Unit 7

Answers: Mechanical Waves

Quick checks

1 b $v = \sqrt{T/\mu}$: four times T gives $\sqrt{4} = 2$ times v. Tightening a string makes its waves faster, and its note higher (Unit 8).

2 b Newton took the changes as isothermal, which makes the bulk modulus P. They are really adiabatic, so the bulk modulus is γP, and the speed $\sqrt{\gamma} = \sqrt{1.4} \approx 1.18$ times larger: 280 × 1.18 ≈ 331 m/s.

3 b The source fixes the frequency. Warmer air carries sound faster ($v \propto \sqrt{T}$), so the wavelength $\lambda = v/f$ grows in the same proportion.

4 c At constant temperature the density doubles with the pressure, so P/ρ is unchanged and $v = \sqrt{\gamma P/\rho}$ is too.

5 c The speed is set by the medium's elasticity and density, $v = \sqrt{E/\rho}$. A change of frequency changes the wavelength, not the speed.

6 d Temperature ($v \propto \sqrt{T}$) and humidity (moist air is less dense) both change it. Pressure alone, at constant temperature, does not.

7 c Iron's great stiffness outweighs its density: about 5000 m/s in an iron rod, against about 1480 m/s in water, 3000–5000 m/s along the grain of wood, and 343 m/s in air.

8 a Laplace's $v = \sqrt{\gamma P/\rho}$ with $P/\rho = RT/M$. Without γ it is Newton's isothermal value, too low.

9 a $v \propto \sqrt{T}$: warmer air, faster sound, about 0.6 m/s more for each degree.

10 b Laplace saw that the gas warms in each compression and cools in each rarefaction, too fast for heat to flow: the changes are adiabatic, so the bulk modulus is γP, not P.

11 b $v = \sqrt{\gamma RT/M}$ grows with T.

12 a $v = \sqrt{Y/\rho} = \sqrt{2 \times 10^{11}/8000} = \sqrt{2.5 \times 10^{7}} = 5000$ m/s, more than three times 1500 m/s.

13 a Water vapor is lighter than dry air, so moist air is less dense, and $v \propto 1/\sqrt{\rho}$. The increase is small.

14 b The medium fixes v, so $\lambda = v/f$ halves when f doubles.

15 a $v_0 = 343\sqrt{\dfrac{273}{293}} = 343 \times 0.965 = 331$ m/s.

16 a The medium fixes the speed; $\lambda = v/f$ halves.

17 **a** The signal goes there and back: $d = \dfrac{vt}{2} = \dfrac{1500 \times 4}{2} = 3000$ m. (6000 m forgets that it travels the distance twice.)

18 **a** The air vibrates at the rod's frequency: $f = \dfrac{v}{\lambda} = \dfrac{340}{0.50} = 680$ Hz. (The rod's speed gives its own wavelength, $5100/680 = 7.5$ m.)

19 **a** Both are diatomic (same γ) at the same temperature, so $v \propto 1/\sqrt{M}$: $v_{O_2} = 1250\sqrt{\dfrac{2}{32}} = \dfrac{1250}{4} = 312.5$ m/s.

20 **b** The sound goes to the cliff and back, 300 m: $v = \dfrac{300}{0.88} = 341$ m/s $\approx$ 340 m/s.

17. a. The speed of sound is $v = \sqrt{E/\rho}$, set by the medium's elasticity and density. It is usually greatest in solids (several thousand m/s: about 5000 m/s in a steel rod), less in liquids (about 1500 m/s in water) and least in gases (about 340 m/s in air). In a solid the particles are held tightly by strong bonds, so its modulus of elasticity is enormous, and it rises far more than the density does. A gas is easily squeezed, so its bulk modulus is tiny (only $\gamma P \approx 1.4 \times 10^5$ Pa for air, against 2×10^{11} Pa for steel's Young's modulus). Although a solid is thousands of times denser than a gas, its modulus is about a million times larger, so $\sqrt{E/\rho}$ is many times greater.

b.

