Unit 9

Acoustic Phenomena

This unit online, with its interactives:
science-playground.com/s/12f9

An ambulance races down the Araniko Highway towards you, siren wailing. The moment it passes, the wail drops, as if someone had turned a knob. Nothing changed in the siren. What changed was the motion between it and your ear. A few seconds later, a fighter jet you can't yet hear tears across the sky, and then, long after it has gone over, a bang like a door slamming in the clouds.

Both are **acoustic phenomena**: things sound does on its way from a source to a listener. This unit measures sound the way your ear does, by its pressure; works out how much energy a sound wave carries and why your ear needs a logarithmic scale, the **decibel**, to cope with it; separates the physical quantities (intensity, frequency) from what we hear (loudness, pitch, quality); and ends with the **Doppler effect**, which shifts the siren's note, measures the speed of a cricket ball, the blood in a baby's heart and the galaxies running away from us.

A STORY FROM SCIENCE

The man who heard the stars move

Christian Doppler (1803–1853) was an Austrian physicist and mathematician, the son of a stonemason in Salzburg. In 1842, in Prague, he presented a paper arguing that the color of a star should depend on how it moves towards or away from us: waves from an approaching source arrive squeezed together, from a receding one stretched apart. His guess about star colors was wrong in detail (stars don't move fast enough to change color visibly), but the principle was right, and it carries his name: the **Doppler effect**.

It was tested with sound first. In 1845 the Dutch scientist **Christophorus Buys Ballot** put a group of trumpeters on an open railway carriage near Utrecht and had them hold one note as the train went by; musicians with a good ear stood beside the track and wrote down the note they heard. As the train came closer they heard it sharp, and as it left, flat, just as Doppler had said. Today the same idea measures the speeds of stars, storms, cars and blood.



Figure 9.1 *Christian Doppler (1803–1853).*
Photo: Unknown author, Public domain

9.1 Introduction

Sound waves are mechanical waves: they need a medium (a solid, a liquid or a gas) and are made by vibrating particles of that medium. As the particles crowd together and spread apart, the pressure of the medium rises and falls a little above and below its normal value, and it is these pressure changes that our ears detect as sound (Units 6 and 7).

Acoustic phenomena are the ways sound behaves: how it is produced, how it travels (is propagated), how it is reflected, refracted, diffracted, interferes and is absorbed, and how it is heard. Some are natural, like an echo across a valley or the rumble of thunder; some are made by people, like the notes of a madal or a bansuri, or the noise of a crowded bus park. They include the **Doppler effect** (a change of pitch when the source and the listener move relative to each other) and the stationary waves of musical instruments (Unit 8).

Every sound has a few measurable properties: its **pressure amplitude** and its **intensity** (which set how loud it is), its **frequency** (which sets its pitch) and its **waveform** (which sets its quality). This unit takes them one by one.

Acoustics is the branch of physics that studies sound. Hearing is one of an animal's most important ways of staying alive, and speech is one of the most distinctive things about human beings and their cultures; both are acoustic. Acoustics has several branches:

- **Architectural acoustics**: designing and controlling sound inside buildings, so that a concert hall, a theater, a classroom or an office sounds clear and

comfortable.

- **Musical acoustics**: how musical instruments make sound, and how their material, shape and size give each its tone and quality.
- **Bioacoustics**: how animals (humans included) make and hear sounds, how they communicate with them, and how sound affects their behavior and their habitats.
- **Medical acoustics**: sound used in health care, mostly ultrasound: in diagnosis (scans), in therapy (physiotherapy) and in treatment, such as **lithotripsy**, breaking up kidney stones with focused sound waves.

9.2 Pressure amplitude [maximum excess pressure] of sound wave

When a sound wave passes through air, each layer of air moves back and forth about its usual place, and the pressure at each point rises and falls above and below the normal (atmospheric) pressure. So a sound wave can be described in two ways:

- by how far the particles move: the greatest distance a particle moves from its mean position is the **displacement amplitude**, a;
- by how much the pressure changes: the greatest rise (or fall) of pressure above (or below) normal is the **pressure amplitude**.

Pressure amplitude is the maximum increase or decrease of pressure in a medium, from its normal value, as a sound wave travels through it. It is also called the **maximum excess pressure**, $(\Delta P)_{\max}$.

Our ears respond to the pressure changes: the eardrum is pushed in and out by them. So although the displacement is a perfectly good description of the wave in the medium, it is the pressure amplitude that matters for hearing, and it is the one microphones measure. The pressure varies sinusoidally, at the same frequency as the displacement. Let's find how big the pressure amplitude is.

Pressure variation in a sound wave

Take a thin cylinder of air of cross-section A and length Δx, lying along the direction in which a sound wave travels with speed v and displacement amplitude a (Figure 9.2). As the wave passes, the layer at the left end of the cylinder is displaced by y_1 and the layer at the right end by y_2.

1. The change of volume. The cylinder's volume was $V = A\,\Delta x$. Its length has changed by $\Delta y = y_2 - y_1$, so its volume has changed by $\Delta V = A\,\Delta y$. If Δy is posi-

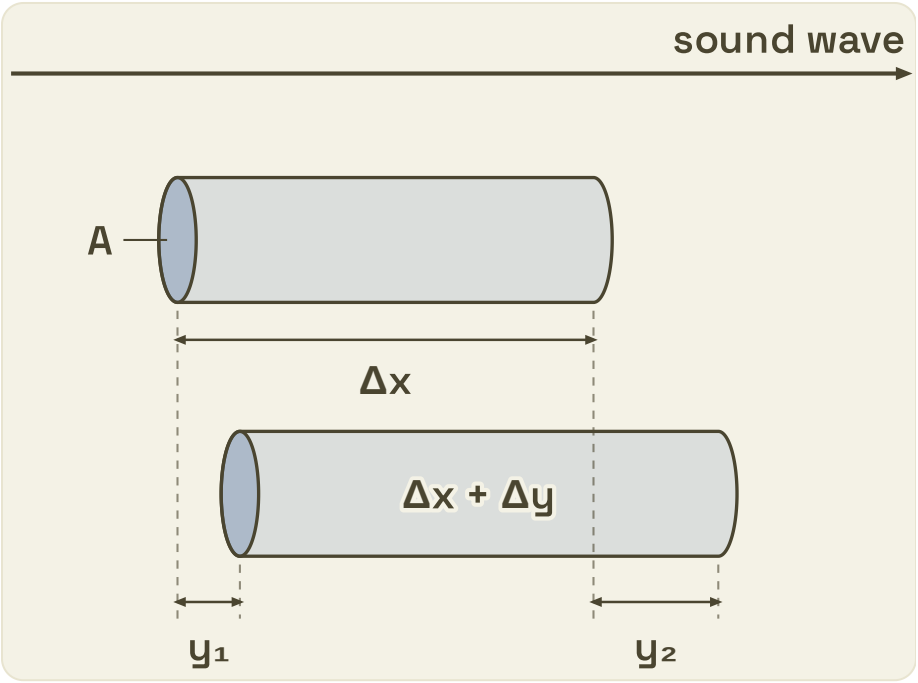


Figure 9.2 *A thin cylinder of air along the path of a sound wave. Its left end moves by y_1 and its right end by y_2, so its length changes by $\Delta y = y_2 - y_1$ and its volume by $A\,\Delta y$.*

tive the cylinder has stretched and the pressure in it falls; if negative, it has been squeezed and the pressure rises.

2. The bulk modulus links the two. The pressure change is a stress and $\Delta V/V$ a volume strain, so by the definition of the **bulk modulus** B of the air,

$$B = -\frac{\Delta P}{\Delta V / V}$$

$$\Rightarrow\ \Delta P = -B\,\frac{\Delta V}{V} \qquad (9.1)$$

The minus sign says that when the volume increases, the pressure decreases, and the other way round.

3. Strain from the displacement.

$$\frac{\Delta V}{V} = \frac{A\,\Delta y}{A\,\Delta x} = \frac{\Delta y}{\Delta x} \qquad (9.2)$$

For a very thin cylinder, $\Delta x \to 0$:

$$\lim_{\Delta x \to 0} \frac{\Delta V}{V} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \frac{dy}{dx} \qquad (9.3)$$

so from (9.1) and (9.3),

$$\Delta P = -B\,\frac{dy}{dx} \qquad (9.4)$$

4. Put in the wave. Let the displacement be

$$y = a\sin(\omega t - kx) \qquad (9.5)$$

where $\omega = 2\pi f$ is the angular frequency (f is the frequency of both the displacement and the pressure variation) and $k = 2\pi/\lambda$ the wave number. Differentiating with respect to x,

$$\frac{dy}{dx} = -ak\cos(\omega t - kx) \qquad (9.6)$$

5. The pressure. From (9.4) and (9.6),

$$\begin{aligned}\Delta P &= -B \times \big(-ak\cos(\omega t - kx)\big)\\ &= Bak\cos(\omega t - kx) \qquad (9.7)\end{aligned}$$

This is how the pressure of the medium varies as the sound wave travels. It is greatest when $\cos(\omega t - kx) = 1$, so the pressure amplitude is

$$(\Delta P)_{\max} = Bak \qquad (9.8)$$

and

$$\Delta P = (\Delta P)_{\max}\cos(\omega t - kx) \qquad (9.9)$$

6. In terms of the air's density. The speed of sound is $v = \sqrt{B/\rho}$, so $B = \rho v^2$ (ρ is the density of the air); and $v = \omega/k$, so $k = \omega/v$. Then

$$(\Delta P)_{\max} = Bak = \rho v^2\, a\,\frac{\omega}{v} = (\rho v\omega)\,a$$

$$\boxed{(\Delta P)_{\max} = \rho v\omega a = 2\pi f \rho v a} \qquad (9.10)$$

For a given medium and frequency, $\rho v\omega$ is constant, so $(\Delta P)_{\max} \propto a$: **the pressure amplitude is directly proportional to the displacement amplitude.**

Compare (9.5) and (9.7): the displacement goes as a sine, the pressure as a cosine. They are **90° ($\pi/2$) out of phase**: where the displacement is greatest the pressure change is zero, and where the displacement is zero the pressure change is greatest (Figure 9.3). Look at the layers in the figure: at a compression, the layers on either side have moved *towards* it (the one on the left moved right, the one on the right moved left), but the layer at its center hasn't moved at all.

