Unit 9

Answers: Acoustic Phenomena

Quick checks

1 c Pressure and displacement are 90° out of phase: $\Delta P = -B\,dy/dx$ is largest where the displacement curve is steepest, which is where it crosses zero. The air on each side has moved in towards the compression; the air at its center stays put.

2 c The distance is 4 times as much, so the intensity is $1/4^2 = 1/16$. The power is the same, but it is spread over 16 times the area.

3 c $\Delta\beta = 10\log_{10} 1000 = 10 \times 3 = 30$ dB. Every factor of 10 adds 10 dB.

4 c Intensities add, not levels: twice the intensity is $10\log_{10} 2 = 3$ dB more, 83 dB. To reach 90 dB you would need ten such machines.

5 c $\sin\theta = 1/M = 1/2$, so $\theta = 30^\circ$. At Mach 1 the "cone" would be flat (90°); the faster the jet, the narrower the cone.

6 c The same note means the same fundamental (pitch); the same loudness means about the same intensity; all sounds travel at the same speed in the same air. What differs is the set of overtones and their strengths, which gives each a different waveform: its quality, or timbre.

7 b Once sent out, sound travels at v, set by the air, however the source moves. The source chases its own waves, so each new wave is sent from a little nearer the listener: they arrive closer together, $\lambda' = (v - v_s)/f$, at the same speed, so more of them arrive each second.

8 c Source and listener (both the bat) approach the image in the wall: $f' = \dfrac{340 + 10}{340 - 10} \times 40 = \dfrac{350}{330} \times 40 = 42.4$ kHz. 41.2 kHz is only one of the two steps (a still bat hearing a moving one, or the other way round). Bats use this shift to sense how fast they are closing in on an insect.

9 b $I = 2\pi^2 f^2 a^2 \rho v$: at a given frequency, $I \propto a^2$. (It also goes as f^2 at a fixed amplitude, not as f.)

10 a A higher frequency is heard as a higher pitch. Intensity and loudness are about how strong the sound is; quality about its overtones.

11 b $\Delta\beta = 10\log_{10} 100 = 10 \times 2 = 20$ dB.

12 c $f' = \dfrac{v}{v - v_s} f = \dfrac{340}{290} \times 500 = 586.2$ Hz.

13 a $f' = \dfrac{v + v_l}{v} f = \dfrac{360}{340} \times 400 = 423.5$ Hz, about 420 Hz.

14 **b** $f' = \dfrac{v}{v - v_s} f = \dfrac{340}{310} \times 600 = 658.1$ Hz, about 660 Hz.

15 **b** Both approach: $f' = \dfrac{v + v_l}{v - v_s} f = \dfrac{360}{320} \times 500 = 562.5$ Hz.

16 **a** $f' = \dfrac{v}{v + v_s} f$, so $f = f' \times \dfrac{v + v_s}{v} = 450 \times \dfrac{370}{340} = 489.7$ Hz, about 490 Hz. The source is receding, so the sound heard is lower than the original: the original must be above 450 Hz.

17 **d** The jet moves faster than its sound, so it arrives before any of the sound it sends while approaching; the formula $vf/(v - v_s)$ gives a negative number, a sign it doesn't apply. The observer hears nothing while the jet approaches, then the sonic boom as the Mach cone passes over. Only after that does the jet's sound arrive: the note it sends as it goes, lowered, and, at the same time, the sound it sent while approaching, which arrives late and backwards (last-sent first).

18 **a** The frequency heard is higher, so the car approaches: $550 = \dfrac{340}{340 - v_s} \times 500$, so $340 - v_s = \dfrac{340 \times 500}{550} = 309.1$ and $v_s = 30.9$ m/s, about 31 m/s.

19 **d** Two steps. The car, as a listener moving towards the detector, receives $\dfrac{370}{340} \times 1000 = 1088$ Hz; as a source moving towards the detector, it sends that back, heard as $\dfrac{340}{310} \times 1088 = 1194$ Hz. In one step, $\dfrac{v + u}{v - u} f = \dfrac{370}{310} \times 1000 = 1193.5$ Hz. 1088 Hz is only the first step. (Real police radar uses radio waves, with the same two-step shift.)

20 **b** The echo from a reflector approached at u is $f' = \dfrac{v + u}{v - u} f$, so the shift is

$$\Delta f = f' - f = \frac{2u}{v - u} f$$

$$\Rightarrow \; 400 = \frac{2u}{1500 - u} \times 40\,000$$

$2u = 0.01(1500 - u)$, $2.01u = 15$, $u = 7.46$ m/s, about 7.5 m/s. (For $u \ll v$, $\Delta f \approx 2uf/v$ gives $u = 400 \times 1500/(2 \times 40\,000) = 7.5$ m/s.)

