Unit 11

Interference of Light

This unit online, with its interactives:
science-playground.com/s/12f11

Blow a soap bubble on a sunny day and look at it against something dark. Swirls of magenta, gold, green and blue chase each other across a film thinner than a thousandth of a millimeter, and change as it thins, until, just before it bursts, the top goes black. Nothing in the soap is colored. The colors are made by light adding to itself in some places and canceling itself in others.

That adding and canceling is **interference**, and it is the clearest sign that light is a wave. In this unit you will see what makes two beams of light able to interfere, follow Thomas Young's famous experiment with two slits, and work out exactly where its bright and dark bands fall: a pattern so regular that you can measure the wavelength of light, less than a thousandth of a millimeter, with a ruler.

A STORY FROM SCIENCE

Thomas Young and the two slits

Thomas Young (1773–1829), whom you met in the unit on surface tension, was an English doctor who read a dozen languages, explained how the eye focuses and how we see color, and helped decipher the Rosetta Stone. Between 1801 and 1803 he showed the Royal Society that light from one source, split into two parts that come together again, makes **bands of light and dark**: in places light added to light gives darkness. He described his two-slit experiment in his lectures of 1807, and argued that only waves could do this.

Newton's particle theory was so respected that Young's work was attacked in print and largely ignored. It took another fifteen years, and Augustin Fresnel's exact wave calculations in France, before most scientists accepted that light is a wave.

Today Young's two slits are the first experiment in almost every book on light, and, done with single electrons, the most famous experiment in quantum physics.



Figure 11.1 *Thomas Young (1773–1829), engraved after a portrait by Thomas Lawrence.*
Photo: Wikimedia Commons, Public domain

11.1 Introduction

Interference is what happens when two or more waves overlap: at each point their displacements add (the principle of superposition, Unit 6), and the result is a new pattern, stronger in some places and weaker in others. It shows in everyday things:

- the colors of **soap bubbles**;
- the colored swirls of **oil on a wet road**;
- the colors of a peacock's feathers and a butterfly's wings, and of the coating on spectacle and camera lenses.

CAREFUL

Rainbows are not interference

The colors of a rainbow in the sky come from **dispersion**: each raindrop refracts sunlight, reflects it once inside, and refracts it again on the way out, and each color leaves at its own angle (red at about 42°, violet at about 40° from the point opposite the Sun). Interference adds only the faint extra pink and green bands sometimes seen just inside the main bow, the **supernumerary bows**. The colors of a bubble or an oil film, on the other hand, are interference.

Soap bubbles

A bubble's wall is a film of soapy water a few hundred nanometers thick, about as thick as a wavelength of light. When light falls on it, a little is reflected from the **outer surface** and a little from the **inner surface** (Figure 11.2). The second part has crossed the film twice, so it comes back behind the first by a small extra path. For some colors that delay puts the two reflections in step: they **reinforce** each other, and that color is reflected strongly. For others it puts them out of step: they **cancel**, and that color is missing from the reflection. What you see is white light with some colors strengthened and others removed, so the film looks colored, and since the extra path depends on the thickness, each thickness of film shows its own color.

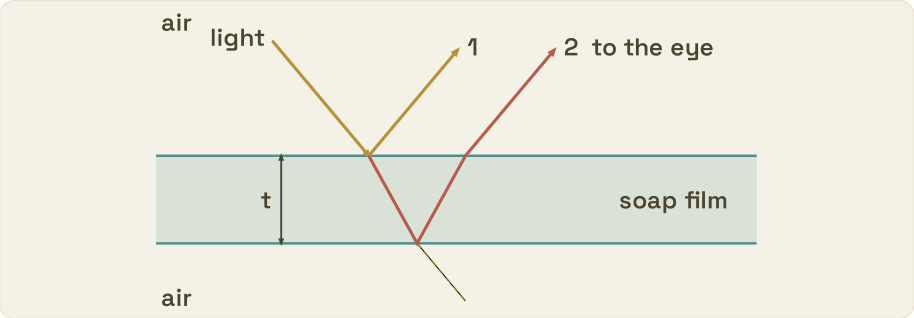


Figure 11.2 *Light reflected from the two surfaces of a thin film. Ray 1 is reflected at the top surface; ray 2 enters the film, is reflected at the bottom and comes out parallel to ray 1, having traveled the extra path inside the film. The two reflected rays overlap in the eye and interfere.*

As a bubble drains and thins, its colors move. Where the film becomes much thinner than a wavelength, the extra path is almost nothing; yet that part of the film looks **black**, not bright. The reason is that the reflection at the outer surface (from air onto water) turns the wave upside down, a phase change of π, while the reflection at the inner surface (from water onto air) doesn't. So the two reflections from a very thin film are always out of step, and cancel: the black spot that appears at the top of a bubble just before it bursts.

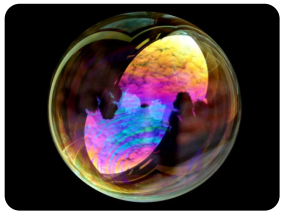


Figure 11.3 *The colors of a soap bubble: each band of color is a band of one thickness of the film.*

Photo: Brocken Inaglory, CC BY-SA 3.0

Oil on water

A drop of petrol or oil on a wet road spreads into a thin film on the water. Light reflected from the top of the oil and from the oil–water boundary below interferes, just as in a bubble. The film is thicker in some places and thinner in others, so it shows colored rings and swirls, each color tracing a line of one thickness, like the contour lines of a map.

22. Interference of light is a fundamental wave effect, which forms alternate bright and dark bands by superposition.
    a. Explain the interference of light. Discuss the conditions needed to observe interference, and distinguish between constructive and destructive interference.
    b. Monochromatic light of wavelength 500 nm passes through a double slit. The distance between the slits is 0.2 mm, and the screen is 1.5 m from the slits. Calculate the fringe separation (β) on the screen.
    c. If the wavelength of the light is changed to 600 nm, how does the fringe separation change? Explain the effect of the wavelength on the fringe width.

