Unit 12

Diffraction

This unit online, with its interactives:
science-playground.com/s/12f12

Stand outside a classroom door and you can hear the teacher, though you can't see her. Look at a street lamp through a fine umbrella cloth at night and it turns into a cross of colored spots. Tilt a CD under a tube light and rainbows slide across it. All three are the same thing: a wave spreading out as it passes an edge or squeezes through a gap. That spreading is **diffraction**, and for light it is so small that it went unnoticed until the 1600s.

This unit looks at diffraction through one narrow slit, finds exactly where its dark and bright bands fall, then lines up thousands of slits in a **diffraction grating**, the tool that splits starlight into its colors. Finally it asks a practical question: why can no telescope or microscope, however perfect its lenses, show details smaller than a certain size?

A STORY FROM SCIENCE

The Jesuit who saw light bend

Francesco Maria Grimaldi (1618–1663) was an Italian Jesuit priest who taught mathematics and physics in Bologna. He let sunlight into a darkened room through a tiny hole and put a thin rod in the narrow cone of light. The rod's shadow on the screen was **wider** than straight-line rays allowed, and its edges weren't sharp: they were fringed with colored bands. Light passing a second small hole spread into a wider patch than it should.

Grimaldi concluded that light doesn't only travel in straight lines, reflect and refract: it can also bend slightly round edges. He called this new behavior *diffractio*, a "breaking apart", the word we still use. His book on light was published in 1665, two years after he died. Few people paid attention: Newton's idea of light as a stream of particles carried the day for more than a century. It took Young and Fresnel, around 1800–1820, to show that diffraction is what waves do.



Figure 12.1 *Francesco Maria Grimaldi (1618–1663), who named diffraction.*
Photo: Wikimedia Commons, Public domain

12.1 Introduction

Diffraction is the bending and spreading of a wave when it passes through a narrow opening or around the edge of an obstacle, into the region a straight-line ray would leave dark (the **geometrical shadow**). Every kind of wave does it: water waves round a harbor wall, sound round a doorway, light round the edge of a razor blade.

How much a wave spreads depends on the size of the gap or obstacle, d, compared with the wavelength, λ:

- If d is **much larger** than λ, the wave goes straight through, and hardly spreads at all. The shadow looks sharp.
- If d is **comparable** to λ (about the same, or a few hundred times larger for light), the spreading is easy to see.

For light, the bending and spreading produces a pattern of **alternating bright and dark bands**, which can only be explained if light is a wave. Diffraction is strong evidence for the wave nature of light.

Think again about three things you may have noticed:

1. You can hear someone talking round a corner, though you can't see them.
2. The shadow of a very thin object, such as a hair or a thread in a laser beam, has soft, fringed edges.

3. Light from a small bright source seen through a narrow gap, such as the gap between two fingers held almost touching, shows thin dark lines.

They are not tricks of the eye or ear. They are diffraction. A voice's waves are about a meter long, the same size as a doorway, so they spread widely round it. Light's waves are about two million times shorter (around 0.5 μm), so light spreads noticeably only through gaps smaller than about a millimeter.



Figure 12.2 *The single-slit pattern of a blue laser: the central band is much the brightest, with fainter bands on either side.*
Photo: Xyqorophibian, CC BY-SA 4.0

Check 1
Light passes through a gap in a wall, and sound passes through the same gap. Which spreads out more on the other side?

a light, because it travels faster
b sound, because its wavelength is much closer to the size of the gap
c both spread the same, because both are waves
d neither: only water waves spread out after a gap

12.2 Types of diffraction

Diffraction is sorted into two types, by **where the source and the screen are** compared with the slit or obstacle (Figure 12.3).

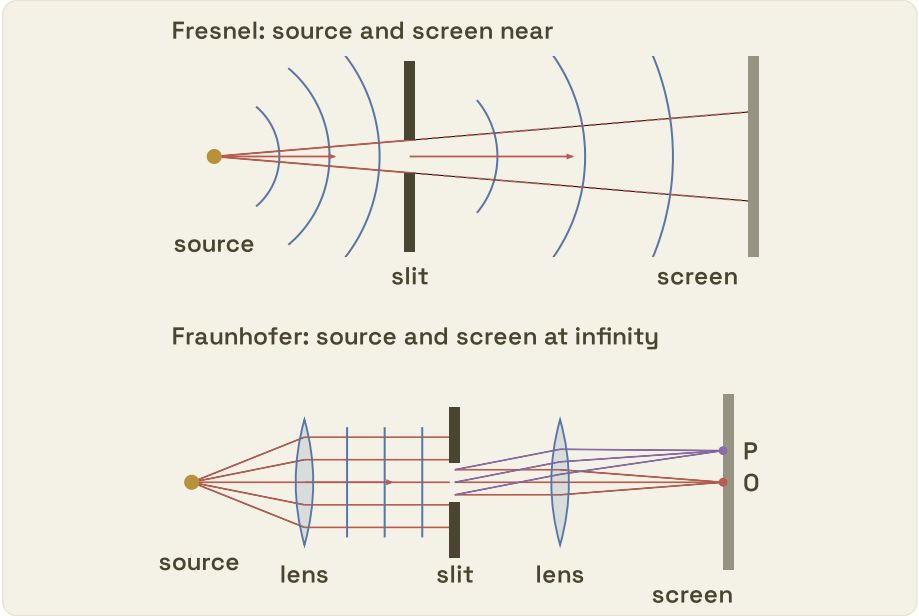


Figure 12.3 *Top: Fresnel diffraction. The source and the screen are close to the slit, so the wavefronts reaching it are curved. Bottom: Fraunhofer diffraction. The source is effectively at infinity (a lens makes the light parallel), so plane wavefronts reach the slit, and a second lens brings each direction to one point on the screen.*

1. Fresnel diffraction (near-field diffraction). The source of light, or the screen, or both, are at a **finite distance** from the slit or obstacle, close enough that the wavefronts reaching it are **curved** (spherical or cylindrical). No lenses are used. The pattern depends on the object's shape and on exactly where the source and the screen are, and working it out needs more advanced mathematics. The colored fringes at the edge of a shadow are Fresnel diffraction.

2. Fraunhofer diffraction (far-field diffraction). The source and the screen are effectively **at infinity** from the slit: the light reaching the slit is a **plane wavefront** (parallel rays), and the light leaving it in each direction is collected as a parallel beam. In a laboratory, two convex lenses do this: the first makes the light from the source parallel, the second brings each parallel

beam to a point on a screen in its focal plane. A laser beam is already nearly parallel, so a laser and a screen a few meters away give a good Fraunhofer pattern without lenses. The pattern is simple (a bright central band with alternating dark and bright bands) and easy to work out, so it is the one used in experiments and in this unit.

	Fresnel diffraction	Fraunhofer diffraction
Source and screen	at a finite distance from the slit	effectively at infinity
Wavefront at the slit	spherical or cylindrical	plane
Lenses	not needed	needed (or a distant screen with a laser)
Pattern	depends on distances; complicated	fixed shape, only its size changes with distance
Mathematics	harder	simpler

DID YOU KNOW?

