Unit 13

Answers: Polarization

Quick checks

1 c Light, radio waves and X-rays are all electromagnetic waves, which are transverse, so all can be polarized. Sound in air is a longitudinal wave: the air vibrates along the direction of travel, so there is no direction across the ray to choose.

2 a Only the part of the light vibrating **along the surface** (perpendicular to the plane of incidence) is reflected at Brewster's angle. For a lake that is horizontal, which is why polarized sunglasses, with a vertical axis, block the glare.

3 c Polarization confines the vibrations of light to a single direction (one plane through the ray).

4 c A Polaroid sheet absorbs the vibrations along its aligned chains and passes those across them. Glass, water and diamond let light vibrating in every direction through.

5 b Plane-polarized light vibrates in one direction across the ray.

6 a By Malus's law, $I = I_0 \cos^2 90^\circ = 0$: crossed polarizers let no light through.

7 b At Brewster's angle the reflected light is completely plane-polarized, and the reflected and refracted rays are at 90°.

8 b $\theta_p = \tan^{-1} 1.49 = 56.1^\circ$, about 56°.

9 c At the polarizing angle, $\theta_p + r = 90^\circ$, so the reflected and refracted rays are at right angles.

10 d Polarized sunglasses cut glare; gas lasers use windows at Brewster's angle; and special **polarization-maintaining** optical fibers carry polarized light for sensors and some communication links. So all three use polarization, though sunglasses are the everyday example.

11 a Sound in air is longitudinal. X-rays, infrared (heat radiation) and light are all transverse electromagnetic waves.

12 b A substance that turns the plane of vibration of polarized light is **optically active**. (The angle turned is the optical rotation; the specific rotation is that angle for a standard length and concentration. Dichroism is absorbing one direction of vibration more than the other, as tourmaline and Polaroid do.)

13 a $\mu = \dfrac{1}{\sin C} = \dfrac{1}{\sin 45^\circ} = \sqrt{2}$, and by Brewster's law $\theta_p = \tan^{-1} \mu = \tan^{-1} \sqrt{2}$ (54.7°).

14 b An ideal polarizer passes half of unpolarized light (the average of $\cos^2 \theta$ is $\frac{1}{2}$), so the other 50 % is stopped.

13. a. **Polarization** of light is the restricting of its vibrations (of its electric field) to a single direction, at right angles to its direction of travel.
b.

	Unpolarized light	Polarized light
Vibrations	in all directions across the ray, at random	in one direction only
Turning an analyzer in front of it	brightness doesn't change	brightness changes, to zero when crossed
Example	sunlight, a candle, a bulb	light through a Polaroid; an LCD screen's light; glare reflected at Brewster's angle

c. At Brewster's angle $\theta_p + r = 90^\circ$, so $\theta_p = 90^\circ - 33.7^\circ = \mathbf{56.3°}$.
By Brewster's law, $\mu = \tan\theta_p = \tan 56.3^\circ = \mathbf{1.50}$.

14. a. Light whose vibrations are in **one direction only**, across the direction of travel (so they all lie in one plane through the ray).

b. As in section 13.5 and Figure 13.6. Unpolarized light falling on a transparent surface has two equal parts: vibrations perpendicular to the plane of incidence (along the surface) and vibrations in the plane of incidence. The surface reflects the first part more strongly than the second at most angles, so reflected light is partially polarized. At the polarizing angle θ_p, where the reflected and refracted rays are at 90°, the part vibrating in the plane of incidence isn't reflected at all, so the reflected light is **completely polarized**, vibrating parallel to the surface, and $\mu = \tan\theta_p$.

c. The reflected and refracted rays are at 90°, so 56° is the polarizing angle.

i. $\mu = \tan 56^\circ = \mathbf{1.48}$.

ii. $v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{1.483} = \mathbf{2.02 \times 10^8\ m/s}$ (2.03×10^8 m/s if μ is rounded to 1.48 first).

15. a. **Polaroids** are thin plastic sheets that make plane-polarized light: a stretched film of polyvinyl alcohol whose long chain molecules are lined up and made conducting with iodine. They absorb the light vibrating along the chains and pass the light vibrating across them.

b. Any two: polarized sunglasses (cut glare), camera filters (remove reflections), 3D film glasses, LCD screens, polarizing microscopes, stress analysis of plastic models.

c. i. The intensity falls to **half** (a little less in a real sheet): the first Polaroid passes only the part of each vibration along its axis, and the average of $\cos^2\theta$ over all directions is $\frac{1}{2}$. The light that comes out is plane-polarized along its axis.

ii. **No light** gets through: the light from the first sheet vibrates along its axis, which is at 90° to the second sheet's axis, and $I = I_0 \cos^2 90^\circ = 0$.

iii. The light gets **brighter gradually**, following Malus's law $I = I_0 \cos^2\theta$ (where θ is the angle between the axes), from zero when crossed to the full I_0, half the original, when parallel. Unpolarized light through a single turning sheet doesn't change in brightness, so the change shows that the light leaving the first sheet has a **direction of vibration** across the ray: it is polarized, and light is a transverse wave.

