Unit 7

Mechanical Waves

This unit online, with its interactives:
science-playground.com/s/12f7

Count the seconds between a flash of lightning over the hills and its thunder: three seconds for every kilometer. Put your ear to a steel railing while a friend taps the far end, and the tap reaches you twice, first through the metal, then through the air. On a −25 °C night at Everest Base Camp sound crawls along at about 315 m/s; in a Nepalgunj afternoon at 40 °C it runs at about 355 m/s. Why does the medium matter so much, and why does the weather?

This unit answers that for **mechanical waves**, the waves that need a medium. It finds what sets their speed, works out the speed of sound in a gas the way Newton did, then shows where Newton went wrong and how Laplace put it right, and finally asks what changes the speed of sound in air: temperature, pressure or humidity.

A STORY FROM SCIENCE

Laplace and the missing sixth

Pierre-Simon Laplace (1749–1827) was a French mathematician and astronomer, sometimes called "the French Newton". His five-volume *Celestial Mechanics* worked out the motions of the planets and moons from Newton's law of gravitation, and his book on probability laid the foundations of that subject. He was elected to the French Academy of Sciences in 1773, served for six weeks as Napoleon's Minister of the Interior in 1799, and was made a count by Napoleon and later a marquis by King Louis XVIII. He was also a foreign member of the Royal Society of London.

For more than a century physicists had been puzzled by a gap in Newton's own work. In his *Principia* (1687), Newton calculated the speed of sound in air from its pressure and density, and got a value about a sixth too low. In the early 1800s Laplace found why: a sound wave squeezes and stretches the air too quickly for heat to flow, so the air in a compression warms a little and pushes back harder. Putting in that one factor, γ, brought the calculated speed into line with what was measured. This unit follows both arguments.



Figure 7.1 *Pierre-Simon Laplace (1749–1827).*
Photo: Paulin Jean-Baptiste Guérin (1783 - 1855), Public domain

7.1 Introduction

A **mechanical wave** is a disturbance that needs a material medium (a solid, a liquid or a gas) to travel through, carrying energy from place to place. A source disturbs the medium, and the energy is passed on by the vibration of the medium's particles, each pushing on the next. Mechanical waves are of two types, **longitudinal** and **transverse**.

The previous unit described wave motion in general. This unit is about mechanical waves, sound above all: how they travel, what decides their speed, and how the conditions of the air change it.

7.2 Speed of mechanical wave

The **speed of a mechanical wave** is the distance the wave travels in unit time. In terms of its wavelength λ and frequency f,

$$v = f\lambda$$

The **frequency** is set by the source: a tuning fork marked 512 Hz makes the air vibrate 512 times a second, whatever the air. The **speed** is set by the medium. The wavelength then follows, $\lambda = v/f$. That is why a note keeps its pitch when it passes from air into water but its wavelength grows more than four times: the frequency stays, the speed changes.

Two properties of the medium decide the speed.

Elastic property. As a mechanical wave passes, each particle of the medium is pulled out of place and the medium is deformed for a moment. A medium that is elastic pulls the particle back, and that pull is what sets the next particle moving. The property that resists deformation and restores the medium is its **elastic property**, measured by its **modulus of elasticity** E. The stiffer the medium, the harder each particle is pulled back, and the faster the disturbance is handed on.

Inertial property. To pass on the wave, each particle must be set moving, so it must have mass: the medium must store and carry kinetic energy. This is its **inertial property**, measured by the mass of a unit volume, its **density** ρ. The denser the medium, the more sluggishly each part responds, and the slower the wave.

So the speed of a mechanical wave is

$$\boxed{v = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{\text{elastic factor}}{\text{inertial factor}}}}$$

The elastic factor and the inertial factor take different forms for different media:

1. **A transverse wave on a stretched string.** The restoring force is the **tension** T, and the inertia is the **mass per unit length** μ:

$$v = \sqrt{\frac{T}{\mu}}$$

2. **Sound (longitudinal) in a solid rod or wire.** A rod is stretched and squeezed along its length, so the modulus is **Young's modulus** Y of the rod's material, and ρ is its density:

$$v = \sqrt{\frac{Y}{\rho}}$$

3. **Sound (longitudinal) in a liquid or a gas.** A fluid has no stiffness against stretching, only against being squeezed, so the modulus is its **bulk modulus** K:

$$v = \sqrt{\frac{K}{\rho}}$$

Medium (about 20 °C)	Speed of sound
Air	343 m/s
Water	about 1480 m/s
Granite	about 6000 m/s
Steel (a thin rod)	about 5100 m/s

Steel is about 6500 times denser than air, but its Young's modulus is over a million times larger than air's bulk modulus (2×10^{11} Pa against about 1.4×10^{5} Pa), so sound runs through it about 15 times faster.