Fresnel and Fraunhofer

Augustin-Jean Fresnel (1788–1827) was a French road and bridge engineer who did optics in his spare time; he also invented the stepped **Fresnel lens** of lighthouses, thin enough to make without cracking. **Joseph von Fraunhofer** (1787–1826) was a German glassmaker's apprentice, orphaned at eleven and nearly killed when the workshop he lived in collapsed. He became the finest lens-maker in Europe, made the first good diffraction gratings, and used one to measure the wavelengths of the dark lines in sunlight, still called **Fraunhofer lines**.

12.3 Fraunhofer diffraction at a single slit

The experiment

Monochromatic light (one wavelength, λ) is made parallel by a lens and falls normally on a narrow slit AB of width d (Figure 12.4). All points of the plane wavefront in the slit are in phase. By **Huygens' principle**, every point across the slit sends out secondary wavelets in all directions. A second convex lens collects the light leaving the slit and focuses it on a screen in its focal plane: all the wavelets that leave the slit in one direction, at an angle θ to the normal, are brought together at one point P on the screen.

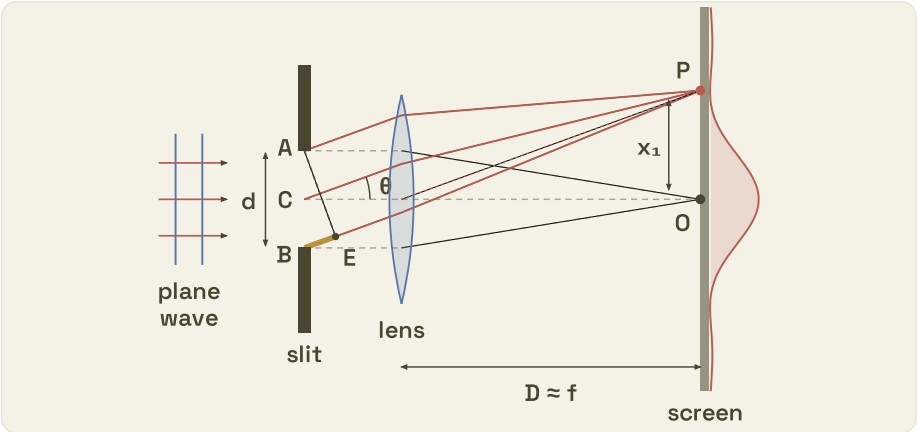


Figure 12.4 *Fraunhofer diffraction at a single slit AB of width d (drawn very much wider than it is). Light leaving at an angle θ is focused at P. AE is drawn at right angles to the rays, so the ray from B travels an extra distance BE = d sin θ. The light going straight on is focused at O, the center of the pattern.*

What happens at P depends on the **path difference** between the wavelets from different parts of the slit.

At O, straight ahead ($\theta = 0$), every wavelet travels the same optical path (a lens adds no path difference between rays it focuses to one point). They arrive in phase and add up: O is bright. This is the **central maximum**.

At P, at an angle θ, draw AE perpendicular to the ray from B. From AE onwards all the rays travel equal paths to P, so the wavelet from B travels farther than the wavelet from A by **BE**. In the right-angled triangle ABE, the angle at A is θ, so

$$\text{path difference between the edges} = BE = d\sin\theta \qquad (12.1)$$

Condition for secondary minima

1. Path difference BE = λ. Imagine the slit divided into two equal halves, AC and CB, each $d/2$ wide (Figure 12.5). Take a point in the upper half and the point $d/2$ below it in the lower half: the top edge A and the middle C, a point just below A and a point just below C, and so on down to C and B. Each such pair is $d/2$ apart, so its path difference is half of BE:

$$\frac{d}{2}\sin\theta = \frac{\lambda}{2}$$

Every pair arrives half a wavelength out of step and cancels. The two halves wipe each other out completely, and P is **dark**. So the first dark band is where

$$d\sin\theta = \lambda \qquad (12.2)$$

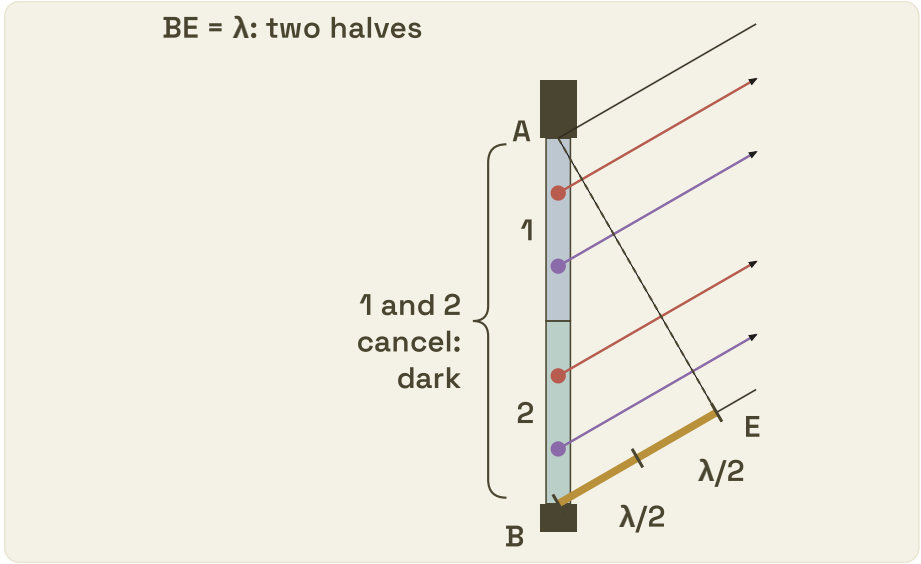


Figure 12.5 *The first minimum: when the edges differ by one wavelength, the slit is two halves whose matching points (the same color) are d/2 apart and differ by λ/2. Each pair cancels, and P is dark.*

2. Path difference BE = 2λ. Now divide the slit into **four** equal parts, each $d/4$ wide. Matching points in the first and second parts are $d/4$ apart, with a path difference $(d/4)\sin\theta = \lambda/2$: they cancel. The third and fourth parts cancel the same way. P is dark again:

$$d\sin\theta = 2\lambda \qquad (12.3)$$

In general, when the path difference is $n\lambda$, the slit can be split into $2n$ equal parts that cancel in pairs, and P is dark. The nth dark band, or **nth secondary minimum**, is at the angle θ_n where

$$\boxed{d\sin\theta_n = n\lambda \quad \Rightarrow \quad \sin\theta_n = \frac{n\lambda}{d}, \qquad n = 1, 2, 3, \ldots} \qquad (12.4)$$

$n = 1$ gives the first minimum, $n = 2$ the second, and so on, on both sides of O. (Not $n = 0$: that is O itself, the brightest point of all.)

> CAREFUL
> **The minima rule looks like Young's bright fringes**
>
> For a **single slit**, $d\sin\theta = n\lambda$ gives the **dark** bands. In **Young's double-slit** experiment (Unit 11), the same-looking formula, with d the distance between the two slits, gives the **bright** fringes. The difference: in Young's experiment there are only two sources, and a path difference of λ brings them in step; in one slit, a path difference of λ between the edges means the two halves cancel. In an exam, say which experiment, and what d is.

