Unit 12

Answers: Diffraction

Quick checks

1 b How much a wave spreads depends on the gap's width compared with the wavelength. Sound waves are centimeters to meters long, like an ordinary gap, so they spread widely. Light's are less than a micrometer, so through a gap of centimeters it goes almost straight. Speed has nothing to do with it.

2 b With a path difference of λ between the edges, the two halves of the slit differ by $\lambda/2$ point for point and cancel: $d \sin\theta = \lambda$. A path difference of $\lambda/2$ between the edges leaves the light bright, only a little dimmer than at the center.

3 c $\sin\theta = \lambda/d$ for the first order: the longer the wavelength, the larger the angle. Red has the longest waves of visible light, so it is turned the most. (A prism does the opposite: it bends violet most.)

4 b The limit is $d\theta = 1.22\lambda/D$: it gets smaller (better) with a larger aperture D or a shorter wavelength. Red light, with longer waves, makes it worse. More magnification or a longer focal length only makes the blurred image bigger.

5 a The first minimum is where the edges of the slit differ in path by one wavelength: $d \sin\theta = \lambda$.

6 b Width of the central maximum $= 2\lambda D/d \propto \lambda$.

7 b Angular width $= 2\lambda/d \propto 1/d$: half the slit, twice the width.

8 b Red light has a longer wavelength than blue, and every band's position, $\sin\theta = n\lambda/d$, is proportional to λ.

9 c The maxima are about $\lambda D/d$ apart on the screen, so they spread out when λ increases **or** d decreases (or the screen moves farther away). Both changes work, which is why they are one option here.

10 b Resolving power is about telling close objects apart. Magnification only makes the image bigger, blur and all.

11 b Light's wavelength (about 0.5 μm) is tiny compared with everyday objects and openings, so it diffracts very little and seems to go straight. Its frequency is very **high**, not small.

12 b The smallest resolvable distance is $\lambda/2\mu\sin\theta$; the resolving power is its reciprocal. (The first option is the limit of resolution, not the resolving power.)

13 c Resolving power $= 2\mu\sin\theta/\lambda$ grows with the numerical aperture $\mu\sin\theta$. A shorter focal length on its own doesn't do it; what counts is the angle of the cone of light the lens takes in.

14 **b** Resolving power $= D/1.22\lambda$. Of the four, only the objective's diameter is part of the telescope's design. (A shorter wavelength would help too, but the wavelength is the light's, not the telescope's.)

15 **a** $d\theta = 1.22\lambda/D$: the wavelength and the aperture.

16 **b** Resolving power is proportional to the numerical aperture.

13. a. **Diffraction** is the bending and spreading of light round the edge of an obstacle or through a narrow opening, into the region of the geometrical shadow. It is significant when the size of the opening or obstacle is **comparable to the wavelength** of light (for visible light, a fraction of a millimeter or less), and when the source is small (a point or a narrow line) and gives monochromatic light.
b.

	Interference	Diffraction
Sources	waves from **two (or more) separate coherent sources**, e.g. two slits	secondary wavelets from **different points of the same wavefront**, across one opening
Fringe width	all the bright fringes **equally wide**	central maximum **twice as wide** as the others
Brightness	all the bright fringes about **equally bright**	brightness **falls quickly** away from the center
Contrast	bright and dark fringes stand out clearly across the whole pattern	the side bands are faint, so they stand out poorly from the dark bands between them
Number of fringes	many fringes	only a few bands are visible

c. $\lambda = 5.0 \times 10^{-7}$ m, $d = 1.0 \times 10^{-4}$ m, $D = 2$ m.
Angular width $= \dfrac{2\lambda}{d} = \dfrac{2 \times 5.0 \times 10^{-7}}{1.0 \times 10^{-4}} =$ **0.01 rad** (about 0.57°).
Width on the screen $= \dfrac{2\lambda D}{d} = 0.01 \times 2 = 0.02$ m **= 2 cm**.

