Unit 24 · Radioactivity and Nuclear Reactions

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Modern Physics (Units 20–25 together): 19 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** long (8, the OR): derive $N = N_0 e^{-\lambda t}$; from a graph of measured activity against time: why the points **scatter** about the curve (decay is random), the half-life read off it, and the curve for a larger decay constant (starts the same, falls **faster**)
- **MCQ (1 mark)**
  - Bq to curie: $\div 3.7 \times 10^{10}$
  - mass left after n half-lives: $(\frac{1}{2})^n$
  - 20% gone in 10 days $\rightarrow$ 64% left after 20
  - 1 rutherford = 10^6 decays a second
  - half-life: set by the nucleus alone
  - activity λN after a time
  - half-life from a fraction left
- **Short and long (5 or 8 marks, in parts)**
  - radioactivity; laws of decay; derive the decay law
  - decay constant, half-life, mean life and their relations
  - α, β, γ: properties, deflection in fields
  - n/p ratio in α decay; time for ¾ to decay
  - half-life and decay constant from a count-rate graph
  - GM tube: construction, working, plateau
  - carbon dating; medical uses; hazards

Key terms

Radioactivity: spontaneous decay of unstable nuclei, giving out α, β or γ radiation (Becquerel, 1896).

Natural radioactivity: of nuclei found in nature; **artificial (induced)**: of nuclei made unstable by a nuclear reaction (Joliot-Curies, 1934).

Decay constant λ: rate of disintegration ÷ number of undecayed atoms; the fraction decaying per second.

Half-life $T_{1/2}$: time for half the atoms of a sample to decay.

Mean life τ: average lifetime of the nuclei of a sample.

Activity R: decays per second, $-dN/dt$.

Becquerel (Bq): 1 decay a second; **curie (Ci)**: 3.7×10^{10} Bq; **rutherford (Rd)**: 10^6 Bq.

Count rate: counts a second at a detector; a fixed fraction of the activity.

Plateau: range of p.d. over which a GM tube's count rate hardly changes.

Tracer: small amount of a radioisotope followed through the body.

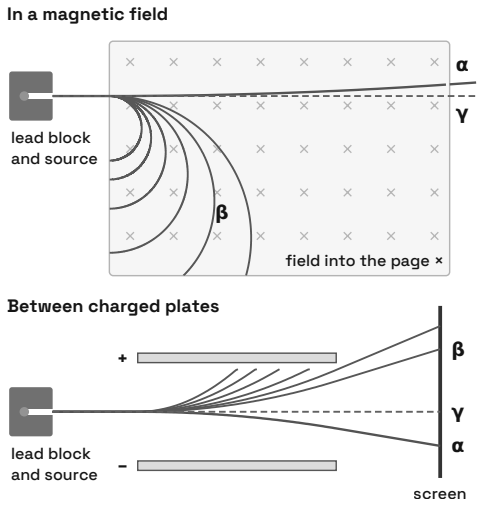
Formulas

$\frac{dN}{dt} = -\lambda N$	rate $\propto$ number left **Use:** decay law
$N = N_0 e^{-\lambda t}$	N_0 at $t = 0$ **Use:** decay equation
$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$	λ in s^{-1} **Use:** half-life
$N = N_0 \left(\frac{1}{2}\right)^n$	$n = t/T_{1/2}$ **Use:** whole half-lives
$\tau = \frac{1}{\lambda} = 1.443\, T_{1/2}$	time to fall to 1/e (37%) **Use:** mean life
$R = \lambda N = R_0 e^{-\lambda t}$	same half-life as N **Use:** activity
$t = \frac{T_{1/2}}{0.693} \ln \frac{R_0}{R}$	C-14: 5730 years **Use:** carbon dating
$V_W = \frac{V_1 + V_2}{2}$	ends of the plateau **Use:** GM working voltage

Radioactivity and its radiations

- **Becquerel** (1896): uranium salts fog plates in the dark; the Curies: polonium, radium
- natural: uranium, radium, thorium, potassium-40; artificial: Al + $\alpha \rightarrow$ radioactive phosphorus

	α	β	γ
is	helium nucleus $^4_2$He	fast electron from the nucleus	electromagnetic wave
charge	+2e	−e	none
speed	5–7% of c	up to 99% of c, every speed	c
stopped by	paper; 3–8 cm of air	a few mm of aluminium	only reduced by cm of lead
ionizing	very strong	less	weak
in fields	bent slightly	bent much more, the other way	not bent



α, β and γ from a source in a lead block. Top: in a magnetic field into the page, β curves sharply, α very slightly the other way, γ straight on. Bottom: between charged plates, α towards −, β towards +.

- all three: affect photographic plates, make zinc sulphide etc. fluoresce
- **cause**: an unstable nucleus; too big (Z > 82) $\rightarrow \alpha$; too many neutrons $\rightarrow \beta$ (a neutron becomes a proton); too much energy left $\rightarrow \gamma$

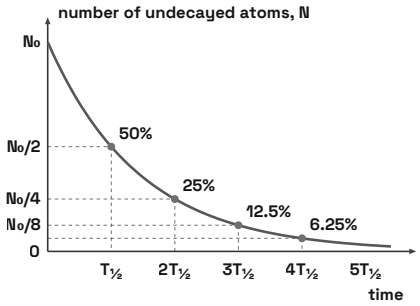
Laws of decay

1. **random and spontaneous**; not changed by temperature, pressure, chemistry or fields
2. a nucleus gives out α **or** β, never both at once
3. α: $A-4$, $Z-2$ (two places back); β: A same, $Z+1$ (one place on), $n \rightarrow p + e + \bar{\nu}$; γ: no change
4. rate $\propto$ number present

- α decay **raises** n/p (e.g. U-238: 146/92 = 1.59 $\rightarrow$ 144/90 = 1.60)
- the β electron is **made** at the decay; not an orbital electron

Derivation of $N = N_0 e^{-\lambda t}$

1. $-dN/dt \propto N \rightarrow dN/dt = -\lambda N$ (minus: N falls)
2. $dN/N = -\lambda\, dt$
3. $\int_{N_0}^{N} dN/N = -\lambda \int_0^t dt$
4. $\ln N - \ln N_0 = -\lambda t \rightarrow \ln(N/N_0) = -\lambda t$
5. $N = N_0 e^{-\lambda t}$



N against t: exponential, halving every half-life (N_0, $N_0/2$, $N_0/4$ …); never quite zero.