22.
   a. Explain the pattern observed on the screen in single-slit diffraction, with a diagram.
   b. Derive the condition for the first minimum in Fraunhofer diffraction.
   c. Monochromatic light of wavelength 650 nm falls on a single slit of width 0.15 mm. The diffraction pattern is observed on a screen 2 m from the slit. Calculate the angle at which the second diffraction minimum occurs.

23. A single-slit diffraction pattern is formed on a screen at a distance D from a slit of width a, as in Figure 12.4.
   a. Write the angular condition for minima in single-slit diffraction.
   b. Derive the expression for the linear position y of the first minimum.
   c. If $a = 0.2$ mm, $D = 2$ m and $\lambda = 600$ nm, calculate the distance y of the first minimum from the central maximum.

24. Diffraction gratings disperse light into its spectral components, using the interference of waves diffracted by many closely spaced slits.
   a. How does a diffraction grating disperse light into its components?
   b. Derive the grating equation used to calculate diffraction angles.
   c. A diffraction grating has 5000 lines per cm. Calculate the angle of the second-order maximum for light of wavelength 500 nm.

25. Diffraction is broadly classified into two types, Fresnel and Fraunhofer, each occurring under specific conditions and producing distinct patterns.
   a. State the conditions for Fresnel and Fraunhofer diffraction.
   b. Discuss the differences in the fringe patterns formed.
   c. In a Fraunhofer diffraction experiment with a single slit, light of wavelength 450 nm produces a central maximum 2 cm wide on a screen 1.5 m from the slit. Calculate the slit width.

26. The width of a slit plays a crucial role in shaping the diffraction pattern, influencing how light spreads and forms maxima and minima on a screen.
   a. How does changing the slit width affect the diffraction pattern?
   b. Derive the relation between the slit width, the wavelength and the width of the central maximum.
   c. Light of wavelength 700 nm passes through a slit of width 0.2 mm. Determine the angular width of the central maximum.

27. Fraunhofer diffraction through a single slit shows the wave nature of light, making a pattern of bright and dark fringes when light passes through a narrow aperture and is observed at a distance or with a lens.
   a. Describe the conditions needed to observe Fraunhofer diffraction through a single slit.
   b. Why is the central maximum wider and brighter than the others?
   c. Derive the condition for the angular positions of the minima.
   d. Monochromatic light of wavelength 600 nm falls on a single slit of width 0.15 mm. Calculate the angular position of the first diffraction minimum.

28. In diffraction grating experiments, light is diffracted through thousands of closely spaced slits, producing sharp maxima at certain angles. These patterns are used to calculate wavelengths, analyze spectra and resolve fine details.
   a. Define "grating element" and its relation to line density.
   b. Why does increasing the number of lines improve a grating's resolution?
   c. Light of wavelength 520 nm falls normally on a grating with 6000 lines/cm.
      i. Find the grating element.
      ii. Calculate the maximum order of diffraction that can be observed with this light.
      iii. Find the angle of diffraction of the second-order maximum.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

16. Diffraction gratings disperse light into its components, helping us analyze wavelengths.
    a. What is a diffraction grating? Mention two practical applications.
    b. State and explain the grating equation for constructive interference.
    c. A diffraction grating has 6000 lines/cm. Find the angle of the second-order maximum for light of wavelength 500 nm.

17. Diffraction limits the resolving power of optical instruments like microscopes and telescopes.
    a. Define resolving power and explain its relation to diffraction.
    b. State Rayleigh's criterion and its relevance to angular resolution.
    c. Calculate the smallest angular separation between two objects that a telescope with an aperture of 0.2 m can resolve, using light of wavelength 550 nm.

18. The effect of diffraction depends on the relative sizes of the aperture and the wavelength.
    a. Why is diffraction insignificant for everyday objects?
    b. How does diffraction affect the performance of optical instruments?
    c. A slit of width 0.05 mm is lit by monochromatic light of wavelength 650 nm. Calculate the angular position of the second diffraction minimum.

19. The behavior of light passing through narrow openings gives insight into the visibility of fringes and diffraction patterns.
    a. What is the effect on the diffraction pattern if the slit width decreases?
    b. Compare the intensity and width of the central maximum in single-slit diffraction with those of the central bright fringe in double-slit interference.
    c. A laser beam of wavelength 650 nm passes through a slit of width 0.1 mm. Calculate the width of the central maximum on a screen 1.5 m away.

20. The single-slit diffraction pattern is determined by the angular spread and the size of the aperture.
    a. Define angular width in single-slit diffraction.
    b. Explain how diffraction influences the resolution of optical instruments.
    c. Monochromatic light of wavelength 550 nm falls normally on a grating having 4000 lines/cm. Find the angle of the first-order maximum.
    d. How does Huygens' principle explain the diffraction of light?

21. The grating element is key to understanding diffraction orders.
    a. What is the grating element? How is it related to the number of lines per cm?
    b. Derive the expression for the angular positions of the maxima in grating diffraction.
    c. Calculate the number of diffraction orders observable with light of wavelength 500 nm and a grating with 5000 lines/cm.
    d. Explain the relationship between the angular width of the central maximum and the wavelength of light in single-slit diffraction.

CAREFUL

"4 orders" or 3?

For question 21 c, some books divide d by λ, get exactly 4, and stop. When d/λ is a whole number, the last order would leave at exactly 90°, along the grating, and can't be seen. Say so: the highest **observable** order is 3. If d/λ isn't a whole number (say 3.2), the highest order is the whole number below it (3).

Condition for secondary maxima

1. Path difference BE = 3λ/2. Divide the slit into **three** equal parts (Figure 12.6). Matching points in the first two parts are $d/3$ apart, with path difference $(d/3)\sin\theta = \lambda/2$: those two parts cancel. The **third part is left over** with nothing to cancel it, and its wavelets make P bright, though much less bright than O, where the whole slit adds up:

$$d\sin\theta = \frac{3\lambda}{2} \qquad (12.5)$$

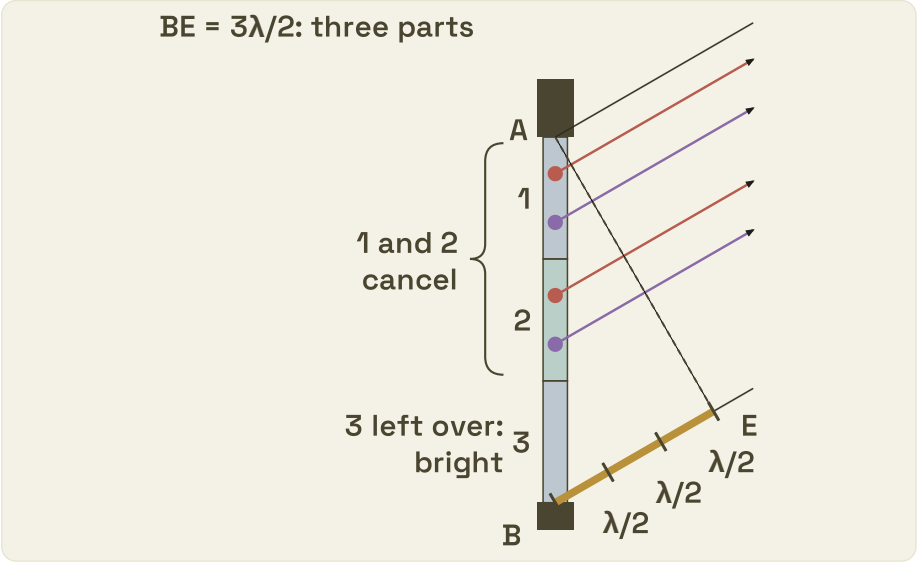


Figure 12.6 *The first secondary maximum: when the edges differ by 3λ/2, the slit is three parts. Two of them cancel in pairs; the third is left over, and its light makes P bright.*