23. The fringe width in the double slit experiment measures the spacing between successive bright or dark bands, and gives an insight into the wave nature of light.
    a. Explain the idea of fringe width in a double slit experiment. Derive the expression for the fringe width, and discuss the factors that affect it.
    b. In a double slit experiment, monochromatic light of wavelength 700 nm passes through slits 0.25 mm apart. The screen is 2 m away. Calculate the distance between the third bright fringe and the central maximum.
    c. If the wavelength of the light is changed to 800 nm, calculate the new position of the third bright fringe. How does changing the wavelength affect the positions of the fringes?

24. The interference of light shows its wave nature, by producing alternate bright and dark fringes by the superposition of coherent waves.
    a. Define interference. Discuss the conditions needed to observe it, and explain the difference between constructive and destructive interference.
    b. How would the interference pattern change if the distance between the slits were halved, keeping the wavelength and screen distance the same? Explain the effect of the slit separation on the pattern.
    c. Monochromatic light of wavelength 450 nm is used in a double slit experiment. The distance between the slits is 0.1 mm, and the screen is 3 m from the slits. Calculate the distance between the first and second dark fringes.

25. A microwave source T is directed towards a metal plate with two narrow slits S_1 and S_2. A receiver is slowly moved along the line PO, as shown in the figure. The wavelength of the microwaves is 2.8 cm, the separation of the slits is 4.0 cm, and the receiver's line is 80 cm from the slits.
    a. Calculate the separation between adjacent maxima.
    b. What is the phase difference between the two waves at P, if P is the first maximum beside O?
    c. Why are well-defined interference patterns harder to observe when the slits are far apart, or when the source is not coherent? Explain.

    The textbook gives the wavelength as 2.8 nm and the slits 4 mm apart, with the answer 5.6×10^{-7} m. But 2.8 nm is an X-ray, not a microwave (microwaves are about 1 mm to 1 m long), and half a micrometer is a spacing no receiver could be moved through. The book's numbers don't work, so here the wavelength is 2.8 cm, that of the microwave kits used in school labs, and the slits 4.0 cm apart.

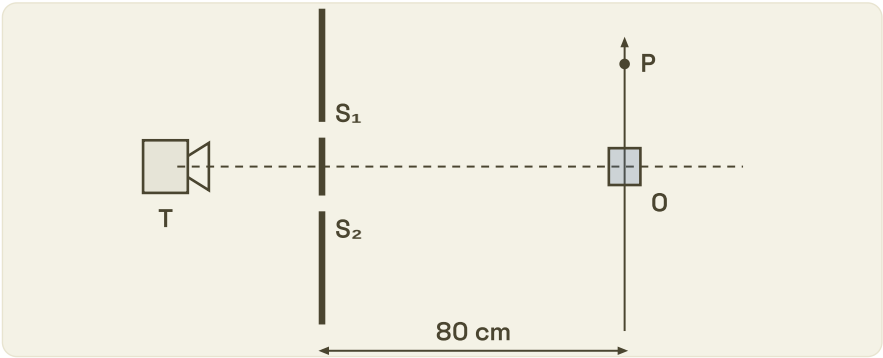


Figure 11.14 *For question 25: the microwave transmitter T, the metal plate with two slits S_1 and S_2, and the receiver at O, moved along the line OP.*

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.

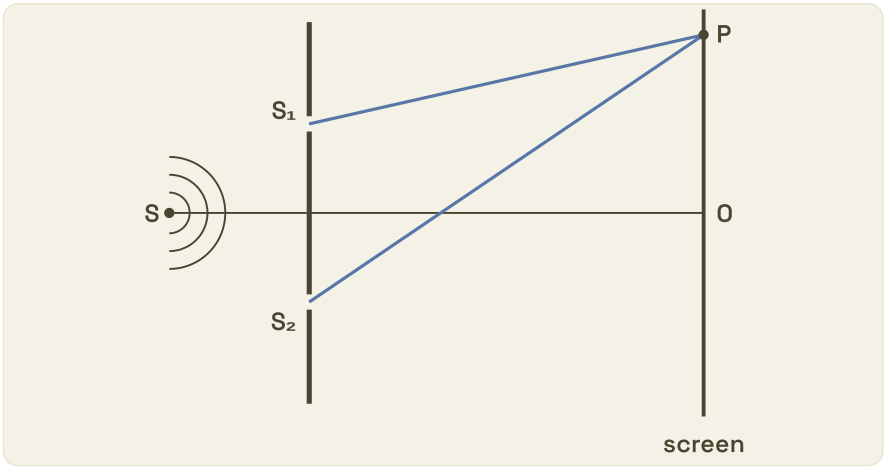


Figure 11.12 *For question 19: a source S, its wavefronts reaching the slits S_1 and S_2, and the paths from the two slits to a point on the screen.*

20. The figure shows a setup of Young's double slit experiment.
    a. Write the condition for constructive interference, and explain its significance.
    b. Derive the expression for the fringe width Δy on the screen, in terms of the slit separation d, the distance x to the screen, and the wavelength λ.
    c. If d = 0.5 mm, x = 1.5 m and λ = 600 nm, calculate the fringe width Δy.

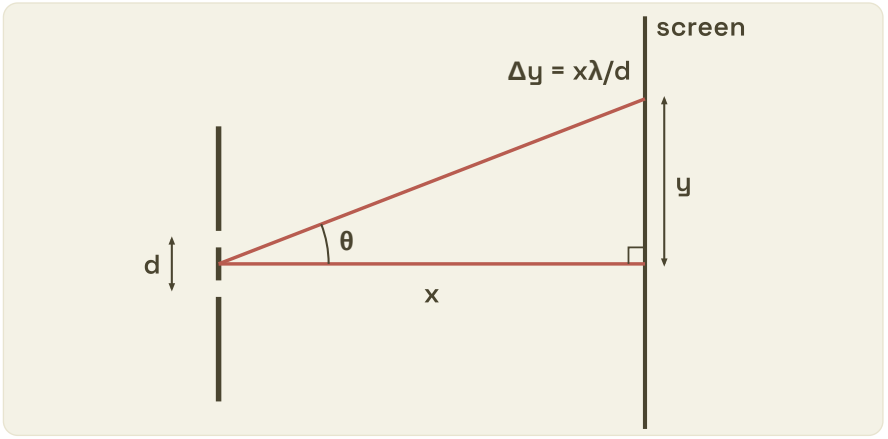


Figure 11.13 *For question 20: two slits a distance d apart, a screen at a distance x, and a point on it at a height y, seen from the slits at an angle θ.*