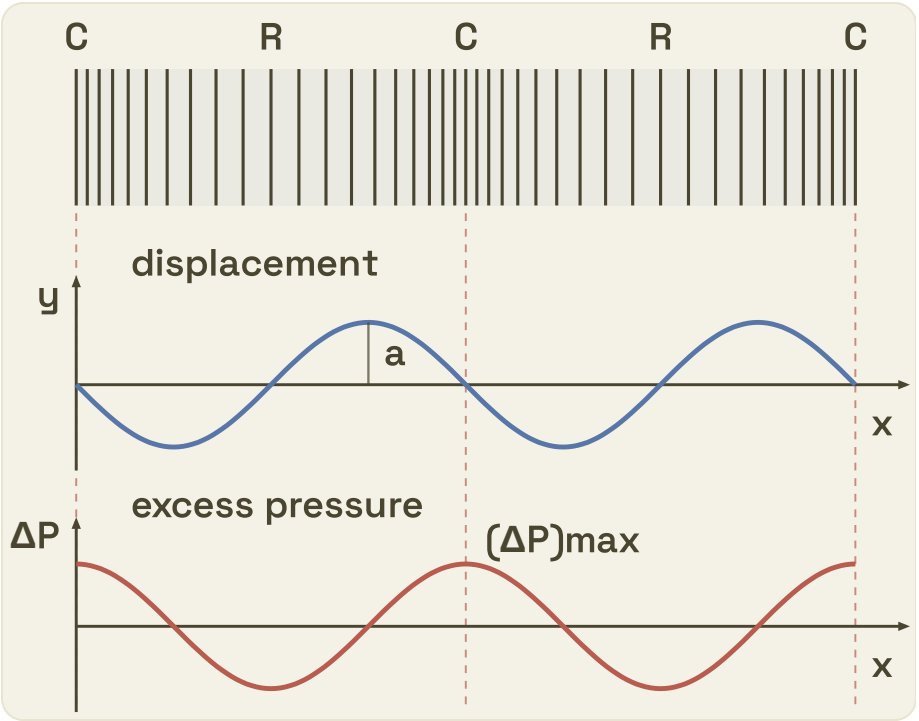


Figure 9.3 *A sound wave at one instant. Top: layers of air, crowding into compressions (C) and spreading into rarefactions (R). Middle: each layer's displacement y (positive to the right). Bottom: the excess pressure ΔP. The pressure is greatest where the displacement is zero, a quarter of a wavelength from where the displacement is greatest.*

Check 1

At a compression in a sound wave, the displacement of the air there is:

a greatest, to the right
b greatest, to the left
c zero
d equal to the pressure amplitude

DID YOU KNOW?

Your ear measures a wiggle smaller than an atom

The faintest sound you can hear at 1000 Hz has an intensity of about 10^{-12} W/m² (section 9.4). Put that into (9.16) below for air ($\rho v \approx 413$ kg/(m² s)) and the displacement amplitude comes out about 1×10^{-11} m: a tenth of the width of a hydrogen atom. Its pressure amplitude is about 3×10^{-5} Pa, three ten-billionths of the air's pressure. Even a sound loud enough to hurt (1 W/m²) has a pressure amplitude of only about 29 Pa, less than three ten-thousandths of an atmosphere.

9.3 Intensity of sound

A loud sound carries more energy than a soft one. How much energy it delivers to a surface each second is its intensity.

Intensity of sound at a point is the average rate at which sound energy flows through unit area held normal (at right angles) to the direction of the wave at that point. Its SI unit is the watt per square meter, W/m².

$$\begin{aligned} I &= \left[\frac{\text{energy/time}}{\text{area}}\right]_{\text{average}} \\ &= \left[\frac{\text{power } P}{\text{area } A}\right]_{\text{average}} \end{aligned} \qquad (9.11)$$

Intensity is proportional to the square of the wave's amplitude (shown below), and, for a source that sends sound out equally in all directions, inversely proportional to the square of the distance from it (section 9.5).

Intensity in terms of the amplitude

1. Power as force times velocity. The air on one side of the area A pushes on the air on the other side with an extra force $F = \Delta P \times A$, and that air moves with the particle velocity v_p. The power delivered is force × velocity, so

$$\begin{aligned} \frac{P}{A} &= \frac{F \times v_p}{A} \\ &= \Delta P \times v_p \\ \Rightarrow\ I &= \left[\Delta P \times v_p\right]_{\text{average}} \end{aligned} \qquad (9.12)$$

2. The pressure and the particle velocity. For the wave

$$y = a\sin(\omega t - kx) \qquad (9.13)$$

the pressure variation is (9.7),

$$\Delta P = Bak\cos(\omega t - kx) \qquad (9.14)$$

and the particle velocity is

$$v_p = \frac{dy}{dt} = a\omega\cos(\omega t - kx) \qquad (9.15)$$

(v_p is how fast the air itself moves back and forth, not the speed v of the wave.)

3. Average the product. From (9.12), (9.14) and (9.15), writing $\theta = \omega t - kx$,

$$\begin{aligned} I &= \left[Bak\cos\theta \times a\omega\cos\theta\right]_{\text{average}} \\ &= Ba^2k\omega\left[\cos^2\theta\right]_{\text{average}} \end{aligned}$$

Over a whole cycle, $\cos^2\theta$ averages $\frac{1}{2}$, so

$$I = \tfrac{1}{2}Ba^2k\omega$$

4. In terms of the air's density. With $B = \rho v^2$ and $k = \omega/v$,

$$I = \tfrac{1}{2}\rho v^2 a^2\frac{\omega}{v}\,\omega = \tfrac{1}{2}\rho v\omega^2 a^2 = \tfrac{1}{2}\rho v(2\pi f)^2 a^2$$

$$\boxed{I = 2\pi^2 f^2 a^2 \rho v} \qquad (9.16)$$

5. In terms of the pressure amplitude. Since $(\Delta P)_{\max} = \rho v\omega a$, $a = (\Delta P)_{\max}/(\rho v\omega)$, and

$$\boxed{I = \frac{(\Delta P)_{\max}^2}{2\rho v}}$$

So **for a given medium and frequency,** $I \propto a^2$: double the amplitude and the intensity becomes four times. The last form, with the pressure amplitude, is the one a microphone or a sound meter uses.

> CAREFUL
> **Two different speeds**
>
> v is the speed of the wave (about 340 m/s in air); v_p is the speed of the air molecules swinging back and forth, which for ordinary sounds is a fraction of a millimeter per second. And $I \propto a^2$ holds only at a fixed frequency: from (9.16), at the same amplitude, a wave of twice the frequency carries four times the intensity.

WORKED EXAMPLE

Example 9.1: amplitude of a note

A note of frequency 300 Hz has an intensity of 1 μW/m². What is the amplitude of the air's oscillation? [Density of air = 1.29 kg/m³, bulk modulus of air = 1.42×10^5 N/m²]

Answer

Given $I = 1\ \mu\text{W/m}^2 = 1 \times 10^{-6}$ W/m², $\rho = 1.29$ kg/m³, $f = 300$ Hz, $B = 1.42 \times 10^5$ N/m².

Since $v = \sqrt{B/\rho}$, $\rho v = \rho\sqrt{B/\rho} = \sqrt{\rho B}$, and (9.16) becomes

$$\begin{aligned} I &= \tfrac{1}{2}\sqrt{\rho B}\,\omega^2 a^2 \\ \Rightarrow\ a^2 &= \frac{2I}{\sqrt{\rho B}\,\omega^2} \\ &= \frac{2I}{\sqrt{\rho B} \times 4\pi^2 f^2} \\ &= \frac{I}{2\pi^2 f^2 \sqrt{\rho B}} \end{aligned}$$

Here $\sqrt{\rho B} = \sqrt{1.29 \times 1.42 \times 10^5} = \sqrt{1.832 \times 10^5} = 428.0$ kg/(m² s), and $2\pi^2 f^2 = 2 \times 9.870 \times 300^2 = 1.777 \times 10^6$ s⁻². So

$$\begin{aligned} a^2 &= \frac{1 \times 10^{-6}}{1.777 \times 10^6 \times 428.0} \\ &= 1.315 \times 10^{-15}\ \text{m}^2 \\ \Rightarrow\ a &= \mathbf{3.6 \times 10^{-8}\ m} \end{aligned}$$

The air moves about 36 nanometers each way, a few hundred times the width of an atom. (Check: $v = \sqrt{B/\rho} = 332$ m/s, the speed of sound in air at 0 °C.)