DID YOU KNOW?

Listening at the rails

In old films, scouts put an ear to the railway line to hear a train long before it could be heard through the air. The sound runs through the steel about fifteen times faster than through the air, and a narrow rail loses much less of it than air spreading in every direction. (Don't try it on a line in use.) Example 7.2 works out the two arrival times.

Check 1

A guitar string is tightened so its tension becomes four times as large. The speed of waves on it:

a becomes four times as large
b doubles
c stays the same
d halves

7.3 Speed of sound in gaseous medium

Sound is usually made by something solid or liquid vibrating, a drum skin, a vocal cord, a bell, but it reaches our ears through the air. So the speed of sound in a gas is the speed that decides when we hear a sound. Working it out took two great scientists.

A sound wave is a **pressure wave**: as it passes, it makes **compressions**, where the air is squeezed to a slightly higher pressure, and **rarefactions**, where it is stretched to a slightly lower one. Squeezing a gas warms it and stretching it cools it, so the wave also changes the gas's temperature a little, back and forth. The passage of sound through a gas is a thermodynamic process, and the question is which kind.

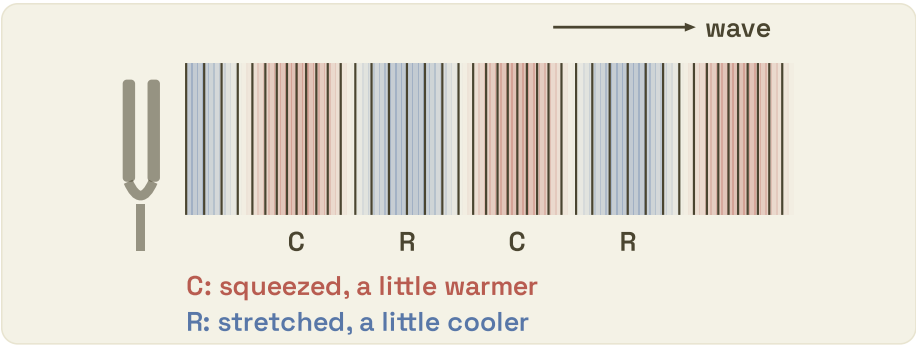


Figure 7.2 *Sound in air: layers of air crowd together in compressions (C) and spread apart in rarefactions (R). The squeezed air is a little warmer and the stretched air a little cooler; the wave moves on before any heat can flow from one to the other.*

Newton's formula

Newton assumed that sound travels **slowly** enough for any difference in temperature between compressions and rarefactions to even out: the gas stays at the same temperature throughout. The process is **isothermal** and obeys **Boyle's law**. For a given mass of gas at pressure P and volume V,

$$PV = \text{constant} \qquad (7.1)$$

1. Differentiating (7.1) by the product rule:

$$P\,dV + V\,dP = 0 \quad \Rightarrow \quad P\,dV = -V\,dP$$

2. Rearranging:

$$P = -\frac{dP}{dV/V} = \frac{\text{stress}}{\text{volumetric strain}} = K \qquad (7.2)$$

The minus sign appears because an increase of pressure ($dP > 0$) makes the volume smaller ($dV < 0$); the bulk modulus itself is positive. So for an isothermal change, **the bulk modulus of a gas equals its pressure**, $K = P$.

