Answers: Interference of Light

Quick checks

1 b The two displacements are equal and opposite at every moment, so they add to zero: no vibration, no light. This happens wherever the two waves arrive half a wavelength (or 3/2, 5/2, … wavelengths) out of step.

2 b $5\lambda/2 = (2 \times 2 + 1)\,\lambda/2$, an odd number of half wavelengths: phase difference 5π, which is the same as π. Crest meets trough: destructive. (Whether it is completely dark does depend on the amplitudes, but the kind of interference doesn't.)

3 c Each torch's light is a jumble of short wave trains with random phases. The phase difference between the two torches changes many millions of times a second, so the fringes move just as fast, and the eye sees only their average, an even patch of light.

4 d $\beta \propto D/d$. D grows 1.5 times and d halves, so β grows $1.5 \times 2 = 3$ times: $2.0 \times 3 = 6.0$ mm.

5 c Coherence needs both: the same frequency (so the same wavelength) and a phase difference that doesn't change. The same frequency alone (a) isn't enough: two separate lamps of one color still have phases that change at random.

6 c At the nth bright fringe the path difference is nλ, and phase difference $= (2\pi/\lambda) \times n\lambda = 2n\pi$. (nλ is the path difference, not the phase difference.)

7 b $\Delta\varphi = \dfrac{2\pi}{\lambda}\Delta x$, so $\Delta x = \dfrac{\lambda}{2\pi}\Delta\varphi$. For constructive interference, $\Delta\varphi = 2n\pi$ gives $\Delta x = n\lambda$.

8 b A constant phase difference is what makes a pattern stay still. (It needs the same frequency too, since waves of different frequencies drift out of step; but the same frequency or wavelength alone, as in a and c, isn't enough.)

9 b Dark fringes are at path differences λ/2, 3λ/2, 5λ/2, …; the first, nearest the center, is at λ/2.

10 c $\varphi = \dfrac{2\pi}{\lambda} \times \dfrac{\lambda}{2} = \pi = 180°$.

11 a Blue light has a shorter wavelength (about 450 nm) than sodium's yellow (589 nm), and $\beta = \lambda D/d \propto \lambda$: the fringes get narrower.

12 d $\varphi = \dfrac{2\pi}{\lambda} \times 2\lambda = 4\pi = 720°$, two whole cycles, which is the same as **0°**: the waves arrive in phase.

13 a $\beta = \lambda D/d$: a larger d gives narrower fringes. More of them then fit on the same width of screen, but that follows from the narrower fringes; the question asks about their width. If d is made very large, the fringes become too close to see.

14 **c** $\beta \propto \lambda$: half the wavelength, half the fringe width.

11.

a. **Coherent sources** are sources that give out light of the same frequency (wavelength) with a constant phase difference between them. They are necessary because the positions of the bright and dark places depend on the phase difference: if it is constant, the pattern stays still and can be seen; if it changes at random, as between two independent lamps, the bright and dark places move about millions of times a second, and the eye sees only an even glow.

b. The **path difference** at a point is the difference between the distances that the two waves have traveled from their (in-phase) sources to that point, $\Delta x = S_2P - S_1P$ (measured as optical paths, μd, if the light passes through different media). It sets the phase difference: $\varphi = (2\pi/\lambda)\Delta x$.

c. If the path difference is a whole number of wavelengths, $\Delta x = n\lambda$, the waves arrive in phase, crest with crest: their amplitudes add, **constructive interference**, a bright point. If it is an odd number of half wavelengths, $\Delta x = (2n + 1)\lambda/2$, they arrive in antiphase, crest with trough: their amplitudes subtract, **destructive interference**, a dark point (completely dark for equal amplitudes).

12.

a. When two or more waves pass through the same point of a medium at the same time, the resultant displacement at that point at any instant is the (vector) sum of the displacements that the individual waves would produce there.

b. i. The sources must be coherent: the same frequency (monochromatic light) and a constant phase difference, which in practice means both derived from one source; the amplitudes should be equal or nearly so; the source should be narrow; the path difference must be less than the coherence length; the two waves must have the same polarization; the slits should be narrow and close together and the screen far away; and the setup must be steady.

ii. A path difference of $\lambda/2$ means a phase difference of π: the waves arrive in antiphase, crest on trough, and cancel. There is **destructive interference**, a dark point (completely dark if the amplitudes are equal).