$$v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.2 \times 10^9}{1000}} = \sqrt{2.2 \times 10^6} = \mathbf{1483\ m/s}$$

18. a. Newton assumed the compressions and rarefactions of a sound wave keep the same temperature (isothermal), so the gas obeys Boyle's law, $PV = \text{constant}$. Differentiating, $P\,dV + V\,dP = 0$, so $P = -\dfrac{dP}{dV/V} = K$, the bulk modulus. With $v = \sqrt{K/\rho}$,

$$v = \sqrt{\frac{P}{\rho}}$$

For air at STP this gives $\sqrt{1.013 \times 10^5 / 1.29} = 280$ m/s, but the measured speed is about 331 m/s, about 15% more. The difference was too large to be an error of measurement, so Newton's assumption had to be wrong. Laplace saw that the changes are too fast for heat to flow, so they are adiabatic: $K = \gamma P$, and $v = \sqrt{\gamma P/\rho} = 331.6$ m/s, in agreement with experiment.

b.

$$v = \sqrt{\frac{K}{\rho}} = \sqrt{\frac{2.25 \times 10^5}{1.5}} = \sqrt{1.5 \times 10^5} = \mathbf{387.3\ m/s}$$

19. a. Laplace argued that a sound wave squeezes and stretches the gas very fast, hundreds of times a second, and a gas conducts heat poorly. So no heat flows between a compression, which warms, and a rarefaction, which cools: the changes are adiabatic, $PV^\gamma = \text{constant}$. Differentiating, $\gamma P V^{\gamma-1} dV + V^\gamma dP = 0$, so $\gamma P = -\dfrac{dP}{dV/V} = K$. The adiabatic bulk modulus γP is larger than the isothermal P, because the warmed gas in a compression pushes back harder. So

$$v = \sqrt{\frac{\gamma P}{\rho}}$$

which is $\sqrt{\gamma} = \sqrt{1.4} = 1.18$ times Newton's value: $280 \times 1.18 = 331.6$ m/s for air at STP, matching the measured speed.

b.

$$v = \sqrt{\frac{\gamma RT}{M}} = \sqrt{\frac{1.4 \times 8.314 \times 300}{0.02897}} = \sqrt{1.205 \times 10^5} = \mathbf{347.2\ m/s}$$

(The molar mass must be in kg/mol: 28.97 g/mol = 0.02897 kg/mol. This gas is air.)

20. a. Using $PV = RT$ for one mole, $P/\rho = RT/M$, so Laplace's formula becomes

$$v = \sqrt{\frac{\gamma RT}{M}}$$

- v: the speed of sound in the gas;
- $\gamma = C_P/C_V$: the ratio of the gas's specific heat capacities, which enters because the changes are adiabatic (1.4 for air);
- R: the universal gas constant, 8.314 J/(mol K);
- T: the absolute temperature of the gas, in kelvin;
- M: the molar mass of the gas, in kg/mol.

For a given gas, $v \propto \sqrt{T}$: the warmer the gas, the faster its molecules move and the faster they pass the disturbance on. So $\dfrac{v_1}{v_2} = \sqrt{\dfrac{T_1}{T_2}}$.

b.

$$v_{27} = 331\sqrt{\frac{300}{273}} = 331 \times 1.0483 = \mathbf{347.0\ m/s}$$

(The rule $v \approx 331 + 0.6t$ gives $331 + 16.2 = 347.2$ m/s, nearly the same.)

21. a. For an ideal gas, $P/\rho = RT/M$. At a constant temperature this ratio is constant: if the pressure rises, the gas is squeezed into a smaller volume and its density rises in the same proportion. So $v = \sqrt{\gamma P/\rho}$ doesn't change: at constant temperature the speed of sound doesn't depend on pressure. (Real gases depart from this only slightly, at very high pressures.)

b.

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.0 \times 10^{11}}{7800}} = \sqrt{2.564 \times 10^{7}} = \mathbf{5064\ m/s}$$

c. At constant temperature, Boyle's law: doubling P halves V, and since the mass is unchanged, it doubles ρ. So

$$v' = \sqrt{\frac{\gamma(2P)}{2\rho}} = \sqrt{\frac{\gamma P}{\rho}} = v$$

The speed of sound doesn't change.