3. The speed of sound in a gas is

$$v = \sqrt{\frac{K}{\rho}} \qquad (7.3)$$

so

$$\boxed{v = \sqrt{\frac{P}{\rho}}} \qquad (7.4)$$

This is **Newton's formula** for the speed of sound in a gas. For air at STP, $P = 1.013 \times 10^5$ N/m² and $\rho = 1.29$ kg/m³:

$$v = \sqrt{\frac{1.013 \times 10^5}{1.29}} = 280 \text{ m/s}$$

But the measured speed of sound in air at STP is about **331 m/s**. Newton's value is about 15% too low, far more than any error of measurement: something in his assumption was wrong.

Laplace's correction

Laplace pointed out that a sound wave is **fast**. A compression and the next rarefaction are only half a wavelength apart and swap places hundreds of times a second, and air is a poor conductor of heat. So there is no time for heat to flow into or out of a compression or a rarefaction: the temperature **rises** in a compression and **falls** in a rarefaction. The process is **adiabatic**, not isothermal, and for a given mass of gas

$$PV^\gamma = \text{constant} \qquad (7.5)$$

where $\gamma = C_P/C_V$ is the ratio of the gas's principal specific heat capacities (the **adiabatic index**, or Laplace's coefficient). It depends on the atomicity of the gas: about 1.67 for a monatomic gas such as helium, 1.40 for a diatomic gas such as nitrogen or oxygen.

1. Differentiating (7.5):

$$P\gamma V^{\gamma-1} dV + V^\gamma dP = 0$$

2. Dividing by $V^{\gamma-1}$:

$$\gamma P\, dV + V\, dP = 0 \quad \Rightarrow \quad \gamma P = -\frac{dP}{dV/V} = K \qquad (7.6)$$

So for an adiabatic change **the bulk modulus is** γP, larger than P: the warmed air in a compression pushes back harder than it would at a steady temperature.

3. With $v = \sqrt{K/\rho}$ (7.7):

$$\boxed{v = \sqrt{\frac{\gamma P}{\rho}}} \qquad (7.8)$$

This is **Laplace's formula** (Newton's formula with Laplace's correction). For air at STP, with $\gamma = 1.4$:

$$v = \sqrt{\frac{1.4 \times 1.013 \times 10^5}{1.29}} = 331.6 \text{ m/s}$$

very close to the measured value, about 331 m/s. Laplace's assumption is the right one: sound in air is adiabatic.

CAREFUL

Two different "temperature" ideas

Laplace's correction is about the tiny temperature swings **inside** the wave, between its compressions and rarefactions, which make the bulk modulus γP instead of P. It is not about the temperature of the air as a whole, which section 7.4 deals with. An exam answer on Laplace's correction should say: the changes are adiabatic, not isothermal, so $K = \gamma P$.

Check 2

Newton's formula gives a speed of sound in air that is too low because he assumed that:

a air has no elasticity

b the temperature of the air stays the same as the sound passes

c the density of air changes with the sound

d sound is a transverse wave

7.4 Factors affecting the speed of sound in gas (air)

i. Temperature

The speed of sound in a gas is

$$v = \sqrt{\frac{\gamma P}{\rho}} \qquad (7.9)$$

1. For one mole of an ideal gas, $PV = RT$. The volume of one mole is its mass divided by its density, $V = M/\rho$, where M is the **molar mass**:

$$\frac{PM}{\rho} = RT \quad \Rightarrow \quad \frac{P}{\rho} = \frac{RT}{M} \qquad (7.10)$$

2. Putting (7.10) into (7.9):

$$\boxed{v = \sqrt{\frac{\gamma RT}{M}}} \qquad (7.11)$$

3. For a given gas, γ, R and M are constant, so

$$v \propto \sqrt{T}$$

If v_1 and v_2 are the speeds of sound in a gas at absolute temperatures T_1 and T_2,

$$\frac{v_1}{v_2} = \sqrt{\frac{T_1}{T_2}} \qquad (7.12)$$

The speed of sound in a gas is directly proportional to the square root of its absolute temperature. In warmer air the molecules move faster and pass the disturbance on faster. (The speed of sound is about 68% of the molecules' root mean square speed, $\sqrt{3RT/M}$, in air.) So a graph of v^2 against **absolute** temperature T is a straight line through the origin (Figure 7.3).