YOUR TURN

Pressure amplitude from the intensity

A sound in air has an intensity of 1.0×10^{-4} W/m² (a loud shout close by). Find its pressure amplitude, taking $\rho = 1.2$ kg/m³ and $v = 340$ m/s.

Answer

$$\begin{aligned}(\Delta P)_{\max} &= \sqrt{2\rho v I} \\ &= \sqrt{2 \times 1.2 \times 340 \times 10^{-4}} \\ &= \sqrt{0.0816} \\ &= \mathbf{0.29\ Pa}\end{aligned}$$

About three millionths of the air's pressure (101 325 Pa).

9.4 Threshold of hearing and threshold of pain

Threshold of hearing is the lowest intensity of sound that the average (normal) human ear can detect, at a frequency of about 1000 Hz. Its value is about

$$I_0 = 10^{-12}\ \text{W/m}^2$$

the quietest sound most people can hear under ideal conditions. It is taken as the zero of the decibel scale (0 dB, section 9.7) and as the reference for measuring sound intensity level. It is also called the **audibility threshold**.

Note: sounds of the same intensity at different frequencies don't sound equally loud: our hearing depends on the frequency as well as the intensity. The ear is most sensitive between about 2000 and 5000 Hz and much less sensitive at very low and very high frequencies, so the threshold of hearing is quoted for a pure tone of about 1000 Hz.

Threshold of pain is the intensity (or intensity level) at which sound becomes physically painful. It is taken as about 1 W/m², an intensity level of 120 dB; for some people sound becomes painful only nearer 130 dB (10 W/m²).

DID YOU KNOW?

A trillion to one

From the faintest sound you can hear (10^{-12} W/m²) to one that hurts (1 W/m²) is a factor of a million million, 10^{12}. No other sense handles such a range. That is why sound levels are measured on a scale of powers of ten, the decibel scale.

9.5 Inverse square law

A source that sends sound out equally in all directions, in open air with nothing to reflect or absorb it (a **free field**), spreads its power over the surface of a sphere that grows as the sound travels. At a distance d the sphere's area is $4\pi d^2$, so if the source's power is P,

$$I = \frac{P}{4\pi d^2}$$

Twice as far, the same power is spread over four times the area, and the intensity is a quarter; three times as far, a ninth (Figure 9.4).

Inverse square law: the intensity of sound at a point is inversely proportional to the square of the distance of the point from the source.

$$I \propto \frac{1}{d^2} \qquad \text{and} \qquad \frac{I_1}{I_2} = \frac{d_2^2}{d_1^2} \qquad (9.17)$$

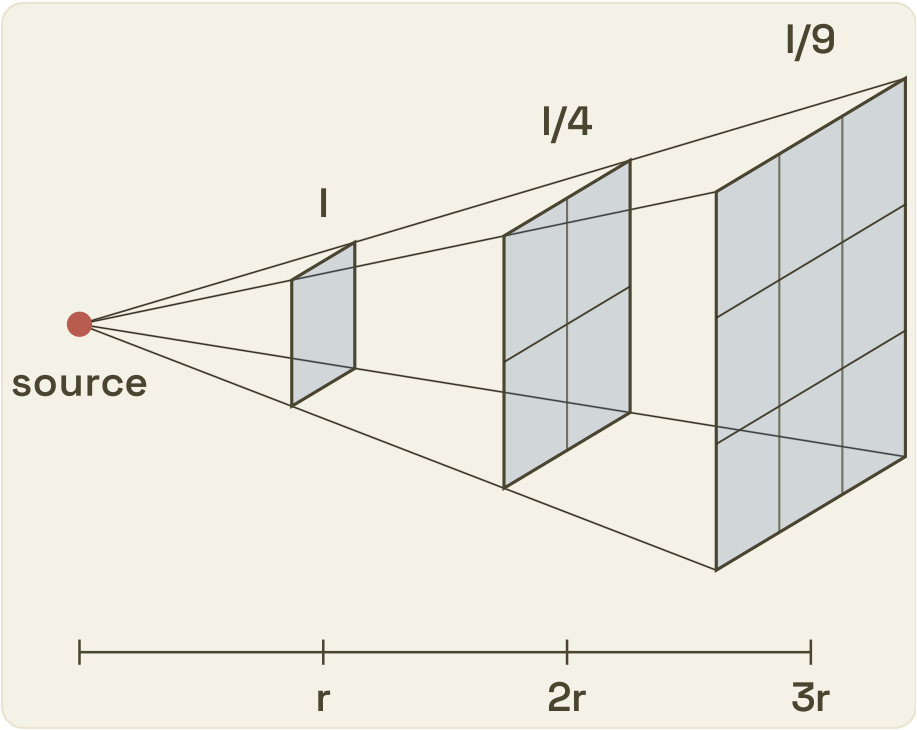


Figure 9.4 *The same sound power through a cone from the source. At distance r it crosses one square; at $2r$ the cone is twice as wide and twice as tall, so the power is spread over 4 such squares; at $3r$, over 9. The intensity at $2r$ is $I/4$, at $3r$, $I/9$.*

Check 2

You stand 2 m from a speaker and then walk back to 8 m. The intensity becomes:

a a quarter
b an eighth
c a sixteenth
d the same, since the speaker's power hasn't changed

9.6 Loudness

Loudness is a perceptual quality (a sensation of hearing) of sound: how strong or weak a sound seems to a human ear. It depends on the sound's intensity, its frequency and the sensitivity of the listener's ear.

Intensity is an objective physical quantity, measured with an instrument. Loudness is subjective: the same sound may seem louder to one person than another, and louder in a quiet room than beside a busy road. Two sounds of the same intensity at different frequencies can seem different in loudness. Loudness grows with intensity, but it is **not** proportional to it (section 9.7).

Difference between intensity and loudness

Intensity	Loudness
a physical quantity: energy per second per unit area	a sensation: how strong the sound seems to a listener
objective: the same for everyone	subjective: depends on the listener's ear and hearing
measured with an instrument (a sound meter), in W/m^2	judged by the ear; compared through the intensity level in decibels
depends only on the wave (amplitude, frequency, medium) and the distance	depends on intensity, frequency, background noise and the listener

9.7 Relation between intensity and loudness [Weber–Fechner law]

If you double the intensity of a sound, it doesn't sound twice as loud: it sounds only a little louder. Our senses respond to *ratios*: going from 1 to 10 units feels like the same step as going from 10 to 100. The **Weber–Fechner law** puts this into a formula: **the**

loudness L of a sound at a point is proportional to the logarithm of its intensity at that point.

$$L \propto \log_{10} I \quad \Rightarrow \quad L = k \log_{10} I$$

where k is a constant. Choosing the unit of loudness so that $k = 1$,

$$L = \log_{10} I \qquad (9.18)$$

If L_0 is the loudness at the threshold of hearing, I_0,

$$L_0 = \log_{10} I_0 \qquad (9.19)$$

Subtracting (9.19) from (9.18),

$$\begin{aligned} L - L_0 &= \log_{10} I - \log_{10} I_0 \\ &= \log_{10} \left(\frac{I}{I_0} \right) \qquad (9.20) \end{aligned}$$

$L - L_0$ says how far a sound's loudness is above the faintest sound we can hear. An absolute loudness L on its own means little; this difference is what is used. It is called the **intensity level** or **sound level**, β.

Intensity level (β) is the logarithm of the ratio of the intensity of a sound, I, to the threshold of hearing, I_0. Being the logarithm of a ratio of two intensities, it has no dimensions; its unit, the **bel** (B), is named after **Alexander Graham Bell**, the inventor of the telephone.

$$\beta = L - L_0 = \log_{10} \left(\frac{I}{I_0} \right) \text{ bel}$$

If $I = 10 I_0$, $\beta = \log_{10}(10 I_0 / I_0) = \log_{10} 10 = 1$ bel. So **a sound has an intensity level of one bel if its intensity is 10 times the threshold of hearing.**

The bel is too large for everyday use, so intensity levels are usually given in **decibels** (dB), one tenth of a bel:

$$\boxed{\beta = 10 \log_{10} \left(\frac{I}{I_0} \right) \text{ dB}}$$

Every **10 dB** more means **10 times** the intensity; every 20 dB, 100 times; every 3 dB, about twice ($10 \log_{10} 2 = 3.01$).

At the threshold of hearing, $I = I_0$ and $\beta = 10 \log_{10} 1 = 0$ dB.
At the threshold of pain, $I = 1$ W/m^2 and

$$\beta = 10 \log_{10} \left(\frac{1}{10^{-12}} \right) = 10 \times 12 = 120 \text{ dB}$$

Sound	Intensity level	Intensity
threshold of hearing	0 dB	10^{-12} W/m²
leaves rustling	about 20 dB	10^{-10} W/m²
a whisper	about 30 dB	10^{-9} W/m²
a quiet room at night	about 40 dB	10^{-8} W/m²
normal conversation	60–70 dB	10^{-6}–10^{-5} W/m²
busy road traffic, at the roadside	about 80 dB	10^{-4} W/m²
a bus's pressure horn close by	about 110 dB	10^{-1} W/m²
a rock concert near the speakers; threshold of pain	about 120 dB	1 W/m²
a jet engine 30 m away	about 140 dB	100 W/m²

A whisper (about 30 dB) seems soft; ordinary conversation (60–70 dB) moderately loud; a rock concert near the speakers (about 120 dB) extremely loud, at the threshold of pain.