22. a. Water molecules ($M = 18$ g/mol) are lighter than the nitrogen and oxygen molecules of dry air (29 g/mol on average). In moist air some of the heavier molecules are replaced by lighter water molecules, so at the same temperature and pressure moist air is less dense than dry air. Since $v = \sqrt{\gamma P/\rho}$, a smaller density means a greater speed: the more humid the air, the faster sound travels in it, though only slightly.

b. First the effect of temperature:

$$v_{30} = 331\sqrt{\frac{303}{273}} = 331 \times 1.0535 = 348.7$$

Then 80% humidity adds $80 \times 0.1\% = 8\%$:

$$v = 348.7 \times 1.08 = \mathbf{376.6\ m/s}$$

(Using $331 + 0.6 \times 30 = 349$ m/s for the first step gives 376.9 m/s.) The question's 0.1% for every 1% is far larger than the real effect: at 30 °C even fully saturated air carries sound less than 1% faster than dry air, about 351 m/s here.

23. a. From $v = \sqrt{\gamma RT/M}$, at a given temperature (and for gases with the same γ), $v \propto \dfrac{1}{\sqrt{M}}$. A gas of lighter molecules is less dense at the same pressure and temperature, so the same pressure pushes it into motion more readily, and sound travels faster. Sound travels four times as fast in hydrogen (2 g/mol) as in oxygen (32 g/mol).

b. At the same temperature,

$$\frac{v}{v} = \sqrt{\frac{\gamma}{\gamma}\cdot\frac{M}{M}} \quad\Rightarrow\quad \frac{M}{M} = \left(\frac{v}{v}\right)^2\frac{\gamma}{\gamma}$$

$$\frac{M}{M} = \left(\frac{1005}{343}\right)^2 \times \frac{1.40}{1.67} = 8.585 \times 0.838 = 7.2$$

So air's molar mass is about **7.2 times** helium's. With $M = 28.97$ g/mol, $M = \dfrac{28.97}{7.2} = \mathbf{4.0\ g/mol}$, helium's real molar mass.

Helium is monatomic ($\gamma = 1.67$) and air diatomic ($\gamma = 1.40$), so the two γs must be kept. Leaving them out, as if $v \propto 1/\sqrt{M}$ alone, gives $28.97 \times (343/1005)^2 = 3.37$ g/mol, which is wrong.

24. a. $v = \sqrt{K/\rho}$: the speed rises with the bulk modulus (a stiffer medium pulls each particle back harder) and falls with the density (a heavier medium responds more sluggishly). Gases have a tiny bulk modulus (about 10^5 Pa) and low density: speeds of hundreds of m/s. Liquids are very hard to compress (water, $K = 2.2 \times 10^9$ Pa) and about a thousand times denser than air: about 1500 m/s. Solids are stiffer still (steel's modulus about 10^{11} Pa) and denser: several thousand m/s. The modulus grows far faster than the density from gas to liquid to solid, so the speed grows too.

b. $350 = 331 + 0.6t \Rightarrow t = \dfrac{19}{0.6} = \mathbf{31.7\ °C}$.

(From $v \propto \sqrt{T}$ instead: $T = 273 \times (350/331)^2 = 305.2$ K, 32.2 °C, nearly the same.)

25. a. **Disagree**, as long as the temperature stays the same. $v = \sqrt{\gamma P/\rho}$, and for an ideal gas $\dfrac{P}{\rho} = \dfrac{RT}{M}$. When the pressure rises at constant temperature, the density rises in the same proportion, so P/ρ, and with it $v = \sqrt{\gamma RT/M}$, is unchanged. Only the temperature (and the kind of gas, and a little the humidity) changes the speed of sound. On a day of high pressure sound may seem to travel differently, but that is because the temperature or the wind differs.

b.

$$\lambda = \frac{v}{f} = \frac{344}{512} = \mathbf{0.672\ m}$$