A handy rule near room temperature: the speed of sound in air rises by about **0.6 m/s for each degree Celsius**, $v \approx 331 + 0.6t$ m/s with t in °C. It comes from (7.12): $\frac{dv}{dT} = \frac{v}{2T} = \frac{331}{2 \times 273} = 0.61$ m/s per kelvin. It is only an approximation; for large changes of temperature use (7.12).

Note: at the same temperature, the speed of sound is inversely proportional to the square root of the molar mass of the gas, $v \propto \frac{1}{\sqrt{M}}$ (for gases with the same γ). So sound travels faster in hydrogen ($M = 2$ g/mol) than in oxygen (32 g/mol) at the same temperature, four times as fast, since $\sqrt{32/2} = 4$.

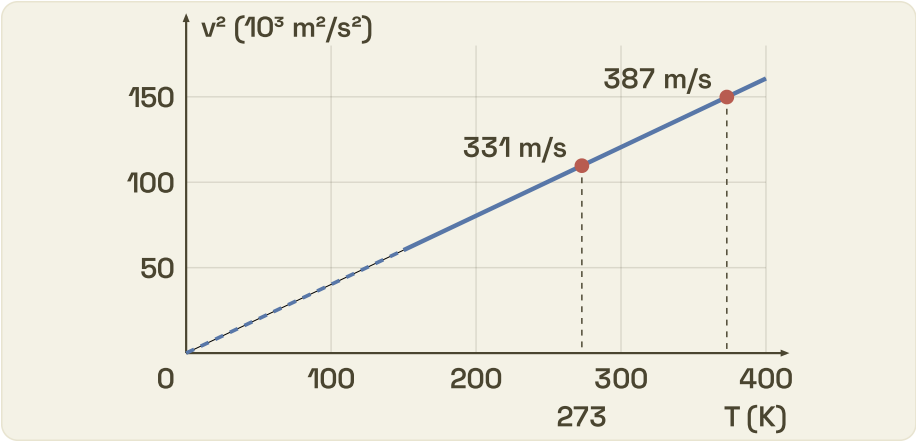


Figure 7.3 v^2 *against absolute temperature* T *for air: a straight line through the origin,* $v^2 = (\gamma R/M)\,T$. *(Below about 150 K it is dashed: the line heads for the origin, but air turns liquid near 80 K, long before.) Plotted against temperature in °C, the same line would not pass through the origin.*

DID YOU KNOW?

Why helium makes your voice squeaky

Breathe in a little helium from a balloon and you sound like a cartoon chipmunk. Your vocal cords vibrate at the same frequency as before, but the air spaces of your throat and mouth, which shape your voice, now hold helium, in which sound travels nearly three times as fast (about 970 m/s). Their resonances move up to higher frequencies, so the high notes in your voice are made louder. (Only a breath from a party balloon, never from a pressurized tank: helium has no oxygen in it.)

ii. Pressure

From (7.11), at a constant temperature $\dfrac{RT}{M}$ is constant, and therefore

$$\frac{P}{\rho} = \text{constant} \qquad (7.13)$$

Then, from (7.9),

$$v = \sqrt{\gamma \times \text{constant}} = \text{constant} \qquad (7.14)$$

At constant temperature, the speed of sound in a gas does not depend on its pressure. When the pressure of a gas changes at constant temperature, its density changes in the same proportion (double the pressure, double the density), so P/ρ stays the same, and so does the speed (Figure 7.4).

So the thin air at the top of Everest, at about a third of sea-level pressure, carries sound no slower for being thin. It does carry it slower because it is cold: at −30 °C (243 K), $v = 331\sqrt{243/273} \approx 312$ m/s.

iii. Humidity

Water vapor in the air is **humidity**. A water molecule ($M = 18$ g/mol) is lighter than the average molecule of dry air (29 g/mol), so moist air is **less dense** than dry air at the same temperature and pressure: $\rho_{\text{moist}} < \rho_{\text{dry}}$. Since $v \propto \dfrac{1}{\sqrt{\rho}}$, **sound travels a little faster in moist air than in dry air**, and the more humid the air, the higher the speed.

