Unit 5

Answers: Second Law of Thermodynamics

Quick checks

1 b Energy is conserved, 1000 J in and 1000 J out, so the first law is satisfied. But it turns all the heat from a single reservoir into work in a cycle, with nothing given to a colder sink, which is exactly what the Kelvin–Planck statement rules out.

2 b $W = Q_1 - Q_2 = 500 - 350 = 150$ J, so $\eta = W/Q_1 = 150/500 = 0.3 = 30\%$. 70% is Q_2/Q_1, the share wasted; 43% is W/Q_2, the wrong heat on the bottom; and 150 J is the work, not an efficiency.

3 c By Carnot's theorem, every reversible engine between the same two reservoirs has the same efficiency, whatever its working substance or size. If one did better, it could drive the other backwards and move heat from cold to hot by itself.

4 b Lowering the sink gives 67% against 57%. The efficiency depends on the **ratio** T_2/T_1, not on the difference: taking 100 K off the smaller temperature changes the ratio more. In practice, though, the sink is usually the air or a river, which we can't cool, so engineers raise the source's temperature instead.

5 c $Q_1 = Q_2 + W = 300 + 100 = 400$ J. The kitchen gets back all the heat taken from inside, plus the work. That is why a refrigerator with its door left open warms the kitchen instead of cooling it.

6 b Squeezing heats the mixture; above about 12, parts of it catch fire by themselves before the flame from the spark reaches them, and the engine knocks. The Otto cycle's efficiency keeps rising with the ratio, as the widget shows; a diesel engine gets its high ratio by squeezing air alone and spraying the fuel in at the end.

7 a $\Delta S = Q/T$: 600/300 = 2 J/K and 600/600 = 1 J/K. The same heat adds more disorder to a cold body, whose particles were more orderly to begin with.

8 c Freezing takes heat out of the water, lowering its entropy. But the freezer gives that heat, plus the compressor's work, to the warmer room, raising the room's entropy by more. The total still rises, as the second law requires. Work done doesn't let the total entropy fall; it only lets entropy be moved from one place to another.

9 c That is the Clausius statement. The first option is the first law; the entropy of an isolated system never **decreases**.

10 d That is the second law in terms of entropy. Entropy measures **dis**order; it increases in the universe; a reversible process produces **no** entropy.

11 b $\eta = 1 - T_2/T_1$: it depends on both temperatures (their ratio), and on nothing else, not even the working substance.

12 **b** $\beta = Q_2/W$: what we want, heat taken from the cold inside, over what we pay, the work. (Heat rejected over work is the COP of a heat pump used for heating.)

13 **c** $\eta = 1 - 1/r^{\gamma-1}$, where $r = V_1/V_2$ is the compression ratio.

14 **c** $\eta = 1 - \dfrac{300}{500} = 0.4 = 40\%$.

15 **c** $\beta = \dfrac{Q_2}{W} = \dfrac{200}{50} = 4$.

16 **c**

$$\begin{aligned}\eta &= 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{\rho_c^{\gamma} - 1}{\gamma(\rho_c - 1)} \\ &= 1 - \frac{1}{18^{0.4}} \cdot \frac{2^{1.4} - 1}{1.4 \times 1} \\ &= 1 - \frac{1.639}{3.178 \times 1.4} = 1 - 0.368 = 0.63\end{aligned}$$

About **63%**. 69% would come from the petrol engine's formula, which treats the heat as added at constant volume.

If a question gives only the compression ratio, as some do, it expects that formula: $\eta = 1 - \dfrac{1}{18^{0.4}} = 1 - \dfrac{1}{3.178} = 0.685 \approx 69\%$ (closest to 70% among options of 55, 60, 65 and 70%). Write that working, and say that it is the Otto-cycle value, the most a diesel engine of this compression ratio could reach.

17 **b** $\beta = \dfrac{T_2}{T_1 - T_2} = \dfrac{250}{50} = 5$, so $W = \dfrac{Q_2}{\beta} = \dfrac{500}{5} = 100$ J.

18 **a** The best is a Carnot engine: $\eta = 1 - 300/500 = 0.4$, so $W = 0.4 \times 600 = 240$ J. (360 J would be the heat rejected.)

19 **b** $\eta = 1 - \dfrac{400}{1000} = 0.6$, so $Q_1 = \dfrac{W}{\eta} = \dfrac{2000}{0.6} = 3333$ J. (5000 J would come from using $T_2/T_1 = 0.4$ as the efficiency.) A real engine, less efficient, would need more.