13. a. **Sustained interference** is an interference pattern whose bright and dark fringes stay in fixed positions for as long as it is watched, so that it can be seen and measured. Conditions: the two sources must be coherent (the same frequency and a constant phase difference, usually from one source); monochromatic light; amplitudes equal or nearly equal; a narrow source; the path difference within the coherence length; the same polarization; the slits narrow and close together and the screen far away; a steady setup.

b. i. Interference happens throughout the space where the two beams overlap, but light passing through air is invisible: you see light only when it enters your eye. The screen scatters light from each point towards the eye, so each point looks as bright as the light arriving there, and the fringes become visible. (You could also look into the beams through a lens, which is how a traveling microscope views them.)

ii. Two independent sources are not coherent: their phase difference changes at random many millions of times a second, so the fringes move as fast, and no stable pattern is seen, only uniform illumination.

14. a. **Constructive interference** is the reinforcement of two waves that meet in phase (path difference $n\lambda$, phase difference $2n\pi$), giving a resultant amplitude equal to the sum of their amplitudes, a bright point. **Destructive interference** is the cancellation of two waves that meet in antiphase (path difference $(2n + 1)\lambda/2$, phase difference $(2n + 1)\pi$), giving a resultant equal to the difference of their amplitudes, a dark point.

b. In a bulb, light is given out by countless atoms independently, each in a short burst of waves lasting a few nanoseconds or less, with a random phase, and over a range of frequencies. The phase difference between the light from two separate bulbs therefore changes at random, very rapidly, and is never constant, so they are not coherent, and their fringes, if any, shift too fast to be seen.

c. $d = 0.5\text{ mm} = 5.0 \times 10^{-4}\text{ m}$, $D = 2\text{ m}$, $\lambda = 600\text{ nm} = 6.0 \times 10^{-7}\text{ m}$.

$$\beta = \frac{\lambda D}{d} = \frac{6.0 \times 10^{-7} \times 2}{5.0 \times 10^{-4}} = 2.4 \times 10^{-3} =$$

15.

i. The completed diagram is Figure 11.7: wavefronts from the lamp pass through the single slit and reach the two slits together; from each slit, waves spread out and overlap on the screen, with bright and dark fringes marked on it. **Constructive interference** (a bright fringe) where the path difference is a whole number of wavelengths, $S_2P - S_1P = n\lambda$ (phase difference $2n\pi$), n = 0, 1, 2, ...

ii. **Destructive interference** (a dark fringe) where the path difference is an odd number of half wavelengths, $S_2P - S_1P = (2n + 1)\lambda/2$ (phase difference $(2n + 1)\pi$).

iii. With the geometry of Figure 11.9, the path difference at a point P a distance x from the center is $d \sin\theta \approx xd/D$. Bright fringes are at $x_n = n\lambda D/d$, so the distance between neighboring bright fringes is

$$\beta = x_{n+1} - x_n = \frac{\lambda D}{d}$$

(the same between dark fringes). The central fringe is where the path difference is zero: at the point of the screen equally far from both slits, on the line through the middle of the two slits, perpendicular to them. Here the single slit is on that line too, so the central fringe lies **neither up nor down**, but straight opposite the middle of the double slit, at the center of the pattern. (If the single slit were moved up, the light would reach S_1 first; the central fringe would then move down, to where the extra path from S_2 to the screen makes up for it.)

16.

a. The **interference of light** is the change in brightness, from bright to dark, produced where two or more coherent light waves overlap, because their displacements add (superposition): bright where they arrive in phase, dark where they arrive in antiphase.

b. **Coherent sources** emit light of the same frequency with a constant phase difference. Example: the two slits of Young's experiment lit by one narrow source (or a laser beam divided in two).

c. One wavefront from a narrow source is divided at two narrow slits close together, which therefore act as two coherent sources in phase. Their light overlaps on a screen, and at each point the waves reinforce or cancel according to their path difference, making evenly spaced bright and dark fringes. Conditions for a clear pattern: monochromatic light; a narrow source slit; the two slits narrow, close together and equally lit; the screen far away; a path difference within the coherence length; a dark room and a steady setup.

d. i. The fringes become **narrower** (closer together), since $\beta = \lambda D/d \propto 1/d$; with a large d they become too close to see.

ii. The fringes become **wider**, $\beta \propto D$, and the pattern dimmer, since the same light is spread over a larger area.

iii. The fringes become **narrower**, by the factor $\mu = 1.33$: in water the wavelength is λ/μ (the frequency is unchanged and the speed is c/μ), so $\beta' = \lambda D/\mu d = \beta/\mu$, about three quarters of the width in air.

e. For a point P on the screen at a distance x from the center, the path difference is $S_2P - S_1P = d\sin\theta \approx d\tan\theta = xd/D$ (θ is small). For a bright fringe $xd/D = n\lambda$, so $x_n = n\lambda D/d$, and the fringe width is

$$\beta = x_{n+1} - x_n = \frac{\lambda D}{d}$$

It is proportional to the wavelength (red light gives wider fringes than blue) and inversely proportional to the slit separation (slits closer together give wider fringes).