The effect is small: at 20 °C, sound in fully saturated air is only about 0.4% faster than in dry air, a little over 1 m/s. Sound carries farther on some days and nights for a different reason: temperature. On a still night the ground cools the air near it, and the warmer air higher up bends sound waves back down towards the ground, so distant sounds,

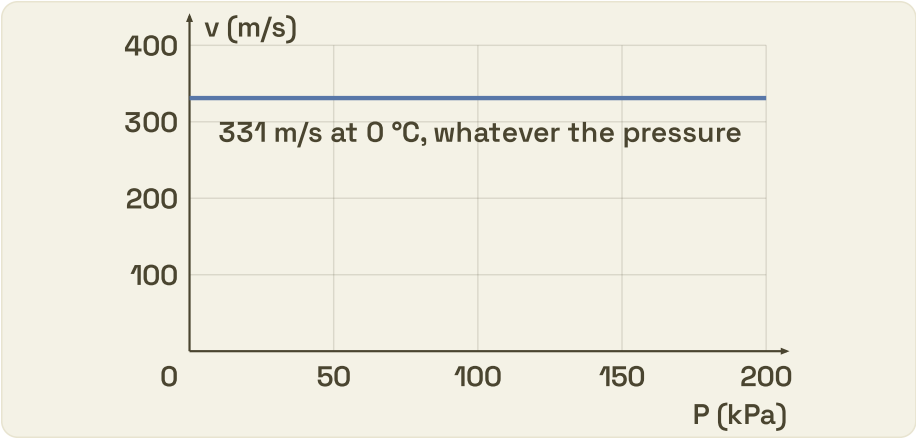


Figure 7.4 *The speed of sound in air against pressure at a fixed temperature, 0 °C: a horizontal line. Doubling the pressure doubles the density too, and the speed is unchanged.*

a temple bell, a far-off dog, are heard more clearly at night than in the heat of the day, when the warm air near the ground bends sound up and away.

CAREFUL

Some books say...

Some books, and some exam questions, say sound is heard farther on a rainy day or at night **because of humidity**. If asked why sound travels faster in humid air, answer as above: moist air is less dense, and $v \propto 1/\sqrt{\rho}$. But the speed changes by less than 0.5%, too little to make a sound noticeably louder. Sound carries far on a still night, or on a cool, still, overcast day after rain, mainly because cool air lies near the ground, under warmer air, and bends the sound back down; the quiet of those times helps too.

Check 3

A sound passes from cold morning air into warm afternoon air. Its speed and wavelength:

a both decrease
b both increase, the frequency staying the same
c the speed increases and the wavelength decreases
d the speed stays the same and the wavelength increases

Check 4

A cylinder of air is compressed slowly to twice its pressure, its temperature kept the same. The speed of sound in it:

a increases by $\sqrt{2}$ times
b doubles
c stays the same
d halves

Try all three factors at once:

WORKED EXAMPLE

Example 7.1: twice as fast

Find the temperature at which the speed of sound in air is twice its value at 0 °C.

Answer

Let v_0 be the speed at 0 °C (273 K) and $v_t = 2v_0$ the speed at T °C. From (7.12),

$$\frac{v_t}{v_0} = \sqrt{\frac{273 + T}{273}} \quad \Rightarrow \quad 2 = \sqrt{\frac{273 + T}{273}}$$

Squaring, $4 \times 273 = 273 + T$, so $T = 3 \times 273 = \mathbf{819\ °C}$ (1092 K, four times the absolute temperature).

WORKED EXAMPLE

Example 7.2: two reports from one bang

A detonator is exploded on a railway line. An observer standing on the line 2 km away hears two reports. What is the time between them? (Young's modulus of steel $= 2.0 \times 10^{11}$ N/m², density of steel $= 8.0 \times 10^{3}$ kg/m³, density of air $= 1.4$ kg/m³, ratio of molar heat capacities of air $= 1.40$, atmospheric pressure $= 10^{5}$ N/m².)

Answer

Speed of sound in air:

$$v_a = \sqrt{\frac{\gamma P}{\rho_{\text{air}}}} = \sqrt{\frac{1.40 \times 10^{5}}{1.4}} = \sqrt{10^{5}} = 316.2\ \text{m/s}$$

Time through 2 km of air: $t_a = \dfrac{2000}{316.2} = 6.32$ s.