21. The double slit experiment is fundamental to understanding interference patterns, in which the fringe width depends on the wavelength, the slit separation and the screen distance.
    a. Derive an expression for the fringe width (β) in the double slit interference pattern. How is the fringe width related to the wavelength of the light, the distance between the slits, and the distance from the slits to the screen?
    b. In a double slit experiment, monochromatic light of wavelength 600 nm falls on two slits 0.5 mm apart. The screen is 2 m from the slits. Calculate the fringe width (β).
    c. What happens to the interference pattern if the distance between the slits is doubled, keeping the wavelength and the screen distance the same? Explain your answer.
    d. Describe the conditions under which constructive and destructive interference occur in the double slit experiment. How do they lead to the bright and dark fringes?

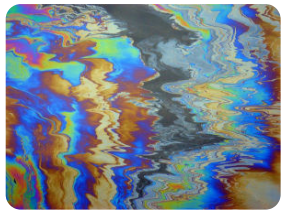


Figure 11.4 *A thin film of oil on water. Each band of color follows one thickness of the film.*
Photo: gambier20, CC BY 2.0

Superposition: what makes the bright and dark places

When two or more trains of waves pass through the same point of a medium at once, the displacement there at any instant is the sum of the displacements each wave would cause alone: the **principle of superposition**. For light, interference is the result of the superposition of two light waves of the **same frequency** whose **phase difference stays constant**. (They don't need equal amplitudes; equal amplitudes only make the dark places completely dark.)

Figure 11.5 shows waves spreading from two sources A and B that vibrate in step. The solid circles are crests and the dashed circles troughs, one wavelength apart.

- Where a crest from A meets a crest from B, or a trough meets a trough, the displacements add: the wave there has twice the amplitude, and the light is **bright**.
- Where a crest from one meets a trough from the other, the displacements cancel: **darkness**.

These places lie along lines spreading out from between the sources, bright and dark in turn. Where the lines meet a screen, you see the **interference pattern**: alternate bright and dark bands, called **fringes**.

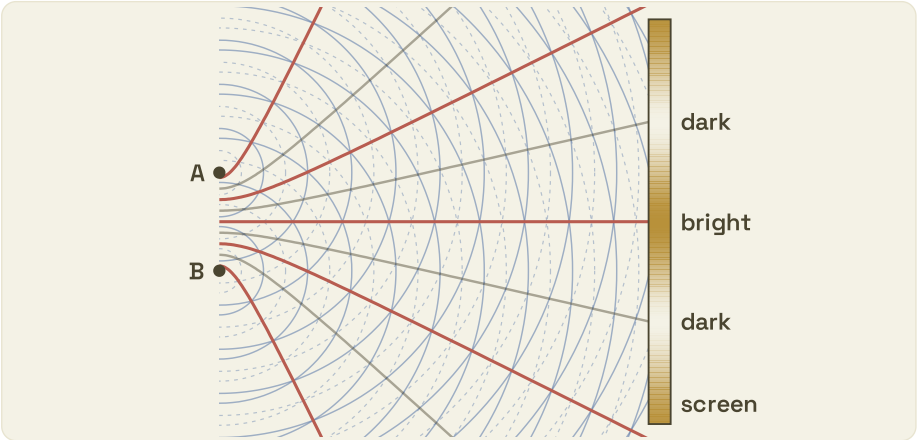


Figure 11.5 *Waves from two sources A and B in step: crests are solid circles, troughs dashed. Along the red lines a crest from one source always meets a crest from the other (and trough meets trough): bright. Along the gray lines a crest meets a trough: dark. On the screen they make alternate bright and dark fringes.*

Check 1
At a point, the crest of a wave from A arrives at the same moment as the trough of a wave of equal amplitude from B. What is there?

a a point twice as bright as one wave alone
b darkness
c a point as bright as one wave alone
d colored light

11.2 Some terms

Coherent sources

Two sources are **coherent** if they send out waves of the **same frequency** (so the same wavelength) with a **constant phase difference** between them. Only then do the bright and dark places stay still, so that the pattern can be seen. (The same amplitude is not part of the definition: two coherent waves of unequal amplitude still interfere, with less contrast.)

A **laser** is the best coherent source: it sends out light of practically one frequency, with a phase that stays steady for a long time, so its beam split in two makes clear fringes even in a lit room.

Two **independent** sources, even two identical bulbs, are never coherent. Light from a bulb or the Sun comes from countless atoms, each sending out a short burst of waves with a random phase, a burst lasting a few nanoseconds or less; and it contains a whole range of frequencies. The phase difference between the light from two bulbs changes at random millions of times a second, so the bright and dark places change just as fast, and the eye sees only the average: an even glow.

So coherent sources are always made from **one** source, by splitting its light in two:

- **By dividing the wavefront:** two parts of the same wavefront are taken from different places, as in **Young's double slit** (two slits in the path of one wavefront), **Lloyd's mirror** (a source and its own image in a mirror), the **Fresnel biprism** and **Fresnel's double mirror** (two virtual images of one source), and **Billet's split lens**.
- **By dividing the amplitude:** a partly reflecting surface sends part of the light one way and part another, as in thin films, and in the **Michelson** and **Rayleigh interferometers**.

Types of interference

How two waves combine depends on their **phase difference**, φ, where they meet.

i) **Constructive interference:** the waves arrive **in phase**, crest on crest and trough on trough, so they reinforce each other. This happens when the phase difference is 0, 2π, 4π, ..., that is $\varphi = 2n\pi$ (0°, 360°, 720°, ...), or in path, a **path difference of a whole number of wavelengths**, $n\lambda$. The resultant amplitude is the sum of the two, $a_1 + a_2$: a bright place.

ii) **Destructive interference:** the waves arrive **in antiphase**, crest on trough, so they oppose each other. This happens when the phase difference is π, 3π, 5π, ..., that is $\varphi = (2n+1)\pi$ (180°, 540°, ...), or a **path difference of an odd number of half wavelengths**, $(2n+1)\lambda/2$. The resultant amplitude is the difference, $|a_1 - a_2|$: a dark place, completely dark when the amplitudes are equal.