2. Path difference BE = 5λ/2. Divide the slit into **five** parts. The first two cancel each other, and so do the next two; the fifth is left over and makes P bright, more faintly still:

$$d\sin\theta = \frac{5\lambda}{2} \qquad (12.6)$$

In general, when the path difference is an odd multiple of $\lambda/2$, the slit splits into $2n+1$ parts; all but one cancel in pairs, and P is bright. The n**th secondary maximum** is at

$$\boxed{d\sin\theta_n = (2n+1)\frac{\lambda}{2} \quad\Rightarrow\quad \sin\theta_n = \frac{(2n+1)\lambda}{2d}, \qquad n = 1, 2, 3, \ldots} \qquad (12.7)$$

$n = 1$ gives the first secondary maximum ($3\lambda/2d$), $n = 2$ the second ($5\lambda/2d$), and so on.

DID YOU KNOW?

Halfway, nearly

The rule "one part left over" is a good picture, and it gives the positions within a few percent. The exact positions, worked out with calculus, are a little nearer the center: the first secondary maximum is at $\sin\theta = 1.43\lambda/d$, not $1.5\lambda/d$, and the second at $2.46\lambda/d$, not $2.5\lambda/d$. The minima, on the other hand, are exactly at $\lambda/d, 2\lambda/d, \ldots$. For exams, use equation (12.7).

Intensity distribution

Figure 12.7 shows how the brightness (intensity) of the pattern changes with the angle. The central peak is the **central maximum**; on either side come the **first-order**, **second-order** … secondary maxima, each fainter than the one before.

Why do the side bands fade so fast? The central maximum gets light from **the whole slit**, all adding up. The first secondary maximum gets light from only the **third left over** (and even within that third the wavelets are a little out of step); the second from only a fifth; and so on. So:

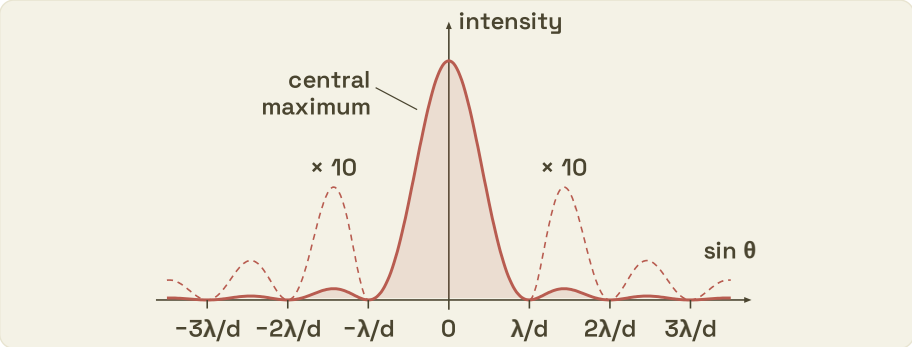


Figure 12.7 *The intensity of a single-slit pattern against sin θ, worked out exactly. The dark bands are at sin θ = ±λ/d, ±2λ/d, ±3λ/d. The central maximum is twice as wide as the others. The side peaks are so faint that the dashed curve shows them ten times taller: the first is only 4.7 % as bright as the center, the second 1.6 %.*

- the **central maximum** is the brightest and **twice as wide** as any other, since it stretches from the first minimum on one side to the first minimum on the other;
- the secondary maxima are about **equally wide**, and get rapidly **fainter** away from the center.

Check 2

In a single-slit pattern, the first dark band on each side is where the path difference between the two edges of the slit is:

a λ/2 b λ
c 3λ/2 d 2λ

Width of the central maximum

The **width of the central maximum** is the distance on the screen between the first minimum on one side of O and the first minimum on the other.

Let the first minimum be at P, a distance x_1 from O, at an angle θ, with the screen a distance D from the slit (Figure 12.4; with the second lens close to the slit and the screen in its focal plane, $D \approx f$).

1. From the triangle formed by the slit's center, O and P: $\tan\theta = \dfrac{OP}{D} = \dfrac{x_1}{D}$.
2. For the first minimum, from (12.2): $\sin\theta = \dfrac{\lambda}{d}$.
3. The angle is small (for light and a slit a fraction of a millimeter wide, θ is well under 1°), so $\sin\theta \approx \tan\theta \approx \theta$ in radians. Then

$$\frac{x_1}{D} = \frac{\lambda}{d} \quad \Rightarrow \quad x_1 = \frac{\lambda D}{d} \qquad (12.8)$$

4. With a lens close to the slit and the screen in its focal plane, $D \approx f$, the focal length:

$$x_1 = \frac{\lambda f}{d} \qquad (12.9)$$

5. The central maximum stretches from $-x_1$ to $+x_1$:

$$\boxed{\text{width of central maximum} = 2x_1 = \frac{2\lambda D}{d}} \qquad (12.10)$$

Its **angular width** (the angle it spans, seen from the slit) is $2\theta = \dfrac{2\lambda}{d}$ radians, and the **angular half-width**, $\theta = \lambda/d$.

Points to remember

1. Width $\propto \dfrac{1}{d}$ (with λ and D fixed): a **narrower slit** gives a **wider** central maximum. Halve the slit, and the central band doubles.
2. Width $\propto \lambda$ (with d and D fixed): **red light** (long waves) gives a wider central maximum than **blue** or **violet** (short waves).
3. Width $\propto D$ (with λ and d fixed): moving the screen farther away makes the pattern larger, though the **angular** width stays the same.
4. With **white light**, the center O is bright for every color at once, so the central maximum is **white**; elsewhere each color's bands fall at different places, so the side bands are **colored**, violet on the inside, red on the outside.

Check 11

7. Which is the best reason why light seems to travel in straight lines?

a Light consists of tiny particles. b The wavelength of light is very small.
c The frequency of light is very small. d The speed of light changes from one medium to another.