Speed of sound in the steel rail:

$$v_s = \sqrt{\frac{Y}{\rho_{\text{steel}}}} = \sqrt{\frac{2.0 \times 10^{11}}{8.0 \times 10^{3}}} = \sqrt{2.5 \times 10^{7}} = 5000\ \text{m/s}$$

Time through 2 km of steel: $t_s = \dfrac{2000}{5000} = 0.40$ s.

Time between the two reports: $6.32 - 0.40 = \mathbf{5.92\ s}$. The first report comes through the rail, the second through the air.

WORKED EXAMPLE

Example 7.3: how far is the storm?

The time between a flash of lightning and the sound of its thunder is 1 s, when the temperature is 10 °C. How far away is the storm, if the speed of sound in air at 0 °C is 330 m/s?

Answer

At 10 °C (283 K), from (7.12):

$$v_{10} = 330\sqrt{\frac{283}{273}} = 330 \times 1.018 = 336\ \text{m/s}$$

The light takes d/c and the sound d/v, so

$$\frac{d}{336} - \frac{d}{3 \times 10^{8}} = 1$$

The light's time is about a millionth of the sound's, far too small to matter, so $d = 336 \times 1 = \mathbf{336\ m}$.
(Hence the rule: count the seconds between flash and thunder and divide by 3 for the distance in kilometers.)

IN SHORT

In short

- A mechanical wave's speed is set by the medium: $v = \sqrt{E/\rho}$, elastic factor over inertial factor.
- String: $v = \sqrt{T/\mu}$; solid rod: $v = \sqrt{Y/\rho}$; liquid or gas: $v = \sqrt{K/\rho}$.
- Newton (isothermal, $K = P$): $v = \sqrt{P/\rho} = 280$ m/s for air at STP, too low.
- Laplace (adiabatic, $K = \gamma P$): $v = \sqrt{\gamma P/\rho} = 331.6$ m/s, which agrees with experiment.
- $v = \sqrt{\gamma RT/M}$: $v \propto \sqrt{T}$ (T in kelvin), $v \propto 1/\sqrt{M}$; independent of pressure at constant temperature; a little faster in moist air.

Exercise

Choose the correct option

Check 5

1. Which of the following is correct about the speed of wave motion in a medium?

a It depends on the frequency of the wave.
b It depends on the wavelength of the wave.
c It depends on the properties of the medium.
d It is constant for all waves in any medium.

Check 6

2. Which of the following factors affect the velocity of sound in air?

a temperature alone
b humidity alone
c pressure alone
d both temperature and humidity

Check 7

3. In which medium does sound travel fastest?

a air
b water
c iron
d wood

Check 8

4. Which of the following expressions represents the velocity of sound in a gas?

a $v = \sqrt{\dfrac{\gamma RT}{M}}$
b $v = \sqrt{\dfrac{RT}{M}}$
c $v = \sqrt{\dfrac{PT}{M}}$
d $v = \sqrt{\dfrac{P}{M}}$

Check 9

5. What is the effect of rising temperature on the velocity of sound in air?

a It increases the velocity.
b It decreases the velocity.
c It has no effect on the velocity.
d It depends on the humidity.

Check 10

6. What does Laplace's correction account for?

a the effect of pressure on the velocity of sound
b the heating and cooling of the gas in compressions and rarefactions (the changes are adiabatic)
c the effect of humidity on the velocity of sound
d the density of the medium

Check 11

7. If the temperature of air increases, how does it affect the speed of sound?

a The speed decreases.
b The speed increases.
c The speed remains constant.
d It depends on the type of wave.

Check 12

8. A sound wave travels through a liquid at 1500 m/s. If the same wave travels through a solid of density 8000 kg/m^3 and Young's modulus 2×10^{11} Pa, how does its speed compare?

a The speed will be higher in the solid.
b The speed will be lower in the solid.
c The speed will be the same in both.
d There is not enough information to decide.

Check 13

9. What is the effect of humidity on the velocity of sound in air?

a It increases the velocity.
b It decreases the velocity.
c It has no effect.
d It depends on the temperature.

Check 14

10. If the frequency of a sound wave is doubled, what happens to its wavelength in a given medium?

a The wavelength is doubled.
b The wavelength is halved.
c The wavelength remains the same.
d The wavelength becomes zero.