14. The bending is prominent when the **opening or obstacle is comparable in size to the wavelength** of the light: the narrower the slit (or the longer the wavelength), the more the light spreads. For light, slits and obstacles must be a fraction of a millimeter or less. A small, bright, monochromatic source, such as a laser, makes the pattern easy to see.
The type of pattern depends on the distances of the source and the screen from the opening:

- If the source or the screen (or both) is **close**, at a finite distance, the wavefronts at the opening are curved, and we get **Fresnel diffraction**: the pattern depends on the distances and is complicated (the fringes at a shadow's edge are an example).
- If both are effectively **at infinity** (in practice, lenses make the light parallel before the slit and focus it after, or a laser and a distant screen are used), the wavefronts are plane, and we get **Fraunhofer diffraction**: a simple pattern of a bright central band with fainter bands on each side, whose shape doesn't change with distance.

15. a. Setups as in Figure 12.3.

	Fresnel	Fraunhofer
Source and screen	at finite distances from the slit	effectively at infinity
Wavefront at the slit	curved (spherical or cylindrical)	plane
Lenses	none	a convex lens before the slit makes the light parallel; one after it focuses each direction on the screen
Pattern	depends on the distances; complicated	simple: central maximum and fainter bands

b. As in section 12.3: a plane wavefront falls on a slit AB of width d. Consider the light leaving at an angle θ, focused at P. Drawing AE perpendicular to the ray from B, the path difference between the edges is $BE = d\sin\theta$. If $BE = \lambda$, divide the slit into two halves; each point in the upper half has a partner $d/2$ below it in the lower half, with path difference $(d/2)\sin\theta = \lambda/2$. Every pair cancels, so P is dark: $d\sin\theta = \lambda$. If $BE = 2\lambda$, divide the slit into four parts, which cancel in pairs: dark again. In general, if $BE = n\lambda$, the slit divides into $2n$ parts that cancel in pairs, so the minima are at

$$d\sin\theta_n = n\lambda, \qquad n = 1, 2, 3, \ldots$$

c. $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{6.0\times10^{-7}}{2.0\times10^{-4}} = 3.0\times10^{-3}$, so $\theta_1 = \mathbf{3.0\times10^{-3}\ rad = 0.17°}$.

d. Unpolarized light has vibrations in all directions across the ray; they can be split into two parts, one vibrating parallel to the surface and one in the plane of incidence. At a transparent, non-metallic surface such as glass or water, the part vibrating **parallel to the surface is reflected more strongly** than the part in the plane of incidence (which, at one angle, Brewster's angle, isn't reflected at all). So the reflected light has more vibrations in one direction than the other: it is **partially polarized**, and completely polarized at Brewster's angle (Unit 13).

16. a. A **diffraction grating** is a plate with a large number of equally spaced, parallel, very narrow slits (or lines), typically hundreds to a thousand per millimeter. Uses: in **spectrometers**, to measure the wavelengths of light and find what stars, flames and chemicals are made of; to measure the wavelength of a laser; in fiber-optic networks to separate signals of different colors; and in everyday things like the color effects of CDs and security holograms.

b. $(a+b)\sin\theta_n = n\lambda$, where $a+b=d$ is the grating element, θ_n the angle of the nth order bright line and λ the wavelength. Light leaving neighboring slits at θ differs in path by $(a+b)\sin\theta$; when this is a whole number of wavelengths, the wavelets from all the slits arrive in step and add up to a bright line. $n=0$ is the central (straight-through) line; $n=1, 2\ldots$ give the orders on both sides.

c. $d = \dfrac{10^{-2}}{6000} = 1.67\times10^{-6}$ m. $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\times5.0\times10^{-7}}{1.67\times10^{-6}} = 0.60$, so $\theta_2 =$ **36.9°**.

17. a. **Resolving power** is the ability of an optical instrument to show two closely spaced objects as separate; it is the reciprocal of the smallest separation (or angle) it can resolve. Diffraction at the instrument's aperture turns the image of every point into a small disc with rings (the Airy pattern). If two such discs overlap too much, the points can't be told apart, so diffraction sets a limit: a wider aperture or a shorter wavelength makes the discs smaller and the resolving power greater.

b. **Rayleigh's criterion:** two point sources are just resolved when the central maximum of one's diffraction pattern falls on the first minimum of the other's. For a circular aperture of diameter D, this gives the smallest resolvable angle, the angular resolution, $d\theta = 1.22\lambda/D$.

c. $d\theta = \dfrac{1.22\lambda}{D} = \dfrac{1.22\times550\times10^{-9}}{0.2} =$ **3.4×10^{-6} rad**, about 0.69 seconds of arc.