> DID YOU KNOW?
> **How long your ears can take it**
>
> Damage to hearing depends on both the level and the time. The US National Institute for Occupational Safety and Health recommends at most 8 hours a day at 85 dB, and for every 3 dB more, half the time: 4 hours at 88 dB, 1 hour at 94 dB, 15 minutes at 100 dB. Earphones turned up high on a noisy bus can pass 100 dB. The damage, to the hair cells of the inner ear, doesn't heal.

Comparison of intensity levels of sound

If I_1 and I_2 are the intensities, and β_1 and β_2 the intensity levels, of a sound at distances d_1 and d_2 from its source,

$$\beta_1 = 10 \log_{10}\left(\frac{I_1}{I_0}\right)$$

$$\beta_2 = 10 \log_{10}\left(\frac{I_2}{I_0}\right) \qquad (9.21)$$

so the difference in level is

$$\begin{aligned}\Delta\beta &= \beta_2 - \beta_1 \\ &= 10\log_{10}\left(\frac{I_2}{I_0}\right) - 10\log_{10}\left(\frac{I_1}{I_0}\right) \\ &= 10\log_{10}\left(\frac{I_2}{I_1}\right) \qquad (9.22)\end{aligned}$$

The threshold I_0 cancels: **a difference of levels depends only on the ratio of the intensities.** By the inverse square law, $I_2/I_1 = d_1^2/d_2^2$, so

$$\begin{aligned}\Delta\beta &= \beta_2 - \beta_1 \\ &= 10\log_{10}\left(\frac{d_1^2}{d_2^2}\right) \\ &= 20\log_{10}\left(\frac{d_1}{d_2}\right)\ \text{dB} \qquad (9.23)\end{aligned}$$

Each time you double your distance from a source, the level falls by $20\log_{10} 2 = 6$ dB.

Move away from a speaker and watch both laws at once:

Check 3

A sound's intensity is made 1000 times as great. Its intensity level rises by:

a 3 dB **b** 10 dB
c 30 dB **d** 1000 dB

Check 4

Two identical machines each make 80 dB at your ear. With both running, the level is about:

a 160 dB **b** 80 dB
c 83 dB **d** 90 dB

Factors affecting loudness

1. **Amplitude of the sound:** loudness increases as the amplitude increases ($I \propto a^2$).
2. **Surface area of the sounding body:** a larger vibrating surface sets more air moving, and the sound is louder. (A tuning fork pressed on a table sounds louder; so does a guitar string, because of the guitar's body.)
3. **Distance between the source and the listener:** loudness decreases as the distance increases.
4. **Motion of the medium:** sound carried along by the wind is louder downwind and quieter upwind.

5. **Reflecting or absorbing bodies nearby:** reflections add to the loudness (a voice is louder in an empty hall), absorption reduces it (curtains, carpets, a crowd).
6. **Frequency of the sound:** at the same intensity, a sound in the range where the ear is most sensitive, about 2000–5000 Hz, seems louder than one at a low or a very high frequency.

CAREFUL

Loudness and frequency

Some books say "loudness increases with increase in frequency". That holds only from the low notes up to about 3000–4000 Hz, where the ear is most sensitive; above that, the ear grows less sensitive, and above about 20 000 Hz we hear nothing at all, however intense the sound. Write: "at the same intensity, the loudness depends on the frequency; the ear is most sensitive at about 2–5 kHz."

WORKED EXAMPLE

Example 9.2: moving closer to a loudspeaker

A loudspeaker produces a sound intensity level 8 dB above a certain reference level at a point 40 m from it. Find the intensity level at a point 30 m from the loudspeaker.

Answer

Let I_1 and I_2 be the intensities 40 m and 30 m from the speaker. By the inverse square law,

$$\frac{I_2}{I_1} = \frac{r_1^2}{r_2^2} = \left(\frac{40}{30}\right)^2$$

The level at the second point is higher than at the first by

$$\begin{aligned}\Delta\beta &= 10\log_{10}\left(\frac{I_2}{I_1}\right)\\ &= 10\log_{10}\left(\frac{40}{30}\right)^2\\ &= 20\log_{10}(1.333)\\ &= 20 \times 0.1249\\ &= 2.5\ \text{dB}\end{aligned}$$

So the intensity level at 30 m is $8 + 2.5 = \mathbf{10.5\ dB}$ above the reference level.

YOUR TURN

Test yourself 9.3: an open window

An open window has dimensions 0.5 m × 2.0 m and the intensity level at the window is 100 dB. Find the acoustic power that enters the window.

Answer

$$100 = 10\log_{10}\left(\frac{I}{10^{-12}}\right)$$
$$\Rightarrow \frac{I}{10^{-12}} = 10^{10}$$
$$\Rightarrow I = 10^{-2}\ \text{W/m}^2$$

Area $A = 0.5 \times 2.0 = 1.0\ \text{m}^2$, so power $P = IA = 10^{-2} \times 1.0 = \mathbf{0.01\ W}$ (10 mW).

YOUR TURN

Test yourself 9.4: a speaker at a rock concert

A loudspeaker at a rock concert produces an intensity of $10^{-2}\ \text{W/m}^2$ at 20 m, at a frequency of 1 kHz. Assume that the speaker spreads its energy uniformly in the forward hemisphere and that none is reflected from the ground or elsewhere.

a. What is the intensity level at 20 m?

b. What is the total acoustic power output of the speaker?

c. At what distance will the intensity be at the threshold of pain, 120 dB?

Answer

a. $\beta = 10\log_{10}\left(\frac{10^{-2}}{10^{-12}}\right) = 10\log_{10} 10^{10} = \mathbf{100\ dB}$.

b. The power spreads over a hemisphere of area $2\pi r^2$:

$$\begin{aligned} P &= I \times 2\pi r^2 \\ &= 10^{-2} \times 2\pi \times 20^2 \\ &= 10^{-2} \times 2513 \\ &= \mathbf{25\ W} \end{aligned}$$

c. 120 dB is $I = 1\ \text{W/m}^2$, 100 times the intensity at 20 m. Since $I \propto 1/r^2$, the distance is $1/\sqrt{100} = 1/10$ as much: $r = 20/10 = \mathbf{2\ m}$. (Or: $r = \sqrt{P/(2\pi I)} = \sqrt{25.13/(2\pi \times 1)} = 2.0$ m.) The frequency doesn't enter.

9.8 Pitch

Pitch is the sharpness or shrillness of a sound: how high or low it sounds. Like loudness, it is a perceptual quality, a sensation rather than a physical quantity; it can't be measured directly the way length or mass can. But it depends mainly on something we can measure, the **frequency**: a high frequency is heard as a high pitch, a low frequency as a low pitch.

Pitch lets us tell two sounds apart by their frequency. A referee's whistle (about 3000 Hz) has a high pitch and a bass drum (about 50 Hz) a low one. A mosquito's whine is high, an elephant's rumble low. Women's voices are usually higher than men's and children's higher than adults', because their vocal cords are shorter and lighter and vibrate faster. A bansuri plays higher notes as more holes are opened (a shorter air column, Unit 8), and on a guitar or sitar a thinner string sounds higher.

Difference between frequency and pitch

Frequency	Pitch
a physical quantity: the number of vibrations per second	a sensation: how high or low the sound seems
objective: the same for every listener	subjective: judged by the listener's ear
measured with instruments, in hertz (Hz)	not measured directly; compared by ear (in music, as notes)
exists for any vibration, even one we can't hear (infrasound, ultrasound)	exists only for sounds we can hear
doubling it always doubles the number	doubling the frequency raises the pitch by one *octave*, the same step whatever the starting note

Infrasonic, audible and ultrasonic sound

Infrasonic (infrasound): mechanical vibrations of frequency **below 20 Hz**, too low for human ears to hear. They are made by very large vibrating things: earthquakes, volcanoes, storms, ocean waves, even the wind over mountains. Some animals use infrasound to communicate over long distances: **elephants** (including those of Chitwan) rumble to one another across several kilometers of forest at frequencies down to about 15 Hz, and large **whales**, the blue and fin whales, call at 10–40 Hz, sounds that travel hundreds of kilometers through the ocean.

Audible sound: sound that human ears can hear, from about **20 Hz to 20 kHz** (20 000 Hz). The upper limit falls with age: many adults can't hear above about 15 kHz.

Ultrasonic (ultrasound): mechanical vibrations of frequency **above 20 kHz**, too high for human ears. They are made by small, fast-vibrating sources, such as a small quartz crystal made to vibrate by an alternating voltage (the piezoelectric effect). Many animals use ultrasound for **echolocation**, navigation, hunting and communication where seeing is hard, in the dark or under water: **bats** catch insects in the dark by their echoes, and **dolphins** and other toothed whales hunt the same way. Ultrasound is used:

1. to find the depth of the sea (echo sounding, sonar);
2. for communication between ships and submarines under water;
3. to see an unborn baby in the womb (an ultrasound scan);
4. to kill bacteria in liquids;
5. to find faults and cracks inside metals;
6. to diagnose diseases inside the body;
7. to weld plastics;
8. to clean hidden parts of instruments and jewelry (ultrasonic cleaners).

Supersonics

An object moving faster than sound in the same medium is **supersonic**: in air, faster than about 340 m/s. Fighter jets such as the F-22 Raptor, and the Concorde airliner (which flew passengers from 1976 to 2003), are supersonic. A rifle bullet and the crack of a whip's tip are too.