Check 12

8. Which is the correct expression for the resolving power of a microscope?

a $R = \dfrac{\lambda}{2\mu\sin\theta}$ b $R = \dfrac{2\mu\sin\theta}{\lambda}$
c $R = \dfrac{\lambda}{2\sin\theta}$ d $R = \dfrac{2\sin\theta}{\lambda}$

Check 13

9. Which of the following increases the resolving power of a microscope?

a increasing the wavelength of light b decreasing the aperture size
c using a lens with a larger numerical aperture d using a lens with a smaller focal length

Check 14

10. Which change in the design of a telescope increases its resolving power?

a decreasing the wavelength of light b increasing the size of the objective lens
c increasing the focal length of the telescope d decreasing the distance between the lens and the object

Check 15

11. What determines the diffraction limit of resolution in an optical instrument?

a the aperture size and the wavelength of light b the lens curvature and the aperture size
c the distance between the object and the lens d the refractive index of the medium

Check 16

12. What happens to the resolving power of a microscope if the objective lens has a larger numerical aperture?

a The resolving power decreases. b The resolving power increases.
c The image becomes blurred. d The image becomes dimmer.

Answer these questions

13. Diffraction is a key wave phenomenon: light bends round obstacles or through small openings, making characteristic patterns on a screen.
    a. Define diffraction of light. Under what conditions is diffraction significant?
    b. Differentiate interference from diffraction, based on their sources and on how the fringes form.
    c. Light of wavelength 500 nm passes through a single slit of width 0.1 mm. Calculate the angular width of the central maximum, and its width on a screen 2 m away.
14. When light meets obstacles or narrow openings, it bends and spreads: diffraction. Under what conditions does this bending become prominent, and how do the positions of the source and the screen decide which type of diffraction pattern is seen?
15. The bending of light becomes more prominent in narrow apertures, and is classified into distinct types.
    a. Compare Fresnel and Fraunhofer diffraction, with diagrams or descriptions of the setup.
    b. Derive the condition for minima in single-slit diffraction using geometry.
    c. A single slit of width 0.2 mm is lit by light of wavelength 600 nm. Calculate the angular position of the first minimum.
    d. Why is light partially polarized when it reflects from a non-metallic surface?

IN SHORT

In short

- **Diffraction** is the bending and spreading of a wave at an edge or through a gap. It is noticeable when the gap is comparable to the wavelength, which for light means a fraction of a millimeter.
- **Fresnel** diffraction: source or screen at a finite distance, curved wavefronts. **Fraunhofer**: both effectively at infinity (lenses), plane wavefronts.
- Single slit of width d: path difference between the edges $d\sin\theta$. **Minima** at $d\sin\theta_n = n\lambda$; **secondary maxima** at about $d\sin\theta_n = (2n+1)\lambda/2$, $n = 1, 2, 3\ldots$
- The central maximum is the brightest and twice as wide as the others: width $2\lambda D/d$, angular width $2\lambda/d$. Narrower slit or longer wavelength: wider pattern.
- **Grating**, grating element $d = a + b = 1/N$: bright lines at $(a+b)\sin\theta_n = n\lambda$, $n = 0, 1, 2\ldots$; highest order $n < d/\lambda$. Red is turned most.
- **Rayleigh's criterion:** just resolved when the central maximum of one image falls on the first minimum of the other.
- Microscope: $d = \lambda/2\mu\sin\theta$, resolving power $2\mu\sin\theta/\lambda$ (numerical aperture $\mu\sin\theta$). Telescope: $d\theta = 1.22\lambda/D$, resolving power $D/1.22\lambda$.

Exercise

Choose the correct option

Check 5

1. What is the angular position of the first diffraction minimum in a single-slit diffraction pattern, if the wavelength of light is λ and the slit width is d?

a $\sin\theta = \lambda/d$ b $\sin\theta = 2\lambda/d$
c $\sin\theta = 3\lambda/d$ d $\sin\theta = \lambda/2d$

Check 6

2. What happens to the diffraction pattern when the wavelength of light is increased in a single-slit diffraction experiment?

a The central maximum becomes narrower. b The central maximum becomes wider.
c The fringes become dimmer. d The pattern becomes sharper.

Check 7

3. If the slit width in a single-slit diffraction experiment is halved, what will happen to the angular width of the central maximum?

a It will remain the same. b It will double.
c It will become half. d It will quadruple.

Check 8

4. What happens if red light is used in place of blue light in diffraction?

a No change. b The bands become broader and farther apart.
c The bands become narrower and crowded. d The bands disappear.

Check 9

5. What makes the distance between consecutive maxima in a single-slit diffraction pattern increase?

a decreasing the wavelength b increasing the slit width
c increasing the wavelength, or decreasing the slit width d moving the screen closer to the slit

Check 10

6. What is the resolving power of an optical instrument?

a the ability to magnify the image b the ability to distinguish two objects as separate
c the ability to focus on an object d the ability to reflect light

WORKED EXAMPLE

Example 12.1: a red laser through a slit

Light from a red laser pointer, of wavelength 650 nm, passes through a slit 0.10 mm wide. Find the angle of the first minimum, and the width of the central maximum on a wall 2.0 m away.

Answer

$\lambda = 650\times10^{-9}$ m, $d = 0.10\times10^{-3}$ m, $D = 2.0$ m.

First minimum: $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{650\times10^{-9}}{1.0\times10^{-4}} = 6.5\times10^{-3}$, so $\theta_1 = 6.5\times10^{-3}$ rad = **0.37°**.

Width of the central maximum: $\dfrac{2\lambda D}{d} = \dfrac{2\times650\times10^{-9}\times2.0}{1.0\times10^{-4}} = 2.6\times10^{-2}$ m = **2.6 cm**.

YOUR TURN

Water in the way

A single-slit experiment gives a central maximum 6.0 mm wide in air. The whole apparatus is then put under water (refractive index 4/3). How wide is the central maximum now?

Hint

In water the wave slows down, and its wavelength becomes λ/μ.

Answer

The wavelength in water is $\lambda_w = \lambda/\mu = \frac{3}{4}\lambda$, three quarters of its value in air. The width $2\lambda D/d$ is proportional to the wavelength, so it becomes $\frac{3}{4}\times6.0$ = **4.5 mm**: the pattern shrinks by a quarter.

12.4 Diffraction grating

A **diffraction grating** is a large number of equally spaced, parallel, very narrow slits, side by side. A typical one has 300 to 1200 slits in every millimeter, made by ruling lines with a diamond point on glass or, more often now, by copying such a ruled master in plastic, or by recording a laser's interference pattern in a light-sensitive layer. The opaque lines are called **rulings**; the clear gaps between them, **slits**.

If each slit is a wide and each ruling b wide, the pattern repeats every

$$d = a + b$$

This repeat distance, from the start of one slit to the start of the next, is called the **grating element** (or grating spacing). If the grating has N lines per unit length, then N slits fit in one unit of length, so

$$d = a + b = \frac{1}{N}$$

A grating of 5000 lines per centimeter has $d = \dfrac{1\text{ cm}}{5000} = 2\times10^{-4}\text{ cm} = 2\times10^{-6}\text{ m} = 2\ \mu\text{m}$.

The grating equation

A plane wavefront of monochromatic light falls normally on the grating (Figure 12.8). Every slit becomes a source of secondary wavelets, all in phase. Consider the light that leaves every slit at an angle θ, which a lens brings together at a point P on the screen.