13. a. The **pressure amplitude** is the greatest increase or decrease of the pressure of the medium, from its normal value, as a sound wave travels through it, $(\Delta P)_{\max}$. As sound propagates, the layers of the medium are displaced, crowding into compressions (pressure above normal) and spreading into rarefactions (below normal); the pressure at each point varies sinusoidally at the sound's frequency, $\Delta P = (\Delta P)_{\max}\cos(\omega t - kx)$, 90° out of phase with the displacement. It is proportional to the displacement amplitude, $(\Delta P)_{\max} = BAk = \rho v \omega a$, and it fixes the energy the wave carries: $I = (\Delta P)_{\max}^2/2\rho v$. Our ears and microphones respond to these pressure changes.

b.
$$\begin{aligned} I &= \frac{(\Delta P)_{\max}^2}{2\rho v} \\ &= \frac{0.5^2}{2 \times 1.2 \times 340} \\ &= \frac{0.25}{816} \\ &= \mathbf{3.06 \times 10^{-4}\ W/m^2} \end{aligned}$$

(An intensity level of $10\log_{10}(3.06 \times 10^{8}) = 85$ dB.)

14. a.

Intensity	Loudness
the average sound energy per second through unit area normal to the wave	the sensation of how strong a sound is
a physical, objective quantity	subjective: depends on the listener and on the frequency
measured in W/m², with a sound meter (or from P/A)	compared through the intensity level, $\beta = 10\log_{10}(I/I_0)$ dB, read on a decibel meter; by Weber–Fechner, $L \propto \log I$
proportional to a^2	grows only as the logarithm of the intensity

b.
$$\begin{aligned} \Delta\beta &= 10\log_{10}\left(\frac{I_2}{I_1}\right) \\ &= 10\log_{10}\left(\frac{1.0 \times 10^{-5}}{1.0 \times 10^{-8}}\right) \\ &= 10\log_{10} 10^{3} \\ &= \mathbf{30\ dB} \end{aligned}$$

15. a. **Pitch** is how high or low (shrill or grave) a sound seems; it depends mainly on the frequency: the higher the frequency, the higher the pitch (a whistle, a mosquito, a child's voice are high; a bass drum, an elephant's rumble, a man's voice low). **Quality** (timbre) is what lets us tell apart two sounds of the same pitch and loudness from different sources. It depends on the overtones that sound with the fundamental and their strengths, which give the wave its shape. Examples: a bansuri's note is nearly a pure tone with weak overtones, soft and smooth; a clarinet (like a pipe closed at one end) has mostly odd harmonics, a hollow sound; a violin or a sarangi, bowed, has strong harmonics of every order, bright and rich; a struck piano or madal has overtones that die away at different rates, so its sound changes as it fades.

b.
$$\begin{aligned} I &= \frac{P}{4\pi r^2} \\ &= \frac{2}{4\pi \times 3^2} \\ &= \frac{2}{113.1} \\ &= \mathbf{0.0177\ W/m^2} \end{aligned}$$

16. a. From a source sending sound equally in all directions, in the open, the power P spreads over a sphere of area $4\pi d^2$, so $I = P/4\pi d^2$: the intensity is **inversely proportional to the square of the distance** (the inverse square law). Twice as far, a quarter the intensity (6 dB less); ten times as far, a hundredth (20 dB less). Indoors, reflections make it fall more slowly, and absorption by the air, faster over long distances.

b.
$$\begin{aligned} \frac{I_2}{I_1} &= \frac{d_1^2}{d_2^2} \\ \Rightarrow\ I_2 &= 1.0 \times 10^{-6} \times \left(\frac{2}{4}\right)^2 \\ &= \frac{1.0 \times 10^{-6}}{4} \\ &= \mathbf{2.5 \times 10^{-7}\ W/m^2} \end{aligned}$$

17.

a. The ear senses the pressure changes of a sound, but the loudness we perceive doesn't grow in proportion to the intensity: it responds to ratios. Ten times the intensity seems about one step louder, and another ten times, another step of the same size. This is the **Weber–Fechner law**: loudness $L \propto \log_{10} I$. This is how the ear copes with a range of 10^{12} between the faintest sound it hears ($I_0 = 10^{-12}$ W/m²) and the threshold of pain (1 W/m²). So sound is measured by the intensity level $\beta = 10 \log_{10}(I/I_0)$ dB, which runs from 0 to about 120 dB over that range: every +10 dB is 10 times the intensity. Loudness also depends on the frequency (the ear is most sensitive at about 2–5 kHz) and on the listener.