17.

a. The **path difference** is the difference between the distances traveled by the two waves from their sources to the point where they meet. It decides their phase difference there, $\varphi = (2\pi/\lambda)\Delta x$, and so whether they reinforce or cancel.

b. The **optical path** is the distance light would travel in a vacuum in the time it takes to travel a given path in a medium: $s = \mu d$, where d is the **geometrical path** (the actual distance) and μ the medium's refractive index. They are equal in a vacuum (and nearly so in air); in a medium the optical path is longer, since the light is slower there and the path holds more wavelengths. Phase differences depend on optical paths.

c. Conditions: coherent sources (the same frequency, a constant phase difference), monochromatic light, nearly equal amplitudes, a narrow source, the path difference within the coherence length, the same polarization, the slits close together with the screen far away, a steady setup. **Temporal coherence** is needed because each source gives wave trains of limited length: if the path difference is longer than a train (the coherence length), the two parts meeting come from different, unrelated trains, their phase difference changes at random, and the fringes vanish. **Spatial coherence** is needed because a wide source is many sources side by side, each giving a pattern slightly shifted; together they blur the fringes away, so the light at the two slits must come from a narrow source, effectively one point.

d. i. If the frequencies differ slightly, the phase difference between the waves changes steadily with time, by 2π every $1/(f_1 - f_2)$ seconds. The fringes therefore **drift** across the screen continuously, one fringe each such period; unless that is very slow, they move too fast to be seen, and the pattern disappears into an even glow (like beats in sound, which you hear as a throbbing).

ii. The fringes stay in the same places, with the same width, but the **contrast falls**: the bright fringes have amplitude $a_1 + a_2$ and the dark ones $a_1 - a_2$, no longer zero, so the dark fringes are not completely dark. The fringe visibility $(I_{max} - I_{min})/(I_{max} + I_{min})$ drops below 1.

e. For a point P at a distance x from the center of the screen, the path difference is $\Delta x = S_2P - S_1P = d\sin\theta \approx xd/D$. A bright fringe forms where $\Delta x = n\lambda$: $xd/D = n\lambda$, so $x_n = n\lambda D/d$. In glass (refractive index μ) the wavelength becomes λ/μ, or equivalently the optical path difference becomes $\mu\,\Delta x$; so the bright fringes are at $x_n = n\lambda D/\mu d$, all closer to the center, and the fringe width falls to β/μ (by a third for μ = 1.5).

18.

a. **Fringe visibility** (contrast) measures how clearly the bright fringes stand out from the dark ones: $V = \dfrac{I_{max} - I_{min}}{I_{max} + I_{min}}$. It is 1 for perfect fringes with completely dark minima, and 0 when there are no fringes.

b. A dark fringe appears where the path difference is an odd number of half wavelengths, $\Delta x = (2n+1)\lambda/2$, that is a phase difference of $(2n+1)\pi$; it is completely dark when the two waves have equal amplitudes.

c. The fringe width depends on the wavelength. With one wavelength, every fringe has the same width, and many sharp fringes are seen. With **white light**, each color makes fringes of its own width; they coincide only at the center, which is white, and on each side are a few colored fringes (blue inside, red outside), after which the colors overlap and the screen looks evenly whitish: only a few fringes are seen. (White light also has a very short coherence length, so the path difference must be nearly zero.)

d. The single slit makes a **narrow source**, so that the light reaching both slits comes from practically one point and its wavefront reaches S_1 and S_2 with a fixed phase difference (zero, if they are equally far from it). Light from a wide source arrives at the slits from many points, each with its own phase difference, giving many shifted patterns that blur together. So the single slit gives the **spatial coherence** that makes S_1 and S_2 coherent.

e. $d = 0.3\text{ mm} = 3.0 \times 10^{-4}\text{ m}$, $D = 1.2\text{ m}$, $\lambda = 480\text{ nm} = 4.8 \times 10^{-7}\text{ m}$.

i. $\beta = \dfrac{\lambda D}{d} = \dfrac{4.8 \times 10^{-7} \times 1.2}{3.0 \times 10^{-4}} = 1.92 \times 10^{-3}\text{ m} = \mathbf{1.92\text{ mm}}$.

ii. $x_4 = 4\beta = 4 \times 1.92 = \mathbf{7.68\text{ mm}}$.