Take two neighboring slits and the corresponding points in them, the top edge A of one slit and the top edge C of the next, a distance $a + b$ apart. Drawing AN perpendicular to the rays (N on the ray from C), as for the single slit, the wavelet from C travels farther by

$$CN = (a+b)\sin\theta$$

The same path difference separates every pair of neighbors, all the way across the grating. If it is a whole number of wavelengths, the wavelets from **all** the slits arrive at P in step, and P is bright. The condition for a **principal maximum** (a bright line) is

$$\boxed{(a+b)\sin\theta_n = n\lambda \quad \text{or} \quad \sin\theta_n = Nn\lambda, \qquad n = 0, 1, 2, \ldots} \qquad (12.11)$$

where θ_n is the angle of diffraction of the nth **order**, and $N = 1/(a+b)$ is the number of lines per unit length. This is the **grating equation** (or grating law). $n = 0$ is the straight-through beam, the **zeroth order**; $n = 1$ gives a first-order bright line on each side of it, $n = 2$ a second-order pair, and so on.

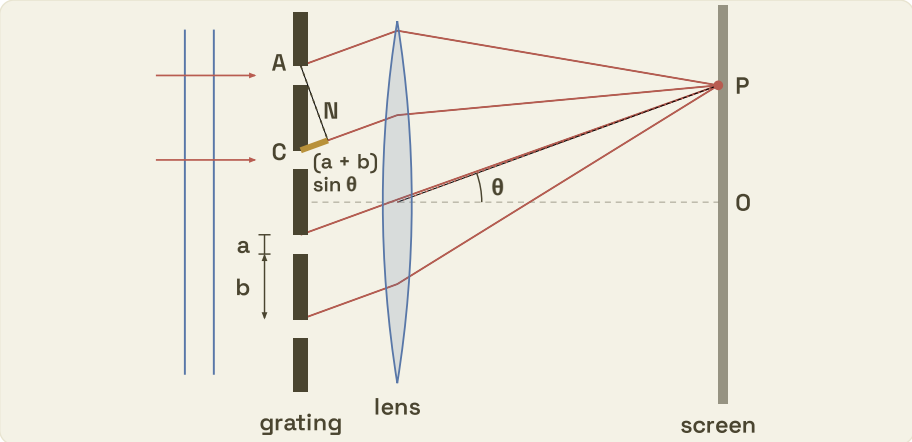


Figure 12.8 *A diffraction grating seen from above (slits and rulings much enlarged). The wavelets leaving neighboring slits at an angle θ differ in path by (a + b) sin θ. When that is a whole number of wavelengths, all of the slits' wavelets arrive at P in step, and P is bright.*

Minima. In between, the wavelets from the many slits arrive with all sorts of phases and cancel. With just two slits, darkness falls exactly halfway between the bright fringes, where the path difference is an odd number of half-wavelengths:

$$(a+b)\sin\theta = (2n+1)\frac{\lambda}{2} \quad \text{or} \quad \sin\theta = N(2n+1)\frac{\lambda}{2}$$

With thousands of slits, light in these directions still cancels (exactly for an even number of slits, almost exactly for an odd number), and so does light in almost every other direction between the orders: with N_S slits there are $N_S - 1$ dark directions between neighboring orders, and the faint maxima between them carry almost no light. A grating doesn't make broad bands with dark gaps, as two slits do: it makes **very sharp, bright lines** on a dark background, and the more lines it has, the sharper they are.

CAREFUL

"The grating law for minima"

Some books give $(a+b)\sin\theta = (2n+1)\lambda/2$ as the condition for minima of a grating. Those directions are dark (or very nearly), but they are only some of the dark directions: with many lines, nearly everything between the orders is dark. If an exam asks for it, write this condition and add that the bright lines of a grating are very sharp because nearly all the light between them cancels.

How many orders? Since $\sin\theta$ can't be more than 1, $n\lambda/d \le 1$, or $n \le d/\lambda$. The highest order that can be seen is the largest whole number **less than** d/λ. (If d/λ is exactly a whole number, that order would leave at 90°, along the grating's surface, and can't be seen on a screen.)

Gratings split colors. For a given order, $\sin\theta_n = n\lambda/d$: each wavelength goes off at its own angle. Violet, with the shortest waves, is turned the least; red, the most. So white light is spread into a **spectrum** in each order on both sides, with a white central line ($n = 0$), where every color goes straight through. This is the reverse of a prism, which bends violet the most.

A plane transmission grating is a flat sheet of glass or plastic with parallel lines ruled or etched on it, which light passes through. A **reflection grating** has its lines on a mirror, and works the same way with the light bouncing off it. The tracks of a CD or DVD are a reflection grating: their spiral is about 1.6 μm from one turn to the next on a CD, 0.74 μm on a DVD, so sunlight or a tube light reflected from them breaks into bright bands of color.

Increasing the focal length or the magnification of the eyepiece makes the image bigger, but not sharper: it only enlarges the blur.

WORKED EXAMPLE

Example 12.5: your eye, and headlights

The pupil of your eye is about 3 mm across in daylight. Taking $\lambda = 550$ nm, find the smallest angle your eye can resolve. How far away could you still tell that a car's two headlights, 1.2 m apart, are two lights?

Answer

$d\theta = \dfrac{1.22\lambda}{D} = \dfrac{1.22 \times 550 \times 10^{-9}}{3 \times 10^{-3}} = 2.2 \times 10^{-4}$ rad, about 0.013° (**0.8 minutes of arc**). At 100 m this angle spans about 2 cm.

The headlights, $s = 1.2$ m apart at a distance L, make an angle s/L. They are just resolved when $\dfrac{s}{L} = d\theta$:

$L = \dfrac{s}{d\theta} = \dfrac{1.2}{2.2 \times 10^{-4}} \approx 5.4 \times 10^{3}$ m, about **5 km**. (In practice a little less: the eye's lens isn't perfect, and at night the pupil opens wider but the eye's other limits take over.)

DID YOU KNOW?

The sharpest eyes in space

The Hubble Space Telescope's mirror is 2.4 m across. In green light its limit is $1.22 \times 550 \times 10^{-9}/2.4 = 2.8 \times 10^{-7}$ rad, about 0.06 seconds of arc: it could tell apart two car headlights from about 4000 km away, farther than from Mechi to Mahakali and back. The James Webb Space Telescope's mirror is 6.5 m across, but it looks in infrared light, whose longer waves cancel part of that advantage. To do better still, radio astronomers link dishes on different continents into one "aperture" as wide as the Earth: in 2019 such a network, the Event Horizon Telescope, made the first picture of a black hole's shadow.