Check 15

11. A student measures the speed of sound in air at 20 °C and finds it to be 343 m/s. What will the speed of sound be at 0 °C?

a approximately 331 m/s
b approximately 320 m/s
c approximately 343 m/s
d approximately 300 m/s

Check 16

12. The frequency of a sound wave is doubled while the medium stays the same. What happens to its speed and wavelength?

a The speed remains the same; the wavelength is halved.
b The speed is doubled; the wavelength is the same.
c Both speed and wavelength are doubled.
d Both speed and wavelength remain the same.

Check 17

13. A sonar signal is sent from a submarine and its echo from an enemy submarine returns after 4 s. If the speed of sound in seawater is 1500 m/s, how far away is the enemy submarine?

a 3000 m
b 2500 m
c 2000 m
d 1500 m

Check 18

14. In a Kundt's tube experiment, a metal rod vibrates in resonance and the wavelength measured in the air is 50 cm. If the speed of sound in air is 340 m/s and in the rod 5100 m/s, find the frequency of vibration.

a 680 Hz
b 1020 Hz
c 1200 Hz
d 5100 Hz

Answer these questions

Check 19

15. The velocity of sound in hydrogen at 0 °C is 1250 m/s. What will it be in oxygen at 0 °C, given that the molar masses of hydrogen and oxygen are 2 g/mol and 32 g/mol?

a 312.5 m/s
b 280.5 m/s
c 331.0 m/s
d 250.5 m/s

Check 20

16. A person shouts 150 m from a tall vertical cliff and hears the echo after 0.88 s. What is the approximate speed of sound in air?

a 320 m/s
b 340 m/s
c 360 m/s
d 380 m/s

18.
a. Derive the formula for the velocity of sound in a gas using Newton's assumption. Then explain why Laplace's correction was necessary.
b. A gas has density 1.5 kg/m^3 and bulk modulus 2.25×10^5 N/m^2. Find the speed of sound in the gas.

19.
a. Describe Laplace's correction to Newton's formula for the velocity of sound in gases. How does considering adiabatic changes instead of isothermal ones correct the result?
b. A gas has adiabatic index $\gamma = 1.4$ and molar mass 28.97 g/mol. Calculate the speed of sound in this gas at 300 K. (Use R = 8.314 J/(mol K).)

20.
a. Explain how temperature affects the velocity of sound in gases. Write the expression for velocity in terms of temperature and explain the physical meaning of each term.
b. Calculate the speed of sound in air at 27 °C. (The velocity of sound in air at 0 °C is 331 m/s.)

21.
a. Explain the effect of pressure on the velocity of sound in a gas. Why is the velocity of sound independent of pressure under ideal conditions?
b. Calculate the velocity of sound in a metal rod whose Young's modulus is 2.0×10^{11} N/m^2 and density 7800 kg/m^3.
c. The pressure of a gas is doubled while its temperature is kept constant. What will be the effect on the speed of sound? Justify your answer mathematically.

22.
a. How does humidity affect the speed of sound in air? Why does moist air allow sound to travel faster than dry air?
b. The velocity of sound in dry air at 0 °C is 331 m/s. Calculate the velocity in air at 30 °C with 80% humidity. Assume the velocity increases by 0.1% for every 1% increase in relative humidity.

23.
a. How does molar mass affect the speed of sound in a gas?
b. If the speed of sound in helium is 1005 m/s and in air is 343 m/s, compare the molar masses of helium and air using the velocity formula.

24.

a. How do the bulk modulus and the density of a medium affect the speed of sound? Explain for the different states of matter.

b. At what temperature will the speed of sound in air be 350 m/s? Use the formula $v = 331 + 0.6t$.

25.

a. A student says that an increase in atmospheric pressure increases the speed of sound in air. Do you agree or disagree? Justify with reasoning and equations.

b. A tuning fork produces a sound wave of frequency 512 Hz in air. If the speed of sound is 344 m/s, calculate its wavelength.

A companion to your textbook, not a replacement for it: study from the book your school uses, and where the two differ, follow your teacher. Found a mistake? Please tell us.