A source moving faster than its own sound outruns the waves it makes. Each wave starts as a sphere from where the source was when it sent it; all these spheres are left behind the source, and they pile up along a **cone** with the source at its tip (Figure 9.5). Along the cone's surface the waves arrive together and add up: a thin, intense pressure jump called a **shock wave**. When the cone sweeps over you on the ground, you hear it as a sudden, very loud bang, a **sonic boom**. The boom isn't made only at the moment the plane "breaks the sound barrier": a supersonic plane drags its cone, and its boom, along the whole of its path.

The speed of a supersonic object is given by its **Mach number**:

$$M = \frac{\text{speed of the object}}{\text{speed of sound}}$$

From Figure 9.5: in a time t the source moves $v_s t$ while its first wave grows to radius vt, and the cone's edge touches that wave, so the cone's half-angle θ has

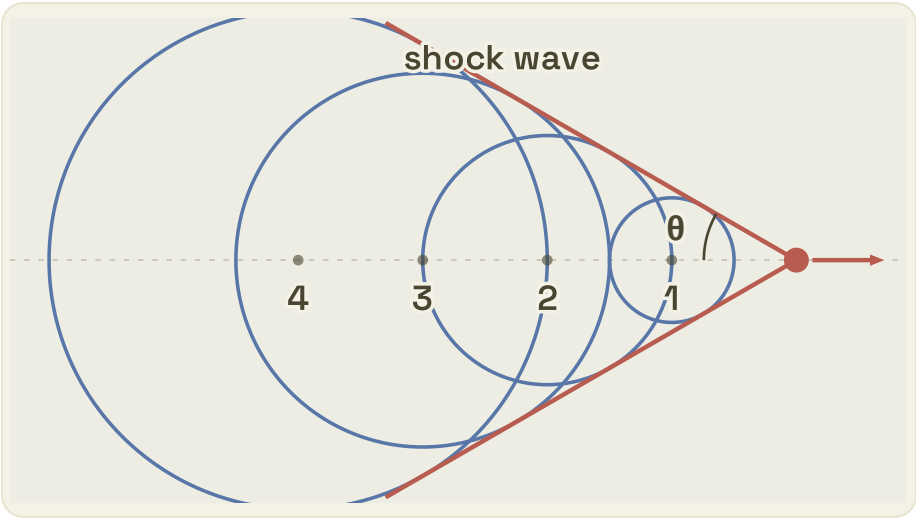


Figure 9.5 *A source moving to the right at twice the speed of sound (Mach 2). Each circle is a wave sent out when the source was at the dot of the same number, 1 to 4, and has grown since at the speed of sound. The circles crowd onto two lines, the edges of the Mach cone, the shock wave. Its half-angle θ has $\sin\theta = v/v_s = 1/M$.*

The faster the object, the narrower the cone. Supersonic objects have Mach numbers from 1 to about 5; at Mach 5 and above an object is called **hypersonic**: hypersonic missiles, and spacecraft and other vehicles re-entering Earth's atmosphere.



Figure 9.6 *An F/A-18 Hornet flying at about the speed of sound. The cloud is water vapor condensing in moist air whose pressure and temperature drop as it rushes round the plane; it clears where the air passes through a shock wave and is squeezed and warmed again.*
Photo: Ensign John Gay, U.S. Navy, Public domain

Check 5

A jet flies at Mach 2. The half-angle of its Mach cone is:

a 90°
b 60°
c 30°
d 2°

9.9 Quality (Timbre)

Why does a piano sound different from a violin when both play the same note? What gives each instrument its own sound?

Quality (or **timbre**) of a sound is the characteristic that lets us tell apart two sounds of the same pitch and loudness coming from different sources. Hardly any instrument makes a single pure frequency. Each sounds its fundamental (which sets the pitch) together with **overtones** (Unit 8), and which overtones it makes, and how strong each is, depends on the instrument, its material and shape, and how it is played (plucked, bowed, struck or blown). Together they give the wave its **shape**, its waveform. A violin and a bansuri playing the same note give different waveforms, because they produce different overtones; we hear that as their different quality.

Quality is the "color" or "texture" of sound. It is how we recognize whether a note comes from a piano, a guitar or a human voice, even when they play the same note at the same loudness; and how we recognize a voice: whether it is a man's, a woman's or a child's, and whose. If a friend rings you from an unknown number, you know who it is from the quality of the voice. **Forensic acoustics** uses the same idea: experts analyze recorded sounds to identify a speaker or a source, and to check whether a recording has been tampered with.

Hear the same note with different overtones:

Check 6

Two instruments play the same note at the same loudness, yet you can tell them apart. They differ in:

a their fundamental frequency
b their intensity
c the overtones sounding with the fundamental
d the speed of their sound in air

9.10 Doppler's effect

Have you noticed that the pitch of an ambulance's siren rises as it comes towards you and falls as it goes away?

When a source of waves and an observer move relative to each other, the frequency the observer receives differs from the frequency the source sends. **The apparent change in the frequency of a wave received by an observer, because of the relative motion of the source and the observer, is called the Doppler effect** (or **Doppler shift**). For sound it is heard as a change of pitch when a source moves towards or away from you. It is noticeable only when the relative speed is a fair fraction of the wave's speed: for sound, tens of meters per second. It happens with all waves, including light and other electromagnetic waves. It was first proposed by the Austrian physicist Christian Doppler in 1842.

When a source of sound or light moves towards an observer, the waves arrive crowded together, with a shorter wavelength and a higher frequency: a higher pitch for sound, a **blue shift** for light. When the source moves away, the waves arrive stretched apart, with a longer wavelength and a lower frequency: a lower pitch, a **red shift**. A train's whistle sounds higher as the train comes towards you, and lower once it has passed. Police radar guns use the Doppler effect to measure the speeds of vehicles: the gun sends a radio wave at a car, and from the change in the frequency of the reflected wave, it works out how fast the car is going.

> DID YOU KNOW?
>
> **Why the siren's note drops at once**
>
> If you stand right beside the road, the siren's pitch jumps from high to low just as the ambulance passes. If you stand well back, it slides down smoothly. Only the part of the ambulance's velocity along the line from it to you shifts the frequency. Beside the road, that part is the ambulance's whole speed towards you, then its whole speed away; from farther back, it changes gradually as the angle turns. Directly opposite you, the ambulance isn't coming closer or going away at all, and for that instant there is no shift.

Doppler's effect in sound

Why does a train whistle seem to change as the train moves? There are three cases of relative motion between a source of sound and a listener. Throughout, v is the speed of sound in the (still) air, f the frequency the source sends, v_s the speed of the source and v_l the speed of the listener, both along the line joining them.

(a) Source in motion, listener at rest

A source (a car blowing its horn) sends out sound of frequency f and wavelength λ, traveling at v:

$$f = \frac{v}{\lambda} \qquad (9.24)$$

The source is at S and the listener at rest at L. Now let the source move with speed v_s.

i. Source moving towards the stationary listener. In one second the source sends out f waves. The first of them travels a distance v towards the listener in that second; meanwhile the source follows it, moving v_s to S′. So the f waves that would have filled the distance v are squeezed into the distance $v - v_s$ between S′ and the front of the first wave (Figure 9.7). The waves are crowded: the wavelength ahead of the source is shorter. They still travel to the listener at the speed of sound v (the speed depends only on the air, not on the source), so if λ' is the new wavelength, the listener receives the frequency

$$f' = \frac{v}{\lambda'} \qquad (9.25)$$

where

$$\lambda' = \frac{v - v_s}{f} \qquad (9.26)$$

From (9.25) and (9.26),

$$f' = \frac{v}{(v - v_s)/f}$$

$$\Rightarrow \boxed{f' = \left(\frac{v}{v - v_s}\right) f} \qquad (9.27)$$

Since $v > v - v_s$, $f' > f$: **when a source of sound moves towards a stationary listener, the frequency heard is higher than the real frequency.**

ii. Source moving away from the stationary listener. Now the source moves away from the listener while it sends the f waves, so they are spread over a distance $v + v_s$ on the listener's side. The waves are stretched, and the listener receives

$$f' = \frac{v}{\lambda'} \qquad (9.28)$$

where the wavelength is now longer:

$$\lambda' = \frac{v + v_s}{f} \qquad (9.29)$$

so

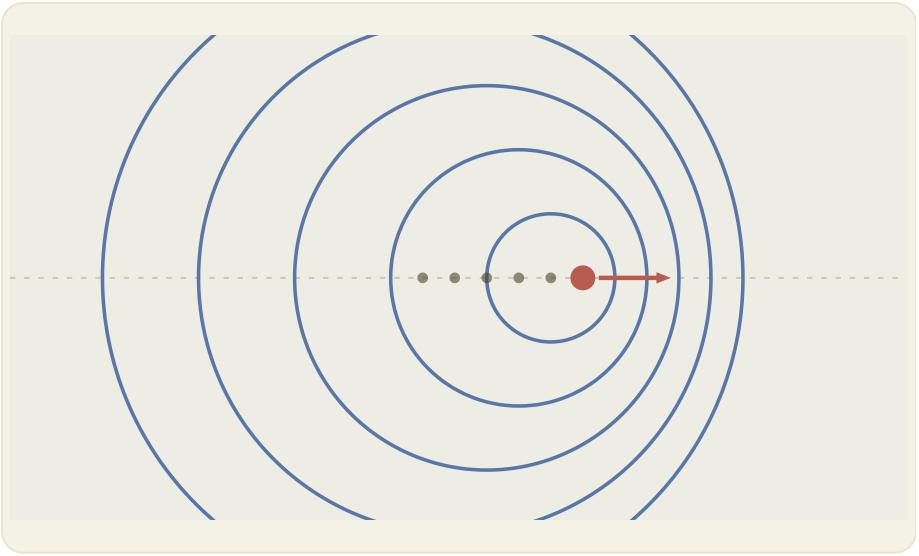


Figure 9.7 *A source moving to the right at half the speed of sound. Each circle is a wave sent out from the place, marked by a dot, where the source was then. Ahead of the source the waves are crowded ($\lambda' = (v - v_s)/f$); behind it they are spread out ($\lambda' = (v + v_s)/f$).*