Check 4

Which change lets a telescope resolve two stars that are closer together in the sky?

a an eyepiece of higher magnification
b an objective lens or mirror of larger diameter
c observing in red light instead of blue
d a longer focal length objective of the same diameter

Points to note

1. **Why diffraction of light is seldom noticed.** Light's wavelength is about 10^{-6} m (a micrometer, or less), and gaps and obstacles that small are rare in daily life, so light's diffraction is too small to notice. Sound's wavelength is around 1 m, the size of doors, windows and people, so sound diffracts all the time. That is why you can hear a person behind a wall but can't see them: sound bends round the wall a lot, light only a tiny amount, far too little to reach your eye.
2. Holding your hand at arm's length blocks the sunlight from your eyes, but not the sound of the traffic from your ears, for the same reason.
3. A chest X-ray shows no diffraction pattern, though the ribs are evenly spaced like a grating. X-rays have a wavelength of about 10^{-10} m, while the ribs are centimeters apart: about a hundred million times too far apart to act as a grating for them. (The rows of atoms in a crystal, about 10^{-10} m apart, *do* diffract X-rays: that is how the structures of salt, DNA and many medicines were found.)
4. If a single-slit apparatus is put under water, with nothing else changed, the wavelength of light falls to λ/μ, about **three quarters** (75 %) of its value in air (a decrease of 25 %), and the width of the central maximum, $2\lambda D/d$, shrinks in the same ratio.

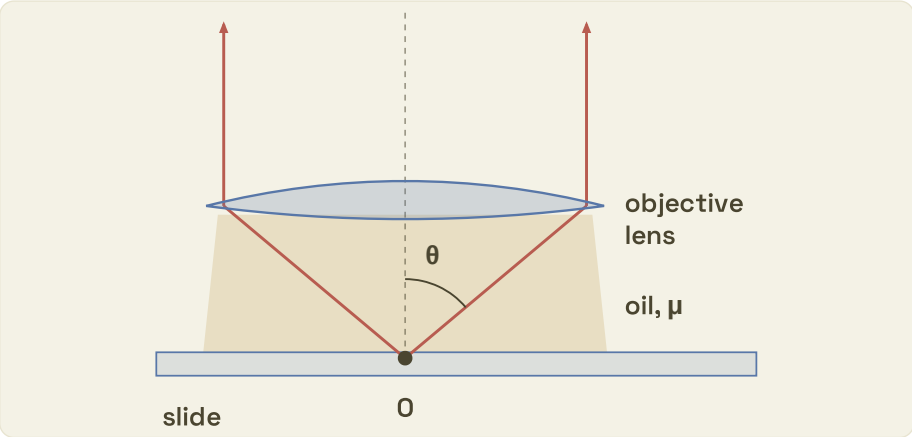


Figure 12.12 *The cone of light that the objective takes in from a point O of the object. Its semi-vertical angle is θ (the cone's full angle is 2θ). Filling the gap with oil of refractive index μ raises the numerical aperture μ sin θ.*

- with **shorter wavelengths**: blue or **ultraviolet** light resolves finer detail than red. An **electron microscope** goes much further: electrons behave as waves too, whose wavelength can be a hundred thousand times shorter than light's.

CAREFUL

Two forms of the microscope's limit

The formula $d = \lambda/2\mu\sin\theta$ is Abbe's, for an object lit from below, the usual case. Some books, using Rayleigh's criterion for two glowing points, write $d = 1.22\lambda/2\mu\sin\theta = 0.61\lambda/\mu\sin\theta$. Both say the same thing: $d \propto \lambda/\text{NA}$. Use the form your course gives.

WORKED EXAMPLE

Example 12.4: the oil-immersion objective

An oil-immersion objective has a semi-vertical angle of 70°, and the oil's refractive index is 1.52. What is the smallest separation it can resolve in green light of wavelength 550 nm? How does this compare with a dry objective of the same angle?

Answer

Numerical aperture with oil: $\mu\sin\theta = 1.52 \times \sin 70^\circ = 1.52 \times 0.940 = 1.43$.

$d = \dfrac{\lambda}{2\mu\sin\theta} = \dfrac{550\times10^{-9}}{2\times1.43} = 1.9\times10^{-7}\ \text{m} \approx \mathbf{0.19\ \mu m}$, about the size of the smallest bacteria.

Dry ($\mu = 1$): $\mu\sin\theta = 0.940$, $d = \dfrac{550\times10^{-9}}{2\times0.940} = 2.9\times10^{-7}\ \text{m} = \mathbf{0.29\ \mu m}$. The oil makes the resolving power 1.52 times larger.

12.5.2 Resolving power of a telescope

A telescope looks at objects very far away, such as two stars of a double star, so what matters is the smallest **angle** between them. By Rayleigh's criterion, the smallest angle that a telescope (or any circular aperture) of diameter D can resolve is the radius of the Airy disc:

$$d\theta = \frac{1.22\lambda}{D}$$

where $d\theta$ is the angular separation (in radians) of two objects that are just resolved, λ is the wavelength of the light and D the diameter of the telescope's objective lens or mirror. The **resolving power** is the reciprocal of this limit:

$$\boxed{\text{resolving power of a telescope} = \frac{1}{d\theta} = \frac{D}{1.22\lambda}} \qquad (12.14)$$

A **smaller** $d\theta$ means **better** resolution. So a telescope resolves finer detail with a **larger objective**. That is the main reason astronomers build ever bigger telescopes (the other is that a larger aperture collects more light, so fainter objects can be seen).



Figure 12.9 *The tracks of a CD, 1.6 μm apart, are a reflection grating. White light reflected from them leaves at angles that depend on its wavelength, so it is spread into colors.*
Photo: Yann Droneaud from Paris, France, CC BY-SA 2.0

Grating and prism

A prism also spreads white light into a spectrum, but by **refraction**: each color has its own refractive index in glass Grade 11. A grating does it by **diffraction and interference**.

	Prism	Grating
How it spreads colors	refraction (the refractive index depends on λ)	diffraction and interference ($\sin\theta = n\lambda/d$)
Bent most	violet	red
Number of spectra	one	several orders on both sides, plus white straight through
Measuring λ	needs the glass's refractive index for each color	from d and θ alone: $\lambda = d\sin\theta/n$

Because a grating turns light through **large angles**, tens of degrees, its angles can be measured precisely, and the wavelength follows straight from $\lambda = d\sin\theta/n$. That is why gratings measure wavelength **more accurately** than a prism, or than interference experiments such as Young's, whose fringes are only a fraction of a degree apart. Gratings are at the heart of the **spectrometers** that find what stars are made of, test medicines and water, and sort colors of light in fiber-optic cables.

WORKED EXAMPLE

Example 12.2: a grating of 5000 lines per centimeter

A diffraction grating with 5000 lines per centimeter is placed at right angles to a beam of monochromatic light of wavelength 600 nm. The pattern is seen on a screen 2.0 m from the grating, parallel to it.

i. Find the angles of diffraction of the first- and second-order maxima.

ii. How far apart are the first- and second-order maxima on the screen, on the same side?

Answer

$d = \dfrac{1}{5000\ \text{per cm}} = \dfrac{10^{-2}}{5000}\ \text{m} = 2.0\times10^{-6}\ \text{m}$; $\lambda = 600\times10^{-9} = 6.0\times10^{-7}\ \text{m}$; $D = 2.0\ \text{m}$.

i. First order ($n = 1$): $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{6.0\times10^{-7}}{2.0\times10^{-6}} = 0.30$, so $\theta_1 = \mathbf{17.5°}$.

