensitive and is damaged by a large current. A shunt protects it: only a small part of the total current passes through the galvanometer.

ii. A shunt converts a galvanometer into an **ammeter**.

iii. A shunt connected across any resistance in a circuit lowers the total resistance, and so **increases the current** in the circuit.

Conversion of a galvanometer into an ammeter

An ammeter measures the current in a circuit, so it is connected in series, and it must have a **low resistance** so as not to change that current. Suppose a galvanometer of resistance G gives full-scale deflection with a current I_g. To make it an ammeter reading 0 to I, a shunt S is connected in parallel with it (Figure 14.17a), chosen so that when a current I flows into the combination, only I_g flows through the galvanometer and the rest, $I - I_g$, through the shunt.

G and S are in parallel, so the p.d. across them is the same:

$$(I - I_g)S = I_g G$$

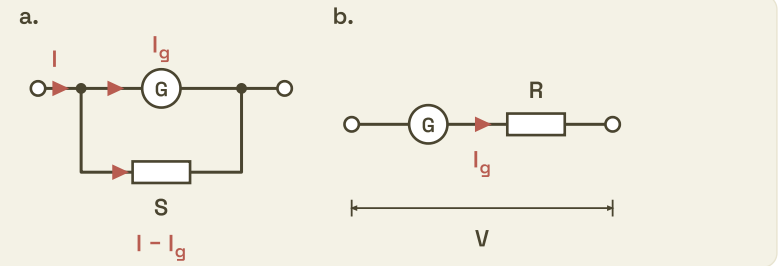


Figure 14.17 *a. A galvanometer G converted into an ammeter: a low resistance S, the shunt, in parallel with it. Of the current I, I_g flows through G and $I - I_g$ through S. b. A galvanometer converted into a voltmeter: a high resistance R in series with it. At the full-scale p.d. V, the current is I_g.*

Knowing I_g, I and G, S is calculated; with this shunt, the galvanometer is an ammeter of range 0 to I. The resistance of the whole ammeter, $\frac{GS}{G+S}$, is less than S: very small, as an ammeter's should be.

Conversion of a galvanometer into a voltmeter

A voltmeter measures the p.d. between two points, so it is connected in parallel with them, and it must have a **high resistance** so as to take as little current as possible. To make a galvanometer a voltmeter reading 0 to V, a **high resistance** R (sometimes called a multiplier) is connected **in series** with it (Figure 14.17b), chosen so that a p.d. V across the combination drives exactly the full-scale current I_g through it:

$$V = V_G + V_R = I_g G + I_g R = I_g(G + R)$$

$$I_g = \frac{V}{G+R} \quad \Rightarrow \quad \boxed{R = \frac{V}{I_g} - G}$$

Knowing V, I_g and G, R is found; with it in series, the galvanometer is a voltmeter of range 0 to V. Its resistance, G + R, is large.

Check 5

The resistance of an ideal ammeter and of an ideal voltmeter are, respectively:

a infinite and zero
b zero and infinite
c both zero
d both infinite

WORKED EXAMPLE

Example 14.10: one galvanometer, two meters

A galvanometer has an internal resistance of 1 Ω and gives full-scale deflection for a current of 50 mA. Show how it can be converted into i. a voltmeter of range 0 to 2.5 V; ii. an ammeter of range 0 to 2.5 A.

Answer

G = 1 Ω, I_g = 50 mA = 0.05 A.

i. **Voltmeter:** a resistance R in series,

$$R = \frac{V}{I_g} - G = \frac{2.5}{0.05} - 1 = 50 - 1 = \mathbf{49\ \Omega}$$

ii. **Ammeter:** a shunt S in parallel,

$$S = \frac{I_g G}{I - I_g} = \frac{0.05 \times 1}{2.5 - 0.05} = \frac{0.05}{2.45} = \mathbf{0.020\ \Omega}$$

So a 49 Ω resistor in series makes it a 2.5 V voltmeter, and a 0.020 Ω shunt in parallel makes it a 2.5 A ammeter.

YOUR TURN

Test yourself 14.12: a voltmeter and an ammeter

A galvanometer can carry a maximum current of 25 mA and has a resistance of 5 Ω. Find the resistances needed to convert it into i. a voltmeter of range 0 to 2 V; ii. an ammeter of range 0 to 10 A.

Answer

i. $R = \frac{V}{I_g} - G = \frac{2}{0.025} - 5 = 80 - 5 = \mathbf{75\ \Omega}$, in series.

ii. $S = \frac{I_g G}{I - I_g} = \frac{0.025 \times 5}{10 - 0.025} = \frac{0.125}{9.975} = \mathbf{0.0125\ \Omega}$, in parallel.

YOUR TURN

Test yourself 14.13: the shunt you have

The coil of a galvanometer has a resistance of 9.36 Ω and needs 0.0224 A for full-scale deflection. We want to make it an ammeter reading 20 A at full scale, but the only shunt available has a resistance of 0.025 Ω. What resistance must be connected in series with the coil?

Hint

The series resistor R and the coil together form one branch, in parallel with the shunt: $I_g(G + R) = (I - I_g)S$.

Answer

The coil branch (G + R) and the shunt have the same p.d.:

$$I_g(G + R) = (I - I_g)S$$

$$0.0224(9.36 + R) = (20 - 0.0224) \times 0.025 = 0.4994$$

$$9.36 + R = \frac{0.4994}{0.0224} = 22.30 \quad \Rightarrow \quad R = \mathbf{12.9\ \Omega}$$

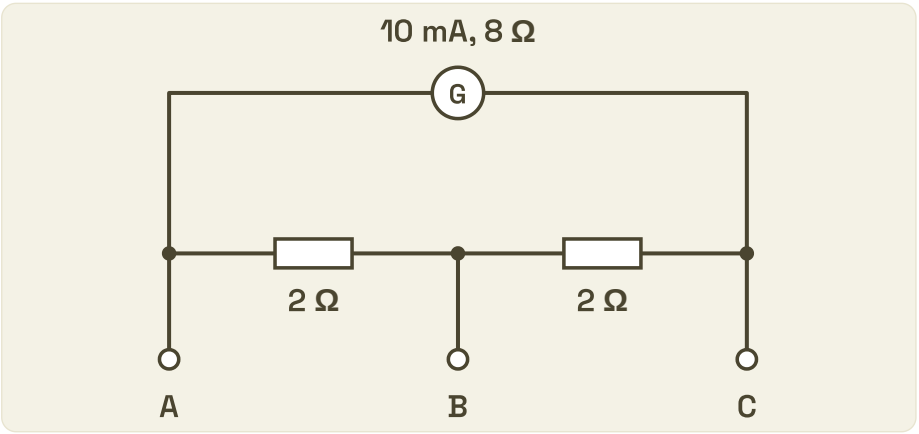


Figure 14.18 *For Test yourself 14.14. A galvanometer (full scale 10 mA, 8 Ω) is connected between the ends of two 2 Ω resistors in series. Terminal A is at the left end, B at the point between the two resistors, and C at the right end.*