Since $v < v + v_s$, $f' < f$: **when a source of sound moves away from a stationary listener, the frequency heard is lower than the real frequency.**

Watch the waves crowd and spread, and hear the note change:

Check 7

A source moving towards a listener changes the frequency the listener receives because:

a the sound travels faster, at $v + v_s$

b the waves are squeezed closer together, so the wavelength is shorter

c the source vibrates faster as it moves

d the listener's ear vibrates faster

(b) Listener in motion, source at rest

Now the source stays at S and sends out sound of frequency f and wavelength λ:

$$f = \frac{v}{\lambda} \qquad (9.31)$$

and the listener moves with speed v_l. The source isn't moving, so the waves in the air are not squeezed or stretched: the **wavelength stays** λ. What changes is how fast the waves pass the listener. In one second, a still listener meets the waves that fill a distance v: $v/\lambda = f$ of them. A listener moving towards the source also runs into the

waves that fill the extra distance v_l he covers (Figure 9.8). So the listener receives the waves at the speed of sound **relative to the listener**, v_R, and the frequency heard is

$$f' = \frac{v_R}{\lambda} \qquad (9.32)$$

more or fewer than f waves a second.

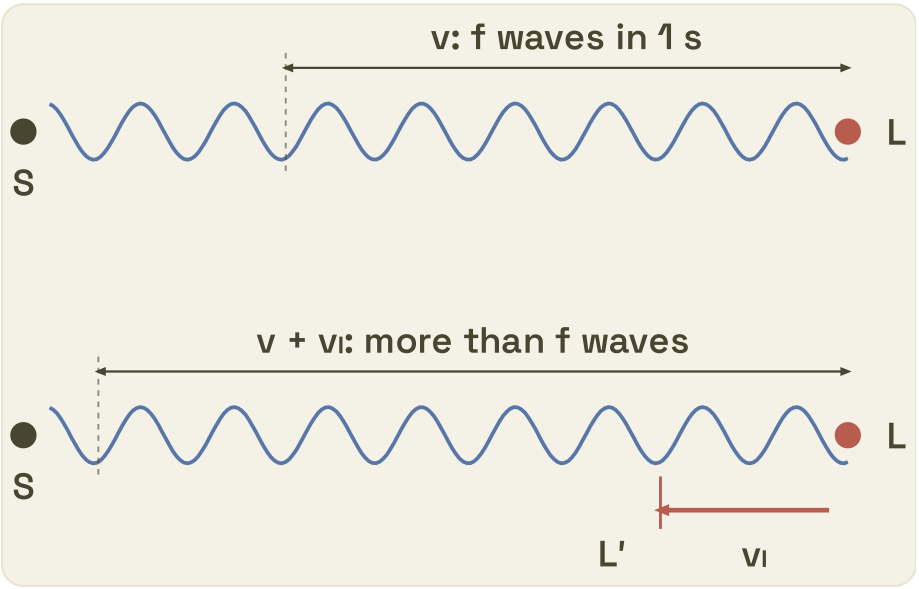


Figure 9.8 *A listener moving towards a source at rest. The waves keep their wavelength λ. In one second, a listener at rest at L is passed by the waves in a length v of air: f waves. Moving towards the source at vl, he reaches L′ in that second and meets the waves in a length v + vl: more than f waves. (vl is drawn a third of v, far faster than any real listener, so that it shows.)*

i. Listener moving towards the stationary source. The sound and the listener move in opposite directions, so the speed of the sound relative to the listener is

$$v_R = v - (-v_l) = v + v_l \qquad (9.33)$$

From (9.31), (9.32) and (9.33),

$$\begin{aligned} f' &= \frac{v + v_l}{\lambda} \\ &= \frac{v + v_l}{v/f} \\ \Rightarrow \boxed{f' &= \left(\frac{v + v_l}{v}\right) f} \end{aligned} \qquad (9.34)$$

Since $v + v_l > v$, $f' > f$: **when a listener moves towards a stationary source, the frequency heard is higher than the real frequency.**

ii. Listener moving away from the stationary source. The sound and the listener move in the same direction, so

$$v_R = v - v_l \qquad (9.35)$$

and

$$\begin{aligned} f' &= \frac{v - v_l}{\lambda} \\ &= \frac{v - v_l}{v/f} \\ \Rightarrow \boxed{f' &= \left(\frac{v - v_l}{v}\right) f} \end{aligned} \qquad (9.36)$$

Since $v - v_l < v$, $f' < f$: **when a listener moves away from a stationary source, the frequency heard is lower than the real frequency.**

> CAREFUL
> **Source moving and listener moving are not the same**
>
> A source moving at 30 m/s towards you and you moving at 30 m/s towards it don't give the same frequency: with $v = 340$ m/s and $f = 500$ Hz, they give 548.4 Hz and 544.1 Hz (Example 9.3). A moving source changes the **wavelength** in the air; a moving listener changes the **speed of the waves relative to him**. Sound has a medium, the air, and what counts is how each moves through it.

(c) Both source and listener in motion

When the source and the listener both move, both changes happen at once: the source's motion changes the wavelength reaching the listener, and the listener's motion changes the speed of the waves relative to him. The frequency heard is then

$$f' = \frac{v_R}{\lambda'} \qquad (9.37)$$

where v_R is the speed of the sound relative to the listener and λ' the wavelength of the waves reaching the listener.

i. Source and listener moving towards each other. The listener moves towards the source, against the sound: $v_R = v - (-v_l) = v + v_l$. The source moves towards the listener, so the waves are crowded: $\lambda' = \dfrac{v - v_s}{f}$. From (9.37),

$$f' = \frac{v + v_l}{(v - v_s)/f} = \left(\frac{v + v_l}{v - v_s}\right) f$$

Since $v + v_l > v - v_s$, $f' > f$: when the source and listener approach each other, the frequency heard is higher than the real frequency.

ii. Source and listener moving away from each other. The listener moves away from the source, with the sound: $v_R = v - v_l$. The source moves away, so the waves are stretched: $\lambda' = \dfrac{v + v_s}{f}$. So

$$f' = \frac{v - v_l}{(v + v_s)/f} = \left(\frac{v - v_l}{v + v_s}\right) f$$

Since $v - v_l < v + v_s$, $f' < f$: when they move apart, the frequency heard is lower.

iii. Source following the listener. Both move the same way, the source behind. The source moves towards the listener (waves crowded, $\lambda' = \dfrac{v - v_s}{f}$); the listener moves away from the source, with the sound ($v_R = v - v_l$):

$$f' = \frac{v - v_l}{(v - v_s)/f} = \left(\frac{v - v_l}{v - v_s}\right) f$$

If $v_l < v_s$ (the source is faster, catching up), then $v - v_l > v - v_s$ and $f' > f$: the frequency heard is higher. If $v_l > v_s$ (the listener is faster, pulling away), then $v - v_l < v - v_s$ and $f' < f$: it is lower.

iv. Listener following the source. Both move the same way, the listener behind. The listener moves towards the source, against the sound ($v_R = v + v_l$); the source moves away from the listener (waves stretched, $\lambda' = \dfrac{v + v_s}{f}$):

$$f' = \frac{v + v_l}{(v + v_s)/f} = \left(\frac{v + v_l}{v + v_s}\right) f$$

If $v_l > v_s$ (the listener is catching up), then $v + v_l > v + v_s$ and $f' > f$. If $v_l < v_s$ (the source is pulling away), then $v + v_l < v + v_s$ and $f' < f$.

In iii and iv, if the two move at the same speed, $f' = f$: the distance between them doesn't change, and there is no Doppler effect.

One formula for every case

All the cases above are one expression:

$$\boxed{f' = \left(\frac{v \pm v_l}{v \mp v_s}\right) f}$$

Use the **upper** signs ($+v_l$, $-v_s$) for motion **towards** the other one, and the **lower** signs ($-v_l$, $+v_s$) for motion **away** from it. A quick check: anything moving towards the other one must raise the frequency.

CAREFUL

Signs in the Doppler formula

The commonest slip is a sign. Don't memorize which sign goes where: ask, for the source and for the listener separately, "is it moving towards the other, or away?" Towards always raises f': a plus on top for the listener, a minus underneath for the source. And the speeds are measured relative to the air: with a wind, use the speeds relative to the moving air.