Between these, the resultant amplitude A of two waves of amplitudes a_1 and a_2 with phase difference φ is

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi$$

and since the intensity (brightness) is proportional to the square of the amplitude,

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$

For two equal waves, each of intensity I_0, this gives $4I_0$ at a bright place and 0 at a dark one. The light isn't created or destroyed: the energy missing from the dark places is in the bright ones, which are twice as bright as the two waves' intensities added.

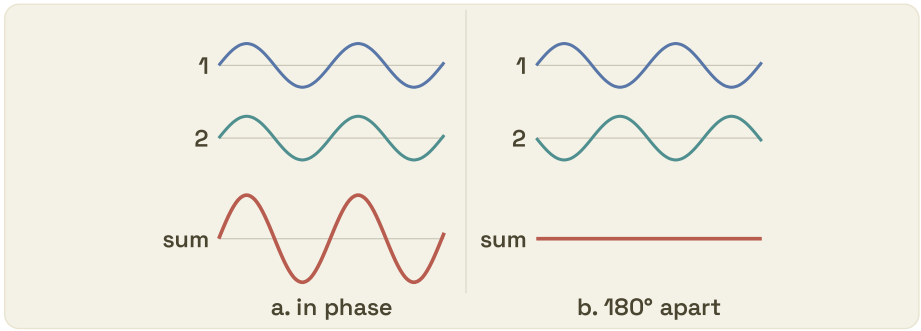


Figure 11.6 *Adding two waves. a. In phase (phase difference 0): the sum has twice the amplitude. b. In antiphase (phase difference 180°, or π): the sum is zero everywhere.*

16. The interference of light is a fundamental effect that shows the wave nature of light, as observed clearly in Young's double slit experiment (YDSE).
    a. What is the interference of light?
    b. What are coherent sources? Give one example.
    c. State the principle of Young's double slit experiment. Mention the conditions for obtaining a clear interference pattern.
    d. What happens to the interference pattern in YDSE when:
    i. the distance between the slits is increased;
    ii. the screen is moved farther from the slits;
    iii. the whole setup is immersed in water?

    Give reasons for each case.
    e. Derive the expression for the fringe width in YDSE. Discuss how it is affected by changing the wavelength and the slit separation.

17. Path difference and coherence are crucial ideas that decide whether an interference pattern forms, and whether it is stable.
    a. What is meant by path difference in wave interference?
    b. Define optical path. How is it different from geometrical path?
    c. State the conditions required for sustained interference. Why are temporal and spatial coherence necessary?
    d. Explain what happens to the interference pattern in the following situations. Give reasons.
    i. The two interfering waves differ slightly in frequency.
    ii. One of the coherent sources becomes slightly brighter than the other.
    e. Two coherent sources produce an interference pattern in air. Derive an expression for the positions of the bright fringes on the screen, in terms of the path difference and the wavelength. Explain what changes if the experiment is done in glass instead of air.

18. Fringe visibility and coherence are key factors in how clear and stable an interference pattern is, especially in Young's double slit experiment (YDSE).
    a. What is meant by fringe visibility?
    b. State the condition for the appearance of dark fringes in an interference pattern.
    c. Why is monochromatic light essential for a clear and steady interference pattern? What would happen if white light were used?
    d. In Young's double slit experiment, why is a single slit placed before the double slit? Explain its role in maintaining coherence.
    e. In a Young's double slit experiment, the distance between the slits is 0.3 mm and the distance from the slits to the screen is 1.2 m. The wavelength of the light used is 480 nm.
    i. Calculate the fringe width.
    ii. How far is the fourth bright fringe from the central maximum?

19. In the double slit experiment, a monochromatic light source is used to produce an interference pattern on a screen.
    a. Derive the expression for the fringe separation (β) in terms of the wavelength (λ) of the light, the distance (d) between the slits, and the distance (D) between the slits and the screen.
    b. Explain how the bright and dark fringes form. How does the pattern change when the distance between the slits is increased, or the wavelength of the light is changed?
    c. What happens to the interference pattern if the monochromatic light is replaced by white light? Explain the change in the pattern and what it means.
    d. Discuss the conditions required for a clear interference pattern in the double slit experiment.

Check 14

10. What happens to the fringe width in Young's experiment if the wavelength is halved?

a it stays the same
b it doubles
c it halves
d it becomes four times as wide

Answer these questions

11. The interference of light depends on the coherent nature of the sources and on the path differences traveled by the light waves.
    a. What are coherent sources of light? Why are they necessary for observing interference?
    b. Define the term "path difference" in the context of the interference of light.
    c. Explain how constructive and destructive interference occur, in terms of path difference.

12. The principle of superposition explains how waves combine to produce interference patterns.
    a. State the principle of superposition of waves.
    b. i. What are the conditions for sustained interference?
    ii. What happens when the path difference is $\lambda/2$?

13. Sustained interference is a key idea in wave optics: it explains the stable, visible patterns formed when waves combine.
    a. What do you mean by sustained interference? State the necessary conditions for it.
    b. i. Two coherent sources produce an interference pattern. Why is a screen necessary to observe it?
    ii. State one limitation of observing interference using two independent light sources.

14. The interference of light makes bright and dark fringes, by the superposition of waves from coherent sources.
    a. Define constructive and destructive interference.
    b. Explain why two independent bulbs are not coherent sources.
    c. Two slits are separated by 0.5 mm, and a screen is placed 2 m away. If the wavelength of the light used is 600 nm, calculate the fringe width.

15. Young's double slit experiment demonstrates the interference of light waves, and helps us understand how the bright and dark fringes form on a screen.
    i. Complete the diagram, and write the condition for constructive interference.
    ii. What is the condition for destructive interference?
    iii. Obtain an expression for the fringe width. Does the central fringe in this setup lie upwards or downwards on the screen?