Second order ($n = 2$): $\sin\theta_2 = \dfrac{2\lambda}{d} = 0.60$, so $\theta_2 = \mathbf{36.9°}$.

ii. These angles are large, so use $\tan\theta$, not θ (the small-angle shortcut would be badly wrong):
$x_1 = D\tan\theta_1 = 2.0\times\tan 17.5^\circ = 2.0\times0.314 = 0.63\ \text{m}$,
$x_2 = D\tan\theta_2 = 2.0\times\tan 36.9^\circ = 2.0\times0.750 = 1.50\ \text{m}$.
Distance between them: $x_2 - x_1 = 1.50 - 0.63 = \mathbf{0.87\ m}$.

WORKED EXAMPLE

Example 12.3: counting the lines

Light of wavelength 550 nm falls normally on a diffraction grating, and the first-order maximum is seen at 20°. How many lines per meter does the grating have?

Answer

$\lambda = 550 \times 10^{-9}$ m, $\theta_1 = 20^\circ$, $n = 1$.

$d = \dfrac{n\lambda}{\sin\theta_1} = \dfrac{550 \times 10^{-9}}{\sin 20^\circ} = \dfrac{5.5 \times 10^{-7}}{0.342} = 1.61 \times 10^{-6}$ m.

Lines per meter: $N = \dfrac{1}{d} = \dfrac{1}{1.61 \times 10^{-6}} = \mathbf{6.2 \times 10^5\ lines/m}$ (6200 lines per centimeter).

YOUR TURN

Test yourself 12.1: a turned grating

A grating with 5000 lines per centimeter is lit normally by light of wavelength 480 nm; the pattern falls on a screen 2 m away.

i. Find the angle of the second-order maximum.

ii. The grating is now turned through 10° about an axis parallel to its lines, so the light meets it at 10° to its normal. Where are the first-order maxima now?

Hint

ii. When the light arrives at an angle i to the grating's normal, neighboring slits are already $d\sin i$ out of step before the light leaves them. Add this to the path difference after the grating: a bright line needs $d(\sin\theta - \sin i) = n\lambda$, where θ is measured from the grating's normal, on the same side as the straight-through beam.

Answer

$d = 2.0 \times 10^{-6}$ m, $\lambda = 4.8 \times 10^{-7}$ m.

i. $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2 \times 4.8 \times 10^{-7}}{2.0 \times 10^{-6}} = 0.48$, $\theta_2 = \mathbf{28.7°}$.

ii. Facing the light squarely, the first order is at $\sin\theta_1 = \lambda/d = 0.24$, $\theta_1 = 13.9^\circ$ on both sides of the beam.

Turned by $i = 10^\circ$: $\sin\theta = \sin i \pm \dfrac{\lambda}{d} = 0.174 \pm 0.240$, so $\theta = 24.4^\circ$ on one side and $\theta = -3.8^\circ$ on the other, measured from the grating's new normal. The straight-through beam still goes along the original direction, at 10° to that normal. Measured from the straight-through beam, then, the two first-order lines are at $24.4^\circ - 10^\circ = \mathbf{14.4°}$ on one side and $10^\circ + 3.8^\circ = \mathbf{13.8°}$ on the other. The pattern does **not** simply turn with the grating: the lines hardly move, and become very slightly unequal. (That is why a spectrometer's grating must be set square to the beam for $d\sin\theta = n\lambda$ to hold.)

Check 3

White light falls on a diffraction grating. In the first-order spectrum, which color is turned through the largest angle?

a violet
b green
c red
d all colors are turned through the same angle

12.5 Resolving power of optical instruments

Look at two car headlights coming towards you on a long straight road at night. Far away they merge into one light; as the car comes closer, there is a moment when you can just see that there are two. Your eye has **resolved** them.

The **resolving power** of an optical instrument is its ability to show two closely spaced objects, or fine details, as separate. It depends on the **wavelength** of the light used, on the **diameter of the aperture** (in a telescope or the eye), and on the **numerical aperture** (in a microscope).

Why no image is a point

Every lens, mirror or pupil is a **circular aperture**, and light passing through it is diffracted, just as through a slit. So even a perfect lens can't make a point image of a point object, such as a star. It makes a small bright disc surrounded by faint rings, the **Airy pattern** (after the astronomer George Airy, who worked it out in 1835). Its bright center is called the **Airy disc**. The angle from the center of the disc to its first dark ring is

$$\theta = \frac{1.22\lambda}{D}$$

for a circular aperture of diameter D. It is like the slit's λ/d, with an extra factor 1.22 because the aperture is round. The smaller the aperture, the bigger the disc: this is the effect noticed "when the size of the aperture is comparable to the wavelength", but it is there for every aperture, only smaller for big ones.

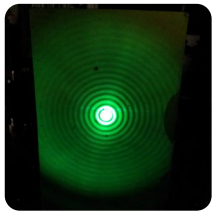


Figure 12.10 *The Airy pattern of a laser beam passed through a tiny pinhole: a bright central disc and faint rings.*
Photo: Anaqreon, CC0

Rayleigh's criterion

Two point sources close together each make their own Airy pattern. If the two patterns overlap too much, they blur into one blob and can't be told apart. In 1879 **Lord Rayleigh** suggested a simple rule for when they just can be:

> **Rayleigh's criterion:** two point sources are **just resolved** when the central maximum of one image falls on the first minimum of the other.

At that separation, the combined brightness dips by about 26 % between the two peaks, just enough for the eye to see two (Figure 12.11). Farther apart, they are clearly **resolved**; closer, the dip disappears and they are **not resolved**.



Figure 12.11 *The brightness across the images of two point sources (thin lines) and their total (thick line). Left: well apart, clearly resolved. Middle: the center of each image lies on the first dark ring of the other, Rayleigh's criterion; there is a small dip between the peaks. Right: closer still, the two make one peak, not resolved.*

The **limit of resolution** is the smallest separation (or angle) at which two objects can still be resolved: the smaller it is, the finer the detail the instrument shows. The **resolving power** is its reciprocal.

12.5.1 Resolving power of a microscope

In a microscope, the object is very close to the objective lens, and what matters is the smallest **distance** d between two points of the object that can be seen separately. The objective takes in a cone of light from each point of the object. If the semi-vertical angle of that cone is θ (Figure 12.12), and the space between the object and the lens is filled with a medium of refractive index μ, the smallest resolvable distance is

$$d = \frac{\lambda}{2\mu\sin\theta}$$

The quantity $\mu\sin\theta$ is called the **numerical aperture** (NA) of the objective; it is engraved on every objective lens (0.25, 0.65, 1.25…). The **resolving power** of a microscope is the reciprocal of the smallest separation:

$$\boxed{\text{resolving power of a microscope} = \frac{1}{d} = \frac{2\mu\sin\theta}{\lambda}} \qquad (12.13)$$

So a microscope resolves finer detail:

- with a **larger numerical aperture**: an objective that is wide and very close to the object takes in a wider cone (larger θ); and putting a drop of **cedarwood or immersion oil** ($\mu \approx 1.5$) between the object and an **oil-immersion objective** raises μ from 1 (air) to about 1.5;