Limitations of Doppler's effect

1. If the source and the observer are both at rest, or move in the same direction at the same speed, there is no change in frequency or wavelength, and no Doppler effect. The same holds whenever the distance between them isn't changing, as when one moves at right angles to the line joining them.
2. For sound, the shift depends on the medium: the speed of sound changes with temperature and humidity, and a wind changes the speeds of the source and listener relative to the air. Changes in these conditions change the shift.
3. The formulas hold only while the source moves slower than sound. An object moving faster than sound makes a shock wave and a sonic boom; a listener ahead of it hears nothing until the cone arrives.
4. The formulas assume steady motion along the line joining the source and the listener. If the motion is along a curve or speeding up, or not along that line (a car passing at a distance, an object moving in a circle), the shift changes from moment to moment and must be worked out at each point from the part of the velocity along the line.

Doppler's effect in electromagnetic waves [light]

Light from a star or a galaxy is shifted too. When it moves away from the Earth, the dark lines in its spectrum are shifted towards the red end (longer wavelengths): a **red shift**. When it moves towards us, they are shifted towards the violet end (shorter wavelengths): a **blue shift**. Measuring the shift gives the speed of the star along our line of sight.

Light needs no medium, and only the motion of the source and the observer *relative to each other* matters. For speeds much less than the speed of light c, the result is like (9.29). Let λ be the wavelength a star would send if it were at rest relative to us, and λ' the wavelength we receive while it moves **away** from us with speed v_s:

WORKED EXAMPLE

Example 9.3: three ways to approach

A source of sound sends out waves that travel at 340 m/s. The frequency of the source is 500 Hz. Find the frequency of the sound heard if:

a. the source moves towards a stationary observer at 30 m/s;
b. the observer moves towards a stationary source at 30 m/s;
c. both source and observer move at 30 m/s and approach one another.

Answer

Given $v = 340$ m/s, $f = 500$ Hz.

a. Source towards a listener at rest, $v_s = 30$ m/s:

$$\begin{aligned} f' &= \left(\frac{v}{v - v_s}\right) f \\ &= \frac{340}{340 - 30} \times 500 \\ &= \frac{340}{310} \times 500 \\ &= \mathbf{548.4\ Hz} \end{aligned}$$

b. Listener towards a source at rest, $v_l = 30$ m/s:

$$\begin{aligned} f' &= \left(\frac{v + v_l}{v}\right) f \\ &= \frac{340 + 30}{340} \times 500 \\ &= \frac{370}{340} \times 500 \\ &= \mathbf{544.1\ Hz} \end{aligned}$$

c. Both approaching, $v_s = v_l = 30$ m/s:

$$\begin{aligned} f' &= \left(\frac{v + v_l}{v - v_s}\right) f \\ &= \frac{340 + 30}{340 - 30} \times 500 \\ &= \frac{370}{310} \times 500 \\ &= \mathbf{596.8\ Hz} \end{aligned}$$

So the frequencies heard are 548.4 Hz, 544.1 Hz and 596.8 Hz.

Knowing λ (from the same line measured in a laboratory) and the shift $\Delta\lambda = \lambda' - \lambda$, we find the star's speed. A positive $\Delta\lambda$ (red shift) means it is receding; a negative one

YOUR TURN

Test yourself 9.7: the train and the cliff

A train approaches a tunnel in a cliff face at 95 km/h, its locomotive's whistle sounding at 100 Hz. What frequency does the driver hear in the echo from the cliff face? And what would it be if the train were coming out of the tunnel at the same speed? (Speed of sound in air = 330 m/s.)

Hint

The cliff acts like a mirror: the echo comes from an image of the whistle, behind the cliff, moving with the train's speed. Or do it in two steps: the cliff first hears the whistle as a listener, then sends that frequency back as a source at rest.

Answer

Speed of the train $u = 95 \times \dfrac{1000}{3600} = 26.39$ m/s.

Approaching the cliff. The cliff, at rest, receives $f_1 = \dfrac{v}{v-u} f$ from the approaching whistle and reflects it; the driver moves towards the cliff and hears $f' = \dfrac{v+u}{v} f_1$. Together,

$$\begin{aligned} f' &= \left(\frac{v+u}{v-u}\right) f \\ &= \frac{330 + 26.39}{330 - 26.39} \times 100 \\ &= \frac{356.39}{303.61} \times 100 \\ &= \mathbf{117.4\ Hz} \end{aligned}$$

Coming out of the tunnel (moving away from the cliff):

$$\begin{aligned} f' &= \left(\frac{v-u}{v+u}\right) f \\ &= \frac{303.61}{356.39} \times 100 \\ &= \mathbf{85.2\ Hz} \end{aligned}$$

(blue shift), approaching, and then the same formula gives a negative v_s. (Only for $v_s \ll c$; near the speed of light, relativity changes the formula.)

Light from distant galaxies is red-shifted ($\lambda' > \lambda$): the galaxies are moving away from us, and the farther away a galaxy, the faster it recedes. This is the evidence that **the universe is expanding**.

YOUR TURN

A receding galaxy

The red line of hydrogen, 656.3 nm in the laboratory, is seen at 662.9 nm in the light of a galaxy. Is the galaxy approaching or receding, and how fast?

Answer

$\lambda' > \lambda$: a red shift, so the galaxy is **receding**.

$$\begin{aligned} v_s &= \frac{\lambda' - \lambda}{\lambda}\, c \\ &= \frac{662.9 - 656.3}{656.3} \times 3 \times 10^8 \\ &= \frac{6.6}{656.3} \times 3 \times 10^8 \\ &= \mathbf{3.0 \times 10^6\ m/s} \end{aligned}$$

about 3000 km/s, 1% of the speed of light.

Doppler's effect in the case of an echo

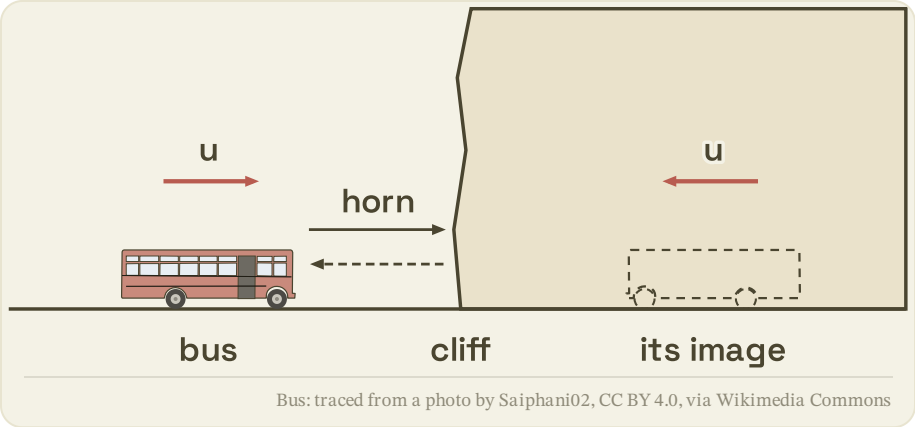
i. Source and listener moving towards the reflector. A bus on a mountain road drives towards a tall cliff, sounding its horn. The source (the horn) and the listener (the driver) are both in the bus. The cliff reflects the sound as a plane mirror reflects light: the echo seems to come from an **image** of the bus behind the cliff face, as far behind it as the bus is in front (Figure 9.9). As the bus moves towards the cliff, this image moves towards the cliff from the other side at the same speed. So it is as if the source (the image) and the listener (the driver) were approaching each other, both at the bus's speed u:

$$f' = \left(\frac{v + v_l}{v - v_s}\right) f = \left(\frac{v + u}{v - u}\right) f$$

The same result comes in two steps: the cliff, at rest, receives $\dfrac{v}{v-u} f$ and sends back that frequency, which the driver, moving towards it, hears as $\dfrac{v+u}{v}$ times as much. **Note:** the source and the listener can also be in different vehicles moving at different speeds; then v_s is the source's speed and v_l the listener's.

ii. Source and listener moving away from the reflector. If the bus drives away from the cliff, the image moves away from the cliff too, at the same speed: it is as if source and listener were moving apart, both at u:

$$f' = \left(\frac{v - v_l}{v + v_s}\right) f = \left(\frac{v - u}{v + u}\right) f$$



Bus: traced from a photo by Saiphani02, CC BY 4.0, via Wikimedia Commons

Figure 9.9 *The echo from a cliff seems to come from an image of the bus, as far behind the cliff face as the bus is in front of it. As the bus drives towards the cliff at speed u, its image comes towards the cliff at u too: source and listener approach each other.*

Applications of the Doppler effect

1. **Radar and speed detection**: police radar guns.
2. **Medical ultrasound**: Doppler ultrasound shows how fast and which way blood flows in arteries and veins.
3. **Astronomy**: measuring the red and blue shifts of stars and galaxies, and so their speeds towards or away from us (and finding planets round other stars from the wobble they give their star).
4. **Weather forecasting**: Doppler radar tracks storms, rain and the winds inside them.
5. **Sonar systems**: detecting objects under water, and their speeds.
6. **Traffic monitoring**: measuring the speeds of vehicles with Doppler radar.
7. **Navigation systems**: satellite navigation (GPS) uses the Doppler shift of the satellites' signals to find a receiver's velocity and help fix its position.
8. **Acoustic studies**: measuring the speed of sound in different conditions, and the speeds of moving air or water.
9. **Echocardiography**: Doppler ultrasound of the heart measures how fast blood flows through its chambers and valves.