- fast at first, then slower; **larger** λ: falls faster (shorter half-life), same N_0

Half-life, decay constant, mean life

Half-life

1. $N_0/2 = N_0 e^{-\lambda T_{1/2}}$
2. $e^{\lambda T_{1/2}} = 2$
3. $\lambda T_{1/2} = \ln 2 \rightarrow T_{1/2} = 0.693/\lambda$

- property of the nucleus alone; radium about 1600 years
- λ: put $t = 1/\lambda \rightarrow N = N_0/e$ (37%); λ is the **reciprocal** of that time
- **mean life**: $\tau = \frac{1}{N_0}\int_0^\infty t\,\lambda N\, dt = 1/\lambda$; $\tau = T_{1/2}/0.693 = 1.443\, T_{1/2}$
- e.g. 30 days $\rightarrow \lambda = 0.0231$ per day; ¾ decayed $\rightarrow$ ¼ left = 2 half-lives = 60 days
- 5 half-lives $\rightarrow$ 1/32 = 3.125% left; 2 months, 1 year $\rightarrow$ M/64

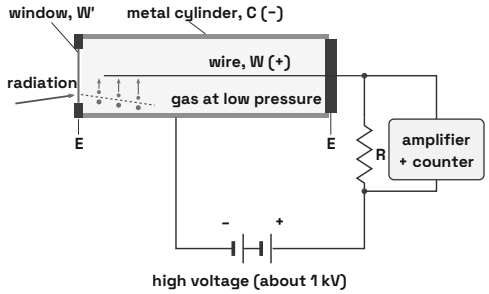
Activity

- $R = -dN/dt = \lambda N \rightarrow R = R_0 e^{-\lambda t}$; falls with the **same half-life**

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-12-physics/radioactivity-and-nuclear-reactions

- from a count-rate graph: read when it halves (160 → 80 → 40: equal times); take off background first
- measured points **scatter** about the smooth curve: decay is random, so each count is off by chance (about $\sqrt{N}$); draw the smooth best-fit curve through them
- e.g. 10^{20} atoms, 130 days, after 260 days: $N = N_0/4$, $R = \lambda N = 1.54 \times 10^{12}$ s^{-1}

GM tube

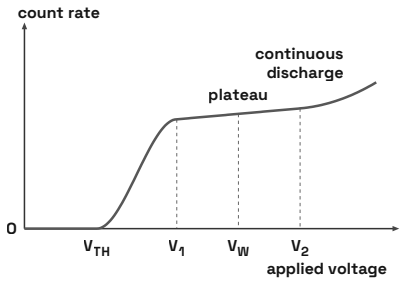


GM tube: metal cylinder (cathode) round a thin tungsten wire (anode) with argon at low pressure; mica window; high voltage through R; pulses to an amplifier and counter.

- cylinder (−), wire (+) through a high resistance R, about 1 kV; argon at about 10 cm of mercury with a little halogen or alcohol vapor

1. a particle ionizes a few gas atoms
2. electrons rush to the wire; near it the field is strong → each knocks out more: an **avalanche**
3. the pulse across R is amplified and counted (a click)

- one count a particle, whatever its energy: can't tell energies apart



Count rate against voltage: nothing below the threshold, a quick rise, a flat plateau from V_1 to V_2, then continuous discharge.

- **plateau**: every particle counted; working voltage at its middle, $V_W = (V_1 + V_2)/2$; beyond it, continuous discharge damages the tube

Carbon dating and uses

1. cosmic-ray neutrons: ${}^{14}_{7}\mathrm{N} + n \rightarrow {}^{14}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H}$
2. C-14 in CO_2; plants and animals take it in at the air's ratio
3. ${}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + e + \bar{\nu}$, half-life 5730 years
4. after death, no new C-14 → activity falls; $N_0/N = R_0/R = e^{\lambda t} \rightarrow t = (T_{1/2}/0.693)\ln(R_0/R)$

- works to about 50,000 years (about ten half-lives)
- **diagnosis** (tracers): Na-24 blood flow, Tc-99m (6 h) bone and heart scans, F-18 PET, Fe-59 anemia; **therapy**: I-131 thyroid, Co-60 cancer, P-32; **sterilizing** packed syringes
- **hazards**: ionization damages cells and DNA; large dose → radiation sickness; small → chance of cancer; outside the body α harmless, inside the **most** dangerous
- **protection**: time, distance (tongs), shielding (lead); no eating; film badge

COMMON MISTAKES

✗ λ is the time to fall to 37%
✓ its **reciprocal** is; λ is in s^{-1}.

✗ carbon-14 decays to carbon-12
✓ to **nitrogen-14**.

✗ the β electron comes from the shells
✓ made in the nucleus (n → p).

✗ α bent as much as β in a magnetic field
✓ α only **slightly**, β much more, opposite ways.

✗ heat or pressure changes the half-life
✓ nothing done to a sample changes it.

✗ the scatter is a mistake in the experiment
✓ decay is **random**; draw the smooth curve.

✗ irradiated food becomes radioactive
✓ γ passes through and leaves nothing.