b. The **Doppler effect** is the apparent change in the frequency of a wave received by an observer because the source and the observer move relative to each other: higher when they approach, lower when they separate. A moving source crowds its waves together in front of it (shorter wavelength) and spreads them out behind; a moving observer meets the waves faster or slower. The frequency heard, $f' = \left(\dfrac{v \pm v_l}{v \mp v_s}\right) f$, depends on the source's frequency f, the speed of the wave in the medium v (and so on the temperature and any wind), the speed of the source v_s, the speed of the observer v_l, and the directions of their motions along the line joining them.

c.

$$\begin{aligned} f' &= \left(\frac{v + v_l}{v}\right) f \\ &= \frac{340 + 20}{340} \times 600 \\ &= \frac{360}{340} \times 600 \\ &= \mathbf{635.3\ Hz} \end{aligned}$$

18. a. Let the source send frequency f and wavelength $\lambda = v/f$, sound traveling at v in still air.

i. **Source moving towards an observer at rest**, speed v_s. In one second the source sends f waves; the first travels v, while the source moves v_s after it. So the f waves lie in a distance $v - v_s$: $\lambda' = \dfrac{v - v_s}{f}$. They reach the observer at speed v, so

$$f' = \frac{v}{\lambda'} = \left(\frac{v}{v - v_s}\right) f$$

Moving away, the waves spread over $v + v_s$: $\lambda' = \dfrac{v + v_s}{f}$ and $f' = \left(\dfrac{v}{v + v_s}\right) f$.

ii. **Observer moving towards a source at rest**, speed v_l. The wavelength is unchanged, λ, but the waves pass the observer at $v_R = v + v_l$, so

$$f' = \frac{v_R}{\lambda} = \frac{v + v_l}{v/f} = \left(\frac{v + v_l}{v}\right) f$$

Moving away, $v_R = v - v_l$ and $f' = \left(\dfrac{v - v_l}{v}\right) f$.

iii. **Both moving towards each other.** The source's motion gives $\lambda' = \dfrac{v - v_s}{f}$; the observer's gives $v_R = v + v_l$. So

$$f' = \frac{v_R}{\lambda'} = \left(\frac{v + v_l}{v - v_s}\right) f$$

In general, $f' = \left(\dfrac{v \pm v_l}{v \mp v_s}\right) f$, the upper signs for motion towards the other.

b.

$$\begin{aligned} f' &= \left(\frac{v + v_l}{v - v_s}\right) f \\ &= \frac{340 + 20}{340 - 30} \times 440 \\ &= \frac{360}{310} \times 440 \\ &= \mathbf{511.0\ Hz} \end{aligned}$$

19. a. The Doppler effect lets us measure the speed of something from a distance, from the frequency of the waves it sends or reflects, without touching it. Applications: **radar speed guns** and traffic monitoring (vehicles' speeds); **Doppler ultrasound** and echocardiography (the speed and direction of blood flow); **astronomy** (the red and blue shifts of stars and galaxies, the expansion of the universe, planets round other stars); **weather radar** (the winds inside storms); **sonar** (the speeds of submarines and shoals of fish); **satellite navigation**.

b. The frequency heard is **higher** than the source's. The source moves after the waves it has sent, so each wave is sent from a point closer to the observer than the one before: the waves in front of it are crowded together, $\lambda' = (v - v_s)/f$. They still travel at the speed of sound, so more of them reach the observer each second: $f' = vf/(v - v_s) > f$.

c.
$$\begin{aligned} f' &= \left(\frac{v}{v + v_s}\right) f \\ &= \frac{340}{340 + 40} \times 400 \\ &= \frac{340}{380} \times 400 \\ &= \mathbf{357.9\ Hz} \end{aligned}$$

20. a. There is no Doppler shift when the **distance between the source and the observer isn't changing**, that is, when neither has any velocity along the line joining them. This happens when the motion is at right angles to that line: a source moving round in a circle with the observer at the center, or a car at the very instant it passes closest to you. (Also, trivially, when both move in the same direction at the same speed: then there is no relative motion.) Only the part of the relative velocity along the line of sight shifts the frequency.

b.
$$\begin{aligned} f' &= \left(\frac{v}{v - v_s}\right) f \\ &= \frac{340}{340 - 30} \times 500 \\ &= \frac{340}{310} \times 500 \\ &= \mathbf{548.4\ Hz} \end{aligned}$$