WORKED EXAMPLE

Example 9.4: a motion detector

A stationary motion detector sends sound waves of 150 kHz towards a truck approaching at 120 km/h. What is the frequency of the wave reflected back to the detector? [Speed of sound in air = 340 m/s]

Answer

Given $f = 150$ kHz $= 150\,000$ Hz, speed of the truck $u = 120 \times \dfrac{1000}{3600} = 33.33$ m/s, $v = 340$ m/s.

First step: the detector is the source and the truck the listener. The truck, moving towards the detector, receives

$$\begin{aligned} f' &= \left(\frac{v+u}{v}\right) f \\ &= \frac{340 + 33.33}{340} \times 150 \\ &= \frac{373.33}{340} \times 150 \\ &= 164.7 \text{ kHz} \end{aligned}$$

Second step: the truck is the source and the detector the listener. The truck reflects this frequency back while moving towards the detector:

$$\begin{aligned} f'' &= \left(\frac{v}{v-u}\right) f' \\ &= \frac{340}{340 - 33.33} \times 164.7 \\ &= \frac{340}{306.67} \times 164.7 \\ &= \mathbf{182.6\ kHz} \end{aligned}$$

(In one step: $f'' = \dfrac{v+u}{v-u} f = \dfrac{373.33}{306.67} \times 150 = 182.6$ kHz.) The detector compares the echo's 182.6 kHz with the 150 kHz it sent; the difference, 32.6 kHz, gives the truck's speed.

Check 8

A bat flying at 10 m/s towards a wall sends out 40 kHz. The echo it hears from the wall is (speed of sound 340 m/s):

a 40 kHz
b 41.2 kHz
c 42.4 kHz
d 37.6 kHz

DID YOU KNOW?

The speed of a cricket ball

A radar speed gun, of the kind long used to time fast bowlers, sends radio waves at the ball and measures the tiny shift in the reflected wave: for a ball at 150 km/h and a radar at 10 GHz, the reflected wave comes back about 2800 Hz higher, a shift of less than one part in three million.

IN SHORT

In short

- Pressure amplitude: $(\Delta P)_{\max} = Bak = \rho v \omega a \propto a$; the pressure is 90° out of phase with the displacement.
- Intensity: $I = 2\pi^2 f^2 a^2 \rho v = \dfrac{(\Delta P)^2_{\max}}{2\rho v}$, in W/m²; $I \propto a^2$, and from a point source $I = P/4\pi d^2 \propto 1/d^2$.
- Threshold of hearing $I_0 = 10^{-12}$ W/m² (0 dB, at about 1 kHz); threshold of pain about 1 W/m² (120 dB).
- Intensity level: $\beta = 10 \log_{10}(I/I_0)$ dB; $\Delta\beta = 10 \log_{10}(I_2/I_1) = 20 \log_{10}(d_1/d_2)$; 10 times the intensity is +10 dB, twice is +3 dB.
- Loudness, pitch and quality are sensations; intensity, frequency and waveform (the overtones) are what they depend on.
- Infrasound below 20 Hz, audible 20 Hz–20 kHz, ultrasound above 20 kHz. Mach number = speed/speed of sound; Mach cone $\sin\theta = 1/M$.
- Doppler effect: $f' = \left(\dfrac{v \pm v_l}{v \mp v_s}\right) f$, upper signs for towards. A moving source changes the wavelength; a moving listener, the speed of the waves relative to him.
- Echo from a reflector approached at u: $f' = \dfrac{v+u}{v-u} f$. Light: $v_s = (\Delta\lambda/\lambda)c$, red shift for receding.

Exercise

Choose the correct option

Check 9

1. The intensity of a sound wave is directly proportional to which of the following?

a amplitude
b square of amplitude
c frequency
d wavelength

Check 10

2. The pitch of a sound depends primarily on:

a frequency
b intensity
c loudness
d quality

Check 11

3. If the intensity of a sound wave is increased by a factor of 100, how much does the sound level in decibels (dB) increase?

a 10 dB
b 20 dB
c 50 dB
d 100 dB

Check 12

4. A source emits a sound of frequency 500 Hz. If the source moves towards a stationary observer with a speed of 50 m/s (speed of sound in air = 340 m/s), what is the frequency heard by the observer?

The textbook prints the options 525, 540, 580 and 620 Hz, but the answer is 586 Hz, so the book's options don't work. Option c is corrected.

a 525 Hz
b 540 Hz
c 586 Hz
d 620 Hz

Check 13

5. An observer moves with a speed of 20 m/s towards a stationary sound source of frequency 400 Hz. What frequency does the observer perceive? (Speed of sound in air = 340 m/s)

a 420 Hz
b 440 Hz
c 460 Hz
d 480 Hz

Check 14

6. A train moving at 30 m/s blows a whistle at 600 Hz. What frequency is heard by a stationary observer standing in front of the train? (Speed of sound in air = 340 m/s)

a 640 Hz
b 660 Hz
c 680 Hz
d 700 Hz

Check 15

7. Two trains approach each other, each moving at 20 m/s. One train blows a whistle of 500 Hz. What is the frequency heard by the driver of the other train? (Speed of sound in air = 340 m/s)

The textbook prints the options 520, 550, 580 and 600 Hz, but the answer is 562.5 Hz, so the book's options don't work. Option b is corrected.

a 520 Hz
b 562.5 Hz
c 580 Hz
d 600 Hz

Check 16

8. A person hears a sound of frequency 450 Hz. If the source is moving away with a speed of 30 m/s, what was the original frequency of the sound? (Speed of sound in air = 340 m/s)

The textbook prints the options 480, 470, 460 and 450 Hz, but the answer is 490 Hz, so the book's options don't work. Option a is corrected.

a 490 Hz
b 470 Hz
c 460 Hz
d 450 Hz

Check 17

9. A supersonic jet moves towards an observer at a speed of 400 m/s while emitting a sound of frequency 1000 Hz. What frequency does the observer hear? (Speed of sound in air = 340 m/s)

a 2000 Hz
b 3000 Hz
c 4000 Hz
d no sound is heard

Check 18

10. A stationary observer hears a frequency of 550 Hz from a moving car. If the actual frequency of the horn is 500 Hz, what is the speed of the car? (Speed of sound in air = 340 m/s)

The textbook prints the options 30, 40, 50 and 60 m/s, but the answer is 30.9 m/s, about 31 m/s, so the book's options don't fit. Option a is corrected.

a 31 m/s **b** 40 m/s
c 50 m/s **d** 60 m/s

Check 19

11. A police speed detector sends a sound wave of frequency 1000 Hz towards a moving car, which reflects the wave back. If the car is approaching at 30 m/s and the speed of sound is 340 m/s, what is the frequency of the reflected wave received by the police?

The textbook prints the options 1088, 1094, 1100 and 1110 Hz, but the answer is 1194 Hz, so the book's options don't work. Option d is corrected.

a 1088 Hz **b** 1094 Hz
c 1100 Hz **d** 1194 Hz

Check 20

12. A ship heading straight towards an underwater cliff sends an ultrasonic wave of frequency 40 kHz ahead of it, towards the cliff. If the reflected wave is received with a frequency shift of 400 Hz due to the motion of the ship, what is the speed of the ship? (Speed of sound in seawater = 1500 m/s)

a 5 m/s **b** 7.5 m/s
c 10 m/s **d** 12.5 m/s

Answer these questions

13.

a. Define pressure amplitude. How does it relate to sound wave propagation?

b. A sound wave has a pressure amplitude of 0.5 Pa and travels through air of density 1.2 kg/m^3 at 340 m/s. Find the intensity of the wave.

14.

a. Differentiate between intensity and loudness of sound. How are they measured?

b. A sound of intensity $1.0 \times 10^{-8}\ \text{W/m}^2$ is increased to $1.0 \times 10^{-5}\ \text{W/m}^2$. By how many decibels has the loudness increased?

15.

a. Explain the concepts of pitch and quality of sound. Give examples of how different instruments produce sound with different quality.

b. A source of sound emits uniformly in all directions with a total power of 2 W. What is the intensity at a distance of 3 m?

16.

a. How does the intensity of sound vary with distance from the source?

b. The sound from a speaker has an intensity of $1.0 \times 10^{-6}\ \text{W/m}^2$ at a distance of 2 m. What is the intensity at 4 m from the source?

17.

a. Discuss how the human ear perceives loudness and its logarithmic relation with intensity.

b. State and explain the Doppler effect. What factors influence the observed frequency?

c. An observer moves towards a stationary source of frequency 600 Hz with a speed of 20 m/s. What is the frequency heard? ($v = 340$ m/s)

18.

a. Derive the expression for the Doppler effect when: i. the source is moving and the observer at rest; ii. the observer is moving and the source at rest; iii. both are moving.

b. The source and the observer move towards each other. The source emits sound of frequency 440 Hz and moves at 30 m/s; the observer moves at 20 m/s. Find the apparent frequency. ($v = 340$ m/s)

19.

a. What is the significance of the Doppler effect in real life? Mention at least three applications.

b. A source of sound approaches a stationary observer. What happens to the frequency heard? Explain why.

c. A source emitting sound of frequency 400 Hz is moving away from an observer at rest with a speed of 40 m/s. What frequency is heard? ($v = 340$ m/s)

20.

a. What is the condition under which no Doppler shift is observed even if there is relative motion?

b. A whistle of frequency 500 Hz moves towards a stationary observer with speed 30 m/s. The speed of sound is 340 m/s. What frequency does the observer hear?

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.