18. a. Diffraction is noticeable only when an object or opening is comparable in size to the wavelength. Light's wavelength is under a micrometer, while everyday objects and openings are millimeters to meters across, millions of times larger; so light spreads by an angle of about λ/d, far too small to notice, and shadows look sharp.

b. Every lens or mirror is an aperture that diffracts light, so even a perfect instrument makes each point of an object into a small disc. Details closer than the size of those discs blur together: diffraction sets the **limit of resolution**, $d = \lambda/2\mu\sin\theta$ for a microscope and $d\theta = 1.22\lambda/D$ for a telescope. Beyond this, more magnification shows no more detail.

c. $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2\times6.5\times10^{-7}}{5.0\times10^{-5}} = 0.026$, so $\theta_2 =$ **1.49°** (0.026 rad).

19.

a. The pattern **spreads out**: every minimum and maximum moves away from the center ($\sin\theta_n = n\lambda/d$), and the central maximum becomes wider (width $2\lambda D/d$). It also gets dimmer, because less light passes through the narrower slit.

b. In **single-slit diffraction**, the central maximum is **twice as wide** as the other bands and **much brighter**: the first side band is under 5 % of its brightness. In **double-slit interference**, the central bright fringe is the **same width** as all the other bright fringes, and (within the slits' own diffraction envelope) about **equally bright**.

c. $\text{Width} = \dfrac{2\lambda D}{d} = \dfrac{2 \times 6.5 \times 10^{-7} \times 1.5}{1.0 \times 10^{-4}} = 1.95 \times 10^{-2}\ \text{m} = \mathbf{19.5\ mm}$.

20.

a. The **angular width** of the central maximum is the angle between the directions of the first minima on either side of it, as seen from the slit: $2\theta = 2\lambda/d$ radians (for small angles). Half of it, λ/d, is the angular half-width.

b. Diffraction at an instrument's aperture spreads the image of every point into a small disc with rings. Two points whose discs overlap too much can't be seen as separate; by Rayleigh's criterion they are just resolved when the center of one disc lies on the first dark ring of the other. This sets the smallest detail an instrument can show: $d\theta = 1.22\lambda/D$ for a telescope, $d = \lambda/2\mu\sin\theta$ for a microscope. Larger apertures and shorter wavelengths resolve finer detail.

c. $d = \dfrac{10^{-2}}{4000} = 2.5 \times 10^{-6}$ m. $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{5.5 \times 10^{-7}}{2.5 \times 10^{-6}} = 0.22$, $\theta_1 = \mathbf{12.7°}$.

d. By **Huygens' principle**, every point of a wavefront acts as a source of secondary wavelets, and the new wavefront is the surface touching them. When a wavefront reaches a slit, only the points in the opening send wavelets on; at the edges, the wavelets spread into the region behind the barrier, so the light bends round into the shadow. Wavelets from different points of the opening travel different distances to a point on the screen, so they interfere: where they arrive in step, the screen is bright; where they cancel (when the edges differ by $\lambda, 2\lambda\ldots$), it is dark. Together these give the diffraction pattern.

21. a. The **grating element** is the distance between corresponding points of neighboring slits: the width of one slit plus one ruling, $d = a + b$. If the grating has N lines per cm, $d = 1/N$ cm.

b. A plane wavefront falls normally on the grating, so all the slits are sources in phase. The wavelets leaving two neighboring slits at an angle θ differ in path by $(a + b)\sin\theta$ (drop a perpendicular from one slit onto the ray from the next). The same path difference separates every neighboring pair. If $(a + b)\sin\theta = n\lambda$, all the wavelets arrive at the focus of the lens in step and add up: a bright principal maximum. So the maxima are at

$$(a + b)\sin\theta_n = n\lambda, \qquad \sin\theta_n = Nn\lambda, \qquad n = 0, 1, 2, \ldots$$

c. $d = \dfrac{10^{-2}}{5000} = 2.0 \times 10^{-6}$ m. $n \le \dfrac{d}{\lambda} = \dfrac{2.0 \times 10^{-6}}{5.0 \times 10^{-7}} = 4$. But $n = 4$ would need $\sin\theta = 1$, $\theta = 90^\circ$: that light would travel along the surface of the grating and can't reach a screen. So the orders that can be seen are $n = 1, 2, 3$: **3 orders** on each side of the central line (7 lines in all, counting the central one).