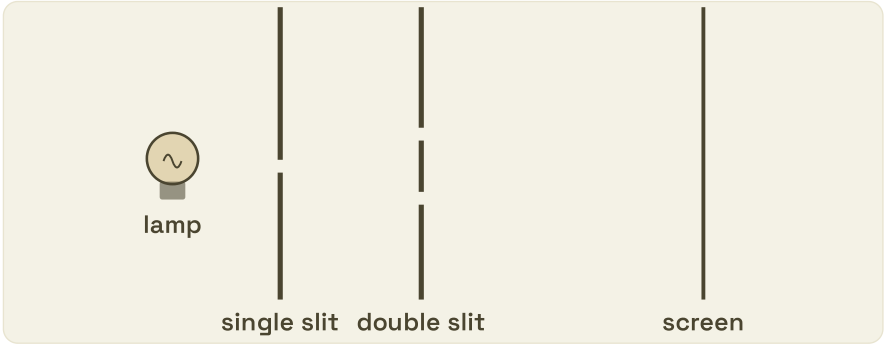


Figure 11.11 *For question 15: a lamp, a single slit, a double slit and a screen.*

Check 2

Two coherent waves meet with a path difference of $5\lambda/2$. What kind of interference is there?

a constructive, since 5 is a whole number
b destructive, since $5\lambda/2$ is an odd number of half wavelengths
c neither, since the path difference is more than a wavelength
d it depends on the amplitudes

Sustained interference

Sustained interference is an interference pattern that stays still and lasts, so it can be seen and measured: the bright and dark fringes keep their places. The conditions for it:

1. **Coherent sources:** the two sources must give waves of the **same frequency** (monochromatic light, of one wavelength) with a **constant phase difference**; in practice, both derived from one source.
2. **Temporal coherence:** each beam's phase must stay steady for long enough. Real light comes in wave trains of limited length, the **coherence length** (about $\lambda^2/\Delta\lambda$ for light with a spread of wavelengths $\Delta\lambda$). White light's is about a micrometer, a few wavelengths; a sodium lamp's a fraction of a millimeter; a laser's centimeters or more. The **path difference** between the two beams must be **less than the coherence length**, so that the two parts meeting are copies of the same wave train; otherwise there are no fringes.
3. **Spatial coherence:** the source must be **narrow** (a point or a fine slit). Each point of a wide source makes its own pattern, slightly shifted, and many shifted patterns blur into an even glow. This is why Young put a **single slit** in front of his two slits.
4. **Equal, or nearly equal, amplitudes:** for good contrast, so that the dark fringes are really dark.
5. **The same polarization:** waves vibrating at right angles to each other can't cancel.
6. **The slits narrow and close together, and the screen far away**, so that the fringes are wide enough to see (the fringe width is $\lambda D/d$, below).
7. **A steady setup:** vibrations, air currents and temperature changes alter the paths and make the fringes shake or drift.

How clear a pattern is, is measured by its **fringe visibility** (contrast):

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

It is 1 when the dark fringes are completely dark (equal amplitudes, perfect coherence) and 0 when there are no fringes at all.

Check 3

Why can't two separate torches, held side by side, make interference fringes on a wall?

a their light is too weak
b torches give polarized light
c the phase difference between their light changes at random, very fast
d the two beams don't overlap

Optical path

Light travels more slowly in glass or water than in air, so a path through a medium contains more waves than the same length in a vacuum. To compare paths in different media, we use the **optical path**: the distance that light would travel in a **vacuum** in the same time that it takes to travel the actual path in the medium.

Let light travel a distance d in a medium in which its speed is v. The time it takes is

$$t = \frac{d}{v}$$

In that time, in a vacuum, it would travel a distance s:

$$s = c\,t = c \times \frac{d}{v} = \frac{c}{v} \times d$$

and since $c/v = \mu$, the absolute refractive index of the medium,

$$s = \mu d$$

Optical path = absolute refractive index × geometrical path.

The optical path counts the waves: the number of wavelengths in a path d in the medium, where the wavelength is λ/μ, is $d \div (\lambda/\mu) = \mu d/\lambda$, the same as in an optical path μd of vacuum. So in interference it is the **optical path difference** that decides the phase difference. A sheet of glass 6 μm thick ($\mu = 1.5$) put in one beam adds $(\mu - 1)d = 3$ μm to that beam's optical path, about six wavelengths of green light: it moves the whole pattern by six fringes.

Path difference and phase difference

The **path difference** between two waves arriving at a point is the difference between the distances they have traveled from their sources (in vacuum or air; the optical paths otherwise). Over one wavelength, a wave goes through one complete cycle, 2π of

phase. So

for a path difference of λ, the phase difference = 2π,

and for a path difference Δx, the phase difference is

$$\varphi = \frac{2\pi}{\lambda} \times \Delta x$$

Phase difference = $\frac{2\pi}{\lambda}$ × path difference.

So a path difference of λ/2 means a phase difference of π (180°): destructive. A path difference of λ, 2λ, 3λ, … means 2π, 4π, 6π, …: the same as 0, constructive.

WORKED EXAMPLE

Example 11.1: from path to phase

Two coherent waves of wavelength 600 nm reach a point after paths of 2.4000 mm and 2.4015 mm. What is their phase difference there, and is the point bright or dark?

Answer

Path difference $\Delta x = 2.4015 - 2.4000 = 0.0015 \text{ mm} = 1.5 \times 10^{-6} \text{ m}$.

$\varphi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi \times 1.5 \times 10^{-6}}{6.0 \times 10^{-7}} = 2\pi \times 2.5 = 5\pi$.

5π is an odd multiple of π (the path difference is 2.5 wavelengths, five half wavelengths), so the waves arrive in antiphase: the point is **dark**.

11.3 Young's double slit experiment

Imagine a dark room with one light shining towards you. In front of you is a barrier with two tiny slits, very close together. You might expect two bright lines on the wall behind. Instead you see a whole row of bright and dark bands, evenly spaced. This is **Young's double slit experiment**, the experiment that showed light is a wave.