19. a. Let P be a point on the screen at a distance x from the center O, making an angle θ with the central line at the slits. The path difference is $S_2P - S_1P = d\sin\theta$; for a small angle, $\sin\theta \approx \tan\theta = x/D$, so the path difference is xd/D. Bright fringes are where it equals $n\lambda$: $x_n = n\lambda D/d$. The separation of neighboring bright fringes is

$$\beta = x_{n+1} - x_n = \frac{(n+1)\lambda D}{d} - \frac{n\lambda D}{d} = \frac{\lambda D}{d}$$

(and the same for dark fringes, at $(2n+1)\lambda D/2d$).

b. The waves from S_1 and S_2 overlap on the screen. Where their path difference is a whole number of wavelengths they arrive in phase and reinforce: bright fringes. Where it is an odd number of half wavelengths they arrive in antiphase and cancel: dark fringes. Since the path difference grows steadily with x, bright and dark fringes alternate at equal spacing. Increasing d makes the fringes narrower ($\beta \propto 1/d$); a longer wavelength makes them wider and a shorter one narrower ($\beta \propto \lambda$).

c. With white light, every color makes its own fringes, of width proportional to its wavelength. At the center all the colors are bright together, so the **central fringe is white**. Beside it are a few **colored fringes**, each blue-violet on its inner edge and red on its outer edge, because blue's fringes are narrower; farther out the colors overlap and the screen is evenly whitish, so only a few fringes are seen. It means the fringe positions depend on the wavelength, which is why the experiment can measure wavelengths, and why the white central fringe can be used to find the point of zero path difference.

d. The two sources must be coherent (from one narrow source slit, or a laser); the light monochromatic; the two slits narrow, close together and equally bright; the screen far away compared with d; the path difference within the coherence length; the room dark and the setup steady.

20. a. The path difference between the waves from the two slits must be a whole number of wavelengths, $d\sin\theta = n\lambda$ (n = 0, 1, 2, ...), so that they arrive in phase. It gives the directions, and so the positions on the screen, of the bright fringes: light is concentrated there, and missing from the dark fringes between.

b. A point on the screen at a height y above the center is seen from the slits at an angle θ with $\tan\theta = y/x$. The path difference is $d\sin\theta \approx d\tan\theta = dy/x$ for a small θ. Bright fringes are where $dy/x = n\lambda$, so $y_n = n\lambda x/d$. The fringe width is

$$\Delta y = y_{n+1} - y_n = \frac{x\lambda}{d}$$

c. $\Delta y = \dfrac{x\lambda}{d} = \dfrac{1.5 \times 6.0 \times 10^{-7}}{5.0 \times 10^{-4}} = 1.8 \times 10^{-3}$ m **= 1.8 mm**.

21.

a. For a point at a distance x from the center of the screen, the path difference is $d\sin\theta \approx xd/D$. Bright fringes satisfy $xd/D = n\lambda$, so $x_n = n\lambda D/d$, and $\beta = x_{n+1} - x_n = \lambda D/d$. The fringe width is directly proportional to the wavelength ($\beta \propto \lambda$) and to the screen distance ($\beta \propto$ D), and inversely proportional to the slit separation ($\beta \propto 1/$d).

b. $\beta = \dfrac{\lambda D}{d} = \dfrac{6.0\times10^{-7}\times 2}{5.0\times10^{-4}} = 2.4\times10^{-3}$ m = **2.4 mm**.

c. The fringe width **halves**, to 1.2 mm, since $\beta \propto 1/$d: the fringes come twice as close together (twice as many in the same width), and the central fringe stays at the center.

d. Constructive interference where the path difference is $n\lambda$ (phase difference $2n\pi$): the waves arrive in phase and reinforce, giving bright fringes at $x = n\lambda D/d$. Destructive interference where it is $(2n+1)\lambda/2$ (phase difference $(2n+1)\pi$): the waves arrive in antiphase and cancel, giving dark fringes at $x = (2n+1)\lambda D/2d$. As x increases the path difference increases steadily, so bright and dark fringes alternate, evenly spaced.

22.

a. When two coherent light waves overlap, their displacements add (superposition). Where they arrive in phase, the light is brighter; where in antiphase, darker or dark: this redistribution of light into bright and dark bands is interference. Conditions: coherent sources (the same frequency and a constant phase difference), monochromatic light, nearly equal amplitudes, a narrow source, the path difference within the coherence length, the same polarization, the sources close together and the screen far away.