d. Angular width $= 2\lambda/d$: for a given slit it is **directly proportional to the wavelength**. Red light (about 700 nm) gives a central maximum nearly twice as wide as violet light (about 400 nm) through the same slit. That is why, in white light, the edges of the central maximum are colored, red outermost.

22. a. (Diagram as Figure 12.4, with the pattern of Figure 12.7.) On the screen there is a **bright central maximum** at O, straight ahead of the slit, where the wavelets from the whole slit arrive in step. On each side of it are dark bands (minima) at $\sin\theta = \lambda/d, 2\lambda/d, \ldots$, with fainter bright bands (secondary maxima) between them, at about $\sin\theta = 3\lambda/2d, 5\lambda/2d, \ldots$ The central maximum is twice as wide as the others, and the secondary maxima get rapidly fainter away from the center.

b. Let a plane wavefront fall normally on a slit AB of width d, and consider the light leaving at θ, focused at P. The path difference between the wavelets from the edges is $BE = d\sin\theta$ (AE perpendicular to the rays). Let $d\sin\theta = \lambda$. Divide the slit into two equal halves, AC and CB. A point in AC and the point $d/2$ below it in CB differ in path by $(d/2)\sin\theta = \lambda/2$, so their wavelets arrive in opposite phase and cancel. Every point in the top half pairs off this way with one in the bottom half, so the whole slit's light cancels at P: the first minimum is at

$$d\sin\theta = \lambda$$

c. $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2 \times 6.5 \times 10^{-7}}{1.5 \times 10^{-4}} = 8.67 \times 10^{-3}$, so $\theta_2 = 8.67 \times 10^{-3}$ rad = **0.497°**. (The screen distance isn't needed for the angle.)

23.

a. With the slit width written a: $a \sin\theta = m\lambda$, $m = 1, 2, 3, \ldots$

b. The first minimum ($m = 1$) is at $\sin\theta = \lambda/a$. On the screen, a distance y from the center O, $\tan\theta = y/D$. For the small angles of light through a slit, $\sin\theta \approx \tan\theta \approx \theta$, so

$$\frac{y}{D} = \frac{\lambda}{a} \quad \Rightarrow \quad y = \frac{\lambda D}{a}$$

(and in general, the mth minimum is at $y_m = m\lambda D/a$).

c. $y = \dfrac{\lambda D}{a} = \dfrac{6.0 \times 10^{-7} \times 2}{2.0 \times 10^{-4}} = 6.0 \times 10^{-3}$ m **= 6 mm**.

24.

a. Each slit diffracts the light, and the light from all the slits interferes. A bright line forms only where neighboring slits' light differs in path by a whole number of wavelengths, $d\sin\theta = n\lambda$. For a given order n, the angle depends on the wavelength: $\sin\theta = n\lambda/d$. So each color in white light goes off in its own direction, violet least and red most, and the light is spread into a spectrum in each order.

b. As in question 21 b: neighboring slits, $a + b$ apart, send light at θ with path difference $(a + b)\sin\theta$; all the slits reinforce when it equals $n\lambda$: $(a + b)\sin\theta_n = n\lambda$.

c. $d = 2.0 \times 10^{-6}$ m. $\sin\theta_2 = \dfrac{2 \times 5.0 \times 10^{-7}}{2.0 \times 10^{-6}} = 0.50$, so $\theta_2 =$ **30°**.