Light from a lamp falls first on a **single narrow slit** S. Spreading out from it by diffraction, the light falls on **two narrow slits** S_1 and S_2, close together and at equal distances from S, so the same wavefront reaches both at once. S_1 and S_2 become two **coherent sources**, in phase. Their light spreads out again and overlaps on a screen (Figure 11.7). Where crests from the two slits arrive together, they reinforce: bright fringes. Where a crest meets a trough, they cancel: dark fringes. This alternate pattern of bright and dark fringes, the **interference pattern**, can't be made by particles fired through two holes: it showed that light behaves as a wave.

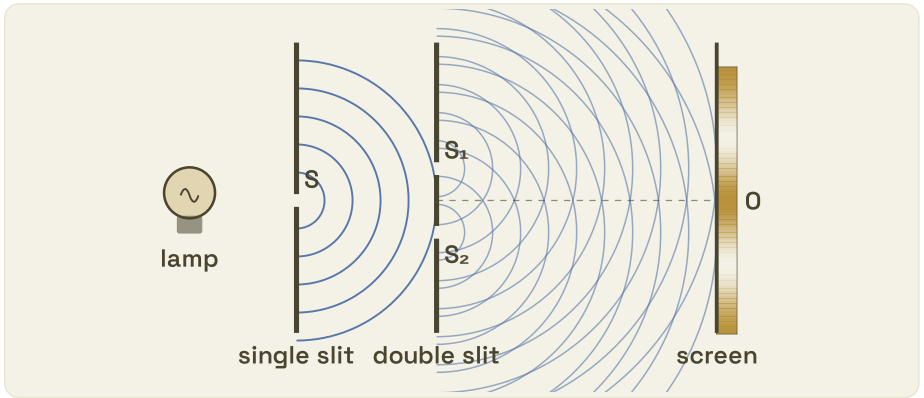


Figure 11.7 Young's double slit experiment. Light from the lamp passes through the single slit S; its wavefronts reach the double slit S_1, S_2 together, and the two slits send out coherent waves that overlap on the screen, making alternate bright and dark fringes. The central fringe O is bright, on the line through the middle of S_1S_2.

Young's experiment did more than settle a dispute about light. A century later the same pattern appeared with electrons, and even with electrons sent through the slits one at a time: each lands as a single dot, yet the dots build up the fringes. Particles too have wave properties, the **wave–particle duality** at the heart of quantum physics. It is remarkable that a light, a card with two slits and a wall can reach so deep.

Exercise

Choose the correct option

Check 5

1. What is required for two light sources to be coherent?

a the same frequency
b the same amplitude
c the same wavelength and a constant phase difference
d the same color

Check 6

2. What is the phase difference between the two waves at the nth bright fringe in a double slit experiment?

a nπ
b nλ
c 2nπ
d 0

Check 7

3. What is the correct relation between the path difference (Δx) and the phase difference (Δφ) for constructive interference?

a $\Delta x = \frac{\Delta\varphi}{2\pi}$
b $\Delta x = \frac{\Delta\varphi\,\lambda}{2\pi}$
c $\Delta x = \frac{\Delta\varphi}{\lambda}$
d $\Delta x = \Delta\varphi$

Check 8

4. Which is the best description of two coherent light sources?

a they have the same frequency and intensity
b they have a constant phase difference over time
c they emit light of the same wavelength
d they have the same polarization

Check 9

5. What is the path difference at the first dark fringe in Young's double slit experiment?

a λ/4
b λ/2
c λ
d 3λ/2

Check 10

6. What is the phase difference between two coherent light waves whose path difference is λ/2?

a 0°
b 90°
c 180°
d 360°

Check 11

7. In Young's double slit experiment, yellow light from a sodium lamp is replaced by blue light of the same intensity. What happens to the fringe width?

a it decreases
b it increases
c it doesn't change
d the fringes become less bright

Check 12

8. What is the phase difference when the path difference between two coherent light waves is 2λ? (Hint: phase difference is periodic; it repeats every 360°.)

a 180°
b 90°
c 540°
d 0°

Check 13

9. What happens to the fringes in Young's double slit experiment if the distance between the slits is increased?

a the fringes become narrower
b the fringes become wider
c more fringes appear
d the fringes disappear

YOUR TURN

Test yourself

Light of wavelength 600 nm falls on two slits 0.2 mm apart, with a screen 4.0 m beyond them. How far from the center of the pattern are the first bright fringe and the first dark fringe?

Answer

$\beta = \dfrac{\lambda D}{d} = \dfrac{6.0 \times 10^{-7} \times 4.0}{2.0 \times 10^{-4}} = 1.2 \times 10^{-2}$ m.
The first maximum (n = 1) is at $x = \lambda D/d =$ **1.2 cm** from the center, on either side; the first minimum (n = 0 in $(2n+1)\lambda D/2d$) is at $\lambda D/2d =$ **0.6 cm**.

DID YOU KNOW?

Measuring a fraction of a wavelength, and a wave of space

Interference measures lengths far smaller than any ruler can. Michelson's interferometer splits a beam in two, sends the halves down two arms and recombines them: moving one mirror by a quarter of a wavelength, about 0.15 μm, turns a bright fringe dark. In 1907 Michelson won the Nobel Prize in Physics for his precise optical instruments and the measurements made with them. The same idea, with arms 4 km long, is LIGO, which in 2015 detected **gravitational waves** from two black holes merging over a billion light-years away: a change in the arms' length of about a thousandth of the width of a proton.

IN SHORT

- **Interference** is the overlapping of two or more waves, which add by superposition: brighter in some places, darker in others.
- **Coherent sources** give waves of the same frequency with a constant phase difference; they are always made from one source (double slit, Lloyd's mirror, biprism, thin films, interferometers). Two independent bulbs are never coherent.
- **Constructive:** path difference $n\lambda$, phase difference $2n\pi$. **Destructive:** path difference $(2n+1)\lambda/2$, phase difference $(2n+1)\pi$.
- **Phase difference** $= \dfrac{2\pi}{\lambda} \times$ path difference. **Optical path** $= \mu d$.
- **Sustained interference** needs coherent, monochromatic sources of nearly equal amplitude, a narrow source, a path difference within the coherence length, the same polarization, and slits close together with a distant screen.
- In **Young's experiment**, path difference $= d\sin\theta \approx xd/D$: bright fringes at $x_n = n\lambda D/d$, dark at $(2n+1)\lambda D/2d$, the center bright.
- **Fringe width** $\beta = \lambda D/d$, the same for all fringes: $\beta \propto \lambda$, $\beta \propto D$, $\beta \propto 1/d$. In a liquid, β/μ. In white light, a white center and colored fringes, blue inside, red outside.