16. a. **Brewster's law:** the refractive index of a medium equals the tangent of its polarizing angle, $\mu = \tan\theta_p$. Derivation as in section 13.5: by Snell's law $\mu = \sin\theta_p / \sin r$; at the polarizing angle the reflected and refracted rays are at 90°, so $\theta_p + r = 90^\circ$ and $\sin r = \cos\theta_p$; hence $\mu = \sin\theta_p / \cos\theta_p = \tan\theta_p$.

b. Diagram as Figure 13.6. The reflected light is sent out by the electrons in the medium, which the refracted light sets vibrating. The part of the refracted light vibrating **in the plane of incidence** makes electrons vibrate at right angles to the refracted ray, which at Brewster's angle is **along the reflected ray**. An oscillating electron sends out no light along its own direction of vibration, so that part isn't reflected. Only the part vibrating **perpendicular to the plane of incidence** (along the surface) is reflected, so the reflected light vibrates in one direction only: completely polarized.

c. Any two: polarized sunglasses cutting glare from water and roads; camera polarizing filters; LCD screens; 3D films; Brewster windows in lasers; stress analysis.

17.

a. **Brewster's angle** (the polarizing angle) is the angle of incidence at which light reflected from a transparent surface is completely plane-polarized. Its significance: at this angle the reflected and refracted rays are at 90°, and $\mu = \tan\theta_p$, so it gives a way to make polarized light by reflection, to cut glare, to build loss-free laser windows, and to measure refractive index.

b. In the figure, the reflected ray is at 90° to the refracted ray. The refracted light's vibrations in the plane of incidence would make the medium's electrons send light out along their own direction of vibration, which is the reflected ray's direction, and an oscillating electron sends no light that way. So only the vibrations perpendicular to the plane of incidence (the dots) are reflected, and the reflected light is completely polarized.

c. $\theta_p = \tan^{-1} 1.5 = \mathbf{56.3°}$.

d. i. $\theta_p = \tan^{-1} 1.66 = \mathbf{58.9°}$.

ii. **90°**.

iii. $v = \dfrac{c}{\mu} = \dfrac{3 \times 10^8}{1.66} = \mathbf{1.81 \times 10^8\ m/s}$.

18.

a. Let the refractive index of the denser medium relative to the rarer one be μ. Going from the denser into the rarer medium, the refractive index of the second relative to the first is $1/\mu$, and i is the polarizing angle, so by Brewster's law

$$\tan i = \frac{1}{\mu}$$

(Check: $r = i$, and $r + 90^\circ + r' = 180^\circ$ gives $i + r' = 90^\circ$, so Snell's law $\mu \sin i = \sin r' = \cos i$ gives the same.) The critical angle C of the denser medium has $\sin C = \dfrac{1}{\mu}$. So

$$\sin C = \tan i, \qquad C = \sin^{-1}(\tan i)$$

For glass of $\mu = 1.5$, $i = 33.7^\circ$ and $C = \sin^{-1} 0.667 = 41.8^\circ$.

b. **Plane-polarized** light vibrates in one direction only across the ray; **unpolarized** light vibrates in all directions across the ray at random. A turning analyzer changes the brightness of plane-polarized light (down to zero when crossed) but not of unpolarized light.

c. Light is an electromagnetic wave whose electric field vibrates across the ray. The reflected light is sent out by the electrons of the medium, set vibrating by the electric field of the light. Split the incident light into the part vibrating perpendicular to the plane of incidence and the part vibrating in it. The second part makes the electrons vibrate at right angles to the refracted ray, which, since the refracted and reflected rays are at 90°, is **along** the reflected ray. Vibrating charges send out no electromagnetic waves along their own line of vibration, so this part isn't reflected. The reflected light is only the part vibrating perpendicular to the plane of incidence: completely plane-polarized.

19.

a. **Polarization** is the restricting of light's vibrations to one direction across the ray. It matters in wave optics because only transverse waves can be polarized: it settles that light is a **transverse** wave (interference and diffraction alone show only that it is a wave). It also explains how double refraction and optical activity work, and gives tools such as polarizing filters and microscopes.

b. **Brewster's law:** $\mu = \tan \theta_p$, where θ_p is the angle of incidence at which the reflected light is completely polarized. At that angle the reflected and refracted rays are at 90°; Snell's law with $r = 90^\circ - \theta_p$ gives $\mu = \sin \theta_p / \cos \theta_p$.

c. Two tourmaline crystals or two Polaroids. Look at a lamp through both. Keep the first fixed and turn the second about the line of sight. The light is brightest when their axes are parallel and disappears when they are crossed at 90°, then returns. If light were longitudinal, turning the second filter about the ray could make no difference; the change shows the vibrations are across the ray, so light is transverse.

d. **Polaroids** are thin plastic sheets of stretched polyvinyl alcohol with iodine, whose long, lined-up, conducting chain molecules absorb the part of the light vibrating along them. Unpolarized light passing through a Polaroid comes out **plane-polarized**, vibrating across the chains (along the transmission axis), at a little under half its intensity.

e. $\theta_p = \tan^{-1} 1.5 = \mathbf{56.3°}$.