12. a. **Kelvin–Planck statement:** it is impossible to make an engine, working in a cycle, that takes heat from a single reservoir and turns all of it into work. Every heat engine must give out part of the heat it takes in to a colder sink, so none can be 100% efficient.
Clausius statement: heat cannot flow by itself from a colder body to a hotter body; for that, external work must be done, as in a refrigerator.
The two are equivalent: a perfect engine (breaking Kelvin–Planck) could drive a refrigerator so that the pair moves heat from cold to hot with no outside work (breaking Clausius), and a perfect refrigerator joined to an ordinary engine would make a perfect engine.
b.

	First law	Second law
Direction of heat flow	not stated: heat could flow either way, as long as energy is conserved	heat flows by itself only from hot to cold; the reverse needs work
Efficiency of turning heat into work	not limited: allows 100%	always less than 100%; at most $1 - T_2/T_1$
About	the quantity of energy (conservation)	the direction of processes, irreversibility, entropy

13.

a. A **source** (a hot reservoir at T_1, from which heat is taken without changing its temperature), a **sink** (a cold reservoir at T_2, to which heat is given without changing its temperature) and a **working substance** (the gas or steam that takes in heat, does work by expanding, and gives out the rest of the heat), in a cylinder or turbine where it does the work.

b. The block diagram is Figure 5.3: Q_1 from the source into the engine, W out as work, Q_2 into the sink. In a cycle the working substance's internal energy doesn't change, so $W = Q_1 - Q_2$, and the efficiency is the work done over the heat taken in:

$$\eta = \frac{W}{Q_1} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$

Measure (or calculate) the heat taken in and the heat given out, or the work done, and use this.

c. i. $W = Q_1 - Q_2 = 1200 - 700 = \mathbf{500\ J}$. ii. $\eta = \dfrac{W}{Q_1} = \dfrac{500}{1200} = 0.417 = \mathbf{41.7\%}$.

d. 100% would need $Q_2 = 0$: all the heat turned into work with none given to a sink. The Kelvin–Planck statement says that is impossible for an engine working in a cycle. Even the ideal Carnot engine has $\eta = 1 - T_2/T_1$, which would be 1 only with a sink at 0 K, and absolute zero can't be reached. Real engines also lose energy to friction and heat leaks.

14.

a. The Carnot cycle is the ideal, reversible cycle of an ideal gas in a cylinder with insulating walls and a conducting base, moved between a source at T_1, an insulating stand and a sink at T_2 (Figure 5.7): **AB**, isothermal expansion on the source at T_1, taking in heat Q_1; **BC**, adiabatic expansion on the stand, the gas cooling to T_2; **CD**, isothermal compression on the sink at T_2, giving out heat Q_2 ; **DA**, adiabatic compression on the stand, back to T_1. On the P–V diagram the isotherms are the gentler curves, the adiabats the steeper ones; the loop is traced clockwise and its area is the net work done in each cycle.

b. For n moles: $Q_1 = W_1 = nRT_1 \ln(V_2/V_1)$; $Q_2 = nRT_2 \ln(V_3/V_4)$; the two adiabatic works, $\pm nR(T_1 - T_2)/(\gamma - 1)$, cancel. From $TV^{\gamma-1} = \text{constant}$ on BC and DA, $T_1V_2^{\gamma-1} = T_2V_3^{\gamma-1}$ and $T_1V_1^{\gamma-1} = T_2V_4^{\gamma-1}$; dividing, $V_2/V_1 = V_3/V_4$. So $W = Q_1 - Q_2 = nR(T_1 - T_2)\ln(V_2/V_1)$ and

$$\eta = \frac{W}{Q_1} = \frac{nR(T_1 - T_2)\ln(V_2/V_1)}{nRT_1 \ln(V_2/V_1)} = 1 - \frac{T_2}{T_1}$$

Significance: the efficiency depends only on the two temperatures (in kelvin), not on the working substance. A hotter source or a colder sink gives a higher efficiency; it could be 100% only with $T_2 = 0$ K, which is impossible. No engine between the same two temperatures can do better.

c. $0.6 = 1 - \dfrac{300}{T_1} \Rightarrow \dfrac{300}{T_1} = 0.4 \Rightarrow T_1 = \mathbf{750\ K}$.