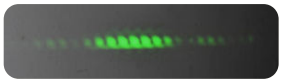


Figure 11.8 *The fringes of a green laser through two slits, on a wall. The bright fringes are evenly spaced; they fade away to the sides because each slit's own light (its diffraction pattern, Unit 12) fades there.*
Photo: Graham Beards, CC BY-SA 4.0

11.4 Young's double-slit experiment and the expression for the nth order maximum

In Young's experiment, monochromatic light falls on two narrow slits S_1 and S_2, close together. They act as **coherent sources**, and their overlapping waves make a pattern of alternate bright and dark fringes on a screen a distance D away. Now we find where the fringes are.

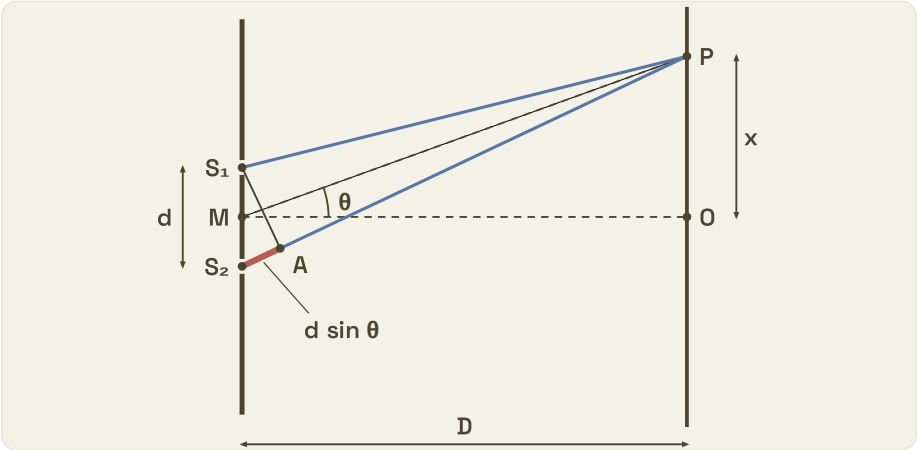


Figure 11.9 *The geometry of Young's experiment. The slits S_1 and S_2 are a distance d apart, the screen a distance D away. P is a point on the screen at a distance x from O, the center of the pattern; θ is the angle between MP and MO, from the middle of the slits. S_1A is perpendicular to S_2P, so the path difference is $S_2A = d\sin\theta$. (The drawing is stretched: really D is thousands of times d, and θ tiny.)*

Theory of interference fringes

Let d be the distance between the two slits, D the distance from the slits to the screen, λ the wavelength of the light, x the distance of a point P on the screen from the central point O, and θ the angle that P makes at the slits with the central line MO (Figure 11.9).

The waves from S_1 and S_2 start in phase. To reach P, the wave from S_2 travels farther. Mark A on S_2P so that $S_1A \perp S_2P$; since P is very far away compared with d, $PS_1 = PA$, and the **path difference** is

$$\Delta x = S_2P - S_1P = S_2A = d\sin\theta$$

The angle θ is very small (D is about a meter, x a few millimeters), so $\sin\theta \approx \tan\theta \approx \theta$, and from the triangle MOP, $\tan\theta = \dfrac{x}{D}$. So

$$\Delta x = d\sin\theta \approx d\,\theta = \frac{x\,d}{D} \tag{11.1}$$

i) Bright fringes: constructive interference

There is a bright fringe at P when the path difference is a whole number of wavelengths:

$$\frac{x\,d}{D} = n\lambda \quad \Rightarrow \quad x_n = \frac{n\lambda D}{d}, \qquad n = 0, 1, 2, 3, \ldots \tag{11.2}$$

This is the position of the **nth order bright fringe** from the center. For $n = 0$, $x = 0$: the center O, equally distant from both slits, is always **bright**, the **central maximum**. The others lie at $\lambda D/d$, $2\lambda D/d$, ... on both sides.

ii) Dark fringes: destructive interference

There is a dark fringe at P when the path difference is an odd number of half wavelengths:

$$\frac{x\,d}{D} = (2n+1)\frac{\lambda}{2} \quad \Rightarrow \quad x_n = \frac{(2n+1)\lambda D}{2d}, \qquad n = 0, 1, 2, 3, \ldots \tag{11.3}$$

This gives the distance of the dark fringes from the center O: $\lambda D/2d$, $3\lambda D/2d$, $5\lambda D/2d$, ..., halfway between the bright ones.

Fringe width (β)

The **fringe width** β is the distance between the centers of two neighboring bright fringes (or two neighboring dark ones). For bright fringes,

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d} \tag{11.4}$$

and for dark fringes, the same:

$$\beta = \frac{(2n+3)\lambda D}{2d} - \frac{(2n+1)\lambda D}{2d} = \frac{\lambda D}{d}$$

So the bright and dark fringes alternate on both sides of the central maximum O, and **all the fringes have the same width**, whatever their order. A bright band and a dark band are each **half the fringe width** wide, $\beta/2 = \lambda D/2d$, and together they make one fringe width, $\beta = \lambda D/d$.

The fringe width is

- (i) directly proportional to the wavelength: $\beta \propto \lambda$ (red fringes are wider than blue);
- (ii) directly proportional to the distance of the screen from the slits: $\beta \propto D$;
- (iii) inversely proportional to the separation of the slits: $\beta \propto \dfrac{1}{d}$.

Figure 11.10 shows how the brightness varies across the screen, with the fringe width marked.