	Constructive	Destructive
Path difference	$n\lambda$	$(2n+1)\lambda/2$
Phase difference	$2n\pi$	$(2n+1)\pi$
Waves meet	crest on crest	crest on trough
Resultant amplitude	$a_1 + a_2$	$a_1 - a_2$
Result	bright fringe	dark fringe

b. $\beta = \dfrac{\lambda D}{d} = \dfrac{5.0\times10^{-7}\times 1.5}{2.0\times10^{-4}} = 3.75\times10^{-3}$ m = **3.75 mm**.

c. $\beta = \dfrac{6.0\times10^{-7}\times 1.5}{2.0\times10^{-4}} = 4.5\times10^{-3}$ m = **4.5 mm**: the fringe separation increases by the same factor as the wavelength, 6/5. Since $\beta \propto \lambda$, longer wavelengths (toward red) give wider fringes and shorter ones (toward violet) narrower fringes.

23.

a. The **fringe width** β is the distance between the centers of two neighboring bright (or dark) fringes; all fringes have the same width, so the positions of the bright fringes are simply 0, β, 2β, ... Derivation: at a point x from the center, the path difference is $d\sin\theta \approx xd/D$; bright fringes at $xd/D = n\lambda$, $x_n = n\lambda D/d$, so $\beta = x_{n+1} - x_n = \lambda D/d$. It increases with the wavelength and the screen distance, and decreases as the slit separation increases; in a medium of refractive index μ it is β/μ.

b. $x_3 = \dfrac{3\lambda D}{d} = \dfrac{3 \times 7.0 \times 10^{-7} \times 2}{2.5 \times 10^{-4}} = 1.68 \times 10^{-2}$ m **= 16.8 mm**.

c. $x_3 = \dfrac{3 \times 8.0 \times 10^{-7} \times 2}{2.5 \times 10^{-4}} = 1.92 \times 10^{-2}$ m **= 19.2 mm**. Every fringe's distance from the center is proportional to the wavelength, so with a longer wavelength all the fringes move outwards, away from the central maximum (which stays at the center), and spread farther apart.

24.

a. **Interference** is the redistribution of light energy, into bright and dark places, where two or more coherent waves overlap and add by superposition. Conditions: coherent sources, monochromatic light, nearly equal amplitudes, a narrow source, a path difference within the coherence length, the same polarization, close slits and a distant screen. In constructive interference the waves meet in phase (path difference nλ) and reinforce, giving bright fringes; in destructive interference they meet in antiphase (path difference (2n+1)λ/2) and cancel, giving dark fringes.

b. The fringe width **doubles**, since $\beta = \lambda D/d \propto 1/d$: the fringes spread twice as far apart, so fewer fit on the screen, and with the slits closer together the central part of the pattern looks larger and easier to measure.

c. Neighboring dark fringes are one fringe width apart:

$$\beta = \frac{\lambda D}{d} = \frac{4.5 \times 10^{-7} \times 3}{1.0 \times 10^{-4}} = 1.35 \times 10^{-2} \; =$$

(The first dark fringe is at $\beta/2 = 6.75$ mm from the center and the second at $3\beta/2 = 20.25$ mm.)

25. a. $\lambda = 2.8$ cm, $d = 4.0$ cm, $D = 0.80$ m. The first maximum is where $d\sin\theta = \lambda$:

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{2.8}{4.0} = 0.70, \qquad \theta_1 = 44.4^\circ$$

This angle is far from small, so $\beta = \lambda D/d$ (which would give 0.56 m) can't be used. On the receiver's line,

$$y_1 = D\tan\theta_1 = 0.80 \times \tan 44.4^\circ = 0.80 \times 0.980 = 0.78$$

The second maximum would need $\sin\theta = 2\lambda/d = 1.4$, which is impossible, so there are only three maxima, at O and 0.78 m on either side. The separation between adjacent maxima is **0.78 m**. (The receiver is twenty slit separations away, far enough for the path difference to be $d\sin\theta$: worked out exactly from the two distances, the first maximum is also 0.78 m from O.)

b. At the first maximum the path difference is one wavelength, so the phase difference is $(2\pi/\lambda) \times \lambda = \mathbf{2\pi}$ (360°): the waves arrive in phase. (At the nth maximum it would be $2n\pi$.)

c. When the slits are far apart, the fringe width $\lambda D/d$ is very small, so the maxima are packed so closely that the receiver (or the eye) can't separate them, and the pattern looks uniform. A wide d also makes the pattern more sensitive to the width of the source (spatial coherence). When the source is not coherent, the phase difference between the waves at the two slits changes at random, so the maxima and minima move about continuously and average out: no steady pattern is seen.