15. a. The P–V diagrams are Figure 5.11 and Figure 5.13.

Otto cycle (petrol engine)	Diesel cycle (diesel engine)
compresses an air–petrol mixture	compresses air only
lit by a spark	fuel lights itself in the hot air
heat added at constant volume (vertical line CD)	heat added at constant pressure (horizontal line CD)
compression ratio 8–12	compression ratio 14–25
$\eta = 1 - 1/r^{\gamma-1}$	$\eta = 1 - \frac{1}{r^{\gamma-1}} \cdot \frac{\rho_c^{\gamma} - 1}{\gamma(\rho_c - 1)}$, lower at the same r, higher in practice because r is higher

Both have adiabatic compression BC and expansion DE, and heat given out at constant volume EB.

b. $Q_2 = Q_1 - W = 5000 - 1200 = \mathbf{3800\ J}$; $\eta = \frac{1200}{5000} = 0.24 = \mathbf{24\%}$.

16. a. A heat engine takes heat from a hot body, gives work out and gives the rest of the heat to a cold body. A refrigerator does the reverse: work W (from the electric motor) is done on its working substance, which takes heat Q_2 from the cold inside and gives out $Q_1 = Q_2 + W$ to the warmer room (Figure 5.8). The refrigerant boils in the evaporator inside, taking in heat; the compressor squeezes the vapor, making it hotter than the room; it gives out heat and condenses in the coils at the back; through the expansion valve its pressure and temperature drop, and it returns to the evaporator (Figure 5.9).

The **coefficient of performance** is the heat taken from the cold body per unit of work: $\beta = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2}$; for an ideal refrigerator $\beta = \frac{T_2}{T_1 - T_2}$.

b. $\beta = \frac{Q_2}{W} = \frac{400}{100} = \mathbf{4}$.

17.

a. **Entropy** S is a state variable that measures the disorder of the particles of a system, or how spread out its energy is. A small change in it is the heat given reversibly divided by the absolute temperature: $dS = dQ/T$, in J/K. The more disordered a system, the more ways its particles can be arranged, and the higher its entropy (Boltzmann: $S = k_B \ln W$): a gas has more entropy than its liquid, and a liquid more than its solid.

b. Almost every natural process is irreversible, and in every irreversible process the total entropy of the system and its surroundings increases; in a reversible process it would stay the same, and it never decreases. The universe is an isolated system, so its entropy keeps rising. *Examples:* heat flowing from a hot cup of tea to the room (1000 J from 400 K to 300 K raises the total entropy by 0.83 J/K); ice melting in a warm room; ink or incense smoke spreading out; friction turning a vehicle's motion into heat in its brakes; fuel burning. None of these undoes itself. The entropy of one part can fall (water freezing in a freezer), but only if the entropy elsewhere rises by more.

18.

a. Suppose an engine X were more efficient than a Carnot engine R between the same reservoirs. R is reversible, so it can be run backwards as a refrigerator, driven by X. X can supply all the work R needs while taking less heat from the hot reservoir than R gives back to it. Together, the two machines then do no net work, yet move heat from the cold reservoir to the hot one: heat flowing from cold to hot by itself, which the Clausius statement forbids. So no engine can beat Carnot's (Carnot's theorem).

b. A real engine can't be reversible: its pistons and bearings have **friction**; heat **leaks** through its walls; its processes are **fast**, not infinitely slow, so the gas isn't in equilibrium; heat must flow across a real **temperature difference** to get in and out in a reasonable time, so the working substance never reaches the source's or the sink's temperature; fuel doesn't burn completely; and real cycles (Otto, Diesel) aren't Carnot cycles, and throw out exhaust that is still hot. A Carnot engine would also run infinitely slowly, giving no power. So real engines reach well under the Carnot efficiency.

c. $\eta = 1 - \dfrac{T_2}{T_1} = 1 - \dfrac{300}{500} = 0.4 = \mathbf{40\%}$.

19. $W = \eta Q_1 = 0.25 \times 600 = 150$ J, so $Q_2 = Q_1 - W = 600 - 150 = \mathbf{450\ J}$.

20.

a. $\eta = 1 - \dfrac{400}{800} = 0.5 = \mathbf{50\%}$.

b. $Q_1 = \dfrac{W}{\eta} = \dfrac{300}{0.5} = \mathbf{600\ J}$ (of which 300 J is rejected to the sink).