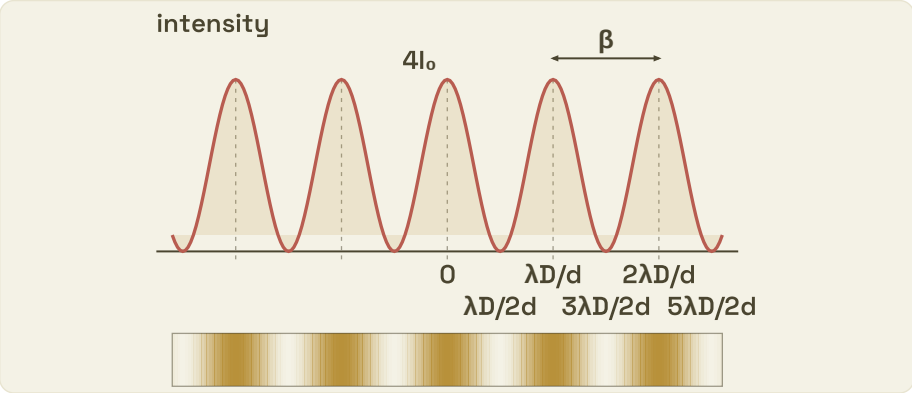


Figure 11.10 *The intensity across the screen for two equal slits (very narrow). The brightness rises and falls smoothly as $\cos^2(\pi x d/\lambda D)$: maxima of $4I_0$ at $x = 0$, $\pm\lambda D/d$, $\pm 2\lambda D/d$, ..., and zero at $\pm\lambda D/2d$, $\pm 3\lambda D/2d$, ... The strip below is the pattern as seen. Every fringe has the same width $\beta = \lambda D/d$.*

Check 4

In a Young's experiment the fringes are 2.0 mm wide. The screen is moved from 1.0 m to 1.5 m from the slits, and the slit separation is halved. What is the new fringe width?

a 1.5 mm
b 3.0 mm
c 4.0 mm
d 6.0 mm

Points to note

i) The path difference in the double slit experiment is $d\sin\theta$. For a maximum at a point P on the screen it must be $n\lambda$, and for a minimum $(2n+1)\lambda/2$.

ii) The fringe width is proportional to the wavelength: the fringes spread out for longer wavelengths (red) and come closer for shorter ones (violet), but the pattern keeps the same shape.

iii) If the whole apparatus is put in a **liquid** of refractive index μ, the light's wavelength in it becomes $\lambda' = \lambda/\mu$ (its frequency is unchanged, its speed falls to c/μ). So the fringes become narrower:

$$\beta' = \frac{\lambda' D}{d} = \frac{\beta}{\mu}$$

In water ($\mu = 1.33$), fringes 4.0 mm wide in air become 3.0 mm wide.

iv) The colors of a soap bubble come from interference between light reflected from the outer and inner surfaces of the film. Different thicknesses reinforce different wavelengths, so the film shows a spread of colors.

v) If the slits are very close together, comparable with the wavelength, the fringes are widely spaced, with only a few on each side (since $\sin\theta = n\lambda/d$ can't exceed 1), and the light through such narrow gaps is faint. If the slits are far apart, the fringes are so close together that they can't be seen. In a typical Young's experiment the slits are about 0.1 mm wide and about 0.5 mm apart.

vi) If the light at the two slits is out of phase by a constant amount (for instance if one slit is a little nearer the single slit), the whole pattern **shifts** sideways, but it stays the same: the fringes are still there, as long as the sources are coherent.

vii) For the maxima, $\sin\theta = \dfrac{n\lambda}{d}$, and for the minima, $\sin\theta = \dfrac{(2n+1)\lambda}{2d}$. These give the angles of the nth order maxima and minima, and hold even when the angles aren't small.

viii) If the monochromatic light is replaced by **white light**, the central fringe is **white**, since every color has its central maximum at O. On each side of it are a few **colored** fringes: since blue's fringes are narrower, each colored fringe is **blue (violet) on its inner edge and red on its outer edge**. Farther out, the bright fringes of different colors overlap, and the screen looks evenly whitish. None of the dark fringes is completely dark, since each color's dark places fall at different points; the darkest are the two beside the white center.

WORKED EXAMPLE

Example 11.2: two slit separations

In a double slit experiment with light of wavelength 500 nm and the screen 2.0 m away, the slit separation is 0.1 mm in one case and 0.05 mm in another. Find the ratio of the fringe widths in the two cases.

Answer

$\lambda = 500\text{ nm} = 5.0\times10^{-7}$ m, $D = 2.0$ m, $d_1 = 0.1\text{ mm} = 1.0\times10^{-4}$ m, $d_2 = 0.05\text{ mm} = 5.0\times10^{-5}$ m.

From $\beta = \dfrac{\lambda D}{d}$, with λ and D the same, $\beta \propto \dfrac{1}{d}$, so

$$\frac{\beta_1}{\beta_2} = \frac{d_2}{d_1} = \frac{0.05}{0.1} = \frac{1}{2}$$

The ratio is **1 : 2**. (In fact $\beta_1 = \dfrac{5.0\times10^{-7}\times 2.0}{1.0\times10^{-4}} = 1.0\times10^{-2}$ m = 10 mm and $\beta_2 = 20$ mm: the slits closer together give the wider fringes.)

WORKED EXAMPLE

Example 11.3: the angle of the first maximum

In a double slit experiment with light of wavelength 450 nm, the distance between the slits is 1.0 mm and the screen is 5.0 m away. Find the angle of the first order maximum.

Answer

$\lambda = 450\text{ nm} = 4.5\times10^{-7}$ m, $d = 1.0\text{ mm} = 1.0\times10^{-3}$ m, $D = 5.0$ m.

For the maxima, $d\sin\theta = n\lambda$, so for the first order ($n = 1$)

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{4.5\times10^{-7}}{1.0\times10^{-3}} = 4.5\times10^{-4}$$

$$\theta_1 = \sin^{-1}(4.5\times10^{-4}) = \mathbf{0.0258°}$$

(that is 4.5×10^{-4} rad). The angle doesn't depend on D; on the screen 5.0 m away, the first maximum is $D\theta_1 = 5.0\times 4.5\times10^{-4} = 2.25\times10^{-3}$ m, about 2.3 mm from the center.