25.

a. **Fresnel:** the source, the screen, or both are at a finite distance from the obstacle or slit, so the wavefront reaching it is curved; no lenses. **Fraunhofer:** the source and the screen are effectively at infinity, so the wavefront at the slit is plane; in practice, convex lenses make the light parallel and focus it, or a laser and a distant screen are used.

b. A Fresnel pattern depends on the distances of the source and screen, and on the object's exact shape: as the screen moves, the pattern changes form (even the center of a small round obstacle's shadow can be bright). A Fraunhofer pattern has a fixed form, a bright central band with regularly placed, rapidly fainter bands on each side, and only its size changes with distance. Fraunhofer's minima and maxima follow simple rules, $d\sin\theta = n\lambda$, so it is the easier one to measure and calculate.

c. Width $w = \dfrac{2\lambda D}{d}$, so $d = \dfrac{2\lambda D}{w} = \dfrac{2 \times 4.5 \times 10^{-7} \times 1.5}{2.0 \times 10^{-2}} = 6.75 \times 10^{-5}$ m = **67.5 μm**.

26.

a. A **narrower** slit spreads the light more: the central maximum and all the bands get wider and farther apart ($\propto 1/d$), and dimmer. A **wider** slit squeezes the pattern towards the center; when the slit is very wide compared with the wavelength, the pattern is just a sharp image of the slit, with no visible spreading.

b. The first minima are at $\sin\theta = \lambda/d$ (d the slit width). On a screen at distance D, the first minimum is at $x_1 = D\tan\theta \approx D\theta \approx \lambda D/d$ (small angles). The central maximum stretches from $-x_1$ to $+x_1$, so its width is

$$w = 2x_1 = \frac{2\lambda D}{d}$$

and its angular width $2\lambda/d$: directly proportional to λ, inversely to d.

c. Angular width $= \dfrac{2\lambda}{d} = \dfrac{2 \times 7.0 \times 10^{-7}}{2.0 \times 10^{-4}} = \mathbf{7.0 \times 10^{-3}\ rad}$, which is **0.40°**.

27.

a. A monochromatic source, made into a parallel beam by a convex lens (or a laser); a slit narrow enough to be comparable to the wavelength (a fraction of a millimeter); the light falling normally on the slit; and a second convex lens after the slit with the screen in its focal plane (or a screen far away), so that the light in each direction is brought to one point.

b. **Brighter:** at the center every wavelet from the whole slit arrives in step, while at a secondary maximum all but one part of the slit cancel (only a third, a fifth… of the slit contributes). **Wider:** the central maximum runs from the first minimum on one side ($\sin\theta = -\lambda/d$) to the first minimum on the other ($+\lambda/d$), an angular width $2\lambda/d$; each secondary maximum lies between two successive minima, only λ/d apart.

c. As in question 15 b: path difference between the edges $d\sin\theta$; when it is $n\lambda$, the slit divides into $2n$ parts that cancel in pairs, so the minima are at $d\sin\theta_n = n\lambda$, $n = 1, 2, 3, \ldots$

d. $\sin\theta_1 = \dfrac{\lambda}{d} = \dfrac{6.0 \times 10^{-7}}{1.5 \times 10^{-4}} = 4.0 \times 10^{-3}$, so $\theta_1 = 4.0 \times 10^{-3}$ rad $=$ **0.23°**.

28.

a. The **grating element** $d = a + b$ is the distance from one slit to the next (one slit plus one ruling). It is the reciprocal of the line density: $d = 1/N$, where N is the number of lines per unit length.

b. The more slits there are, the more completely the light cancels at any angle even slightly away from $d\sin\theta = n\lambda$: with N slits, each bright line is only about $1/N$ as wide (in angle) as the gap between orders. Narrower lines mean that two wavelengths very close together make two separate lines instead of one blurred one. The resolving power of a grating is $\lambda/\Delta\lambda = nN$: more lines (or a higher order) separate closer wavelengths, such as the two yellow lines of sodium, 589.0 and 589.6 nm, which need $nN \geq 1000$.

c. i. $d = \dfrac{10^{-2}}{6000} = \mathbf{1.67 \times 10^{-6}\ m}$.

ii. $\dfrac{d}{\lambda} = \dfrac{1.67 \times 10^{-6}}{5.2 \times 10^{-7}} = 3.2$, so the highest observable order is **3**.

iii. $\sin\theta_2 = \dfrac{2\lambda}{d} = \dfrac{2 \times 5.2 \times 10^{-7}}{1.67 \times 10^{-6}} = 0.624$, $\theta_2 = \mathbf{38.6°}$.