Unit 23 · Quantization of Energy

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Modern Physics (Units 20–25 together): 19 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** short (5): state Bohr's postulates; from the Lyman series' diagram, the frequency of the photon when an electron drops from the **second excited state** ($n = 3$) to the ground level: $\Delta E = 13.6 - 1.51 = 12.09$ eV $\rightarrow f = \Delta E/h \approx 2.9 \times 10^{15}$ Hz
- **MCQ (1 mark)**
  - $1/\lambda$: wave number
  - angular momentum in orbit 2: h/π
  - electrons a second: I/e
  - He^+: $13.6Z^2 = 54.4$ eV
  - $E_5 = -0.54$ eV; ionize from $n = 3$: 1.51 eV
  - 4860 Å: Balmer
  - $v_n \propto Z/n$
  - $\lambda_{min} = hc/eV$; $f_{max} = eV/h$
  - Bragg: d from λ and θ
- **Short and long (5 or 8 marks, in parts)**
  - Bohr's postulates; derive r_n, v_n, E_n; Rydberg's formula; limitations
  - spectral series; energy level diagram; how many lines from a level; shortest wavelength
  - excitation and ionization potentials (13.6 V); emission and absorption spectra; why cold hydrogen absorbs only Lyman lines
  - de Broglie wavelength; proton or electron of the same KE; orbit = whole waves
  - X-rays: production, Coolidge tube, quality and intensity, target metal, heat; properties, uses
  - Bragg's law (derive) and numericals

Key terms

Stationary orbit: allowed orbit in which the electron goes round without radiating.

Principal quantum number n: 1, 2, 3, … labelling the orbits.

Ground state: lowest level ($n = 1$); higher levels are **excited states**.

Wave number: $1/\lambda$, waves per meter.

Excitation energy (potential): energy (p.d.) to raise an electron from the ground state to a higher level; hydrogen's first 10.2 eV (10.2 V).

Ionization energy (potential): energy (p.d.) to remove the electron from the ground state to infinity; hydrogen's 13.6 eV (13.6 V).

Emission spectrum: light given out as electrons drop: continuous, line or band.

Absorption spectrum: dark lines where a cooler gas absorbs its own wavelengths from white light.

Matter (de Broglie) wave: wave that goes with a moving particle, $\lambda = h/mv$.

Uncertainty principle: position and momentum (or energy and time) can't both be known exactly.

X-rays: electromagnetic waves of very short wavelength (0.06–100 Å), made when fast electrons hit a heavy target.

Glancing angle: angle between a ray and the crystal's planes (not the normal).

Formulas

$mvr = \dfrac{nh}{2\pi}$	$n = 1, 2, 3, \ldots$ **Use:** Bohr's quantization
$hf = E_2 - E_1$	photon out or in **Use:** frequency condition
$r_n = \dfrac{n^2h^2\varepsilon_0}{\pi me^2} = 0.53n^2$ Å	$\div Z$ for ions **Use:** radius
$v_n = \dfrac{e^2}{2\varepsilon_0 nh}$	$\times Z$ for ions **Use:** speed
$E_n = -\dfrac{me^4}{8\varepsilon_0^2 n^2h^2} = -\dfrac{13.6}{n^2}$ eV	$\times Z^2$ for ions **Use:** energy
$\dfrac{1}{\lambda} = R\left(\dfrac{1}{n_1^2} - \dfrac{1}{n_2^2}\right)$	$R = 1.097 \times 10^7$ m^{-1} **Use:** Rydberg
$\lambda = \dfrac{h}{mv} = \dfrac{h}{p}$	any particle **Use:** de Broglie
$\lambda = \dfrac{h}{\sqrt{2meV}} = \dfrac{12.27}{\sqrt{V}}$ Å	electron through V **Use:** electron's wavelength
$\Delta x\,\Delta p \geq \dfrac{h}{2\pi}$, $\Delta E\,\Delta t \geq \dfrac{h}{2\pi}$	order of size **Use:** uncertainty
$\lambda_{\min} = \dfrac{hc}{eV}, f_{\max} = \dfrac{eV}{h}$	tube p.d. V **Use:** Duane–Hunt
$2d\sin\theta = n\lambda$	θ glancing **Use:** Bragg's law

Bohr's model

- Rutherford's nuclear atom failed: an orbiting (accelerating) electron should **radiate**, spiral in (atoms unstable) and give a **continuous** spectrum; hydrogen gives **lines**
- Planck's quanta (1900) → **Bohr** (1913) kept the nucleus, added:

Postulates

1. **stationary orbits**: only certain circular orbits, each of fixed energy; no radiation in them
2. **angular momentum quantized**: $mvr = nh/2\pi$
3. **frequency condition**: a jump from higher to lower gives one photon, $hf = E_2 - E_1$; absorbing hf lifts it

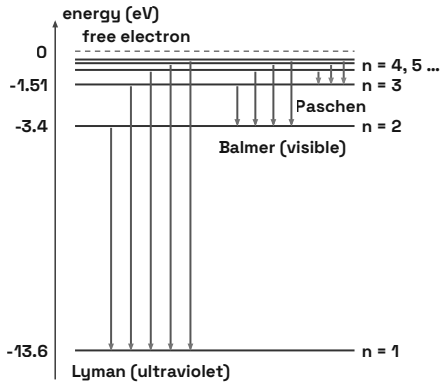
Derivations (force $e^2/4\pi\varepsilon_0 r^2$ = centripetal mv^2/r)

1. $\dfrac{e^2}{4\pi\varepsilon_0 r_n^2} = \dfrac{mv_n^2}{r_n}$; from postulate 2, $v_n = nh/2\pi mr_n$
2. substitute: $r_n = n^2h^2\varepsilon_0/\pi me^2 \to \propto n^2$ $(1:4:9)$; $r_1 =$ 0.53 Å, the Bohr radius
3. back into v_n: $v_n = e^2/2\varepsilon_0 nh \to \propto 1/n$
4. KE $= \frac{1}{2}mv^2 = e^2/8\pi\varepsilon_0 r_n$; PE $= -e^2/4\pi\varepsilon_0 r_n$
5. $E_n =$ KE + PE $= -e^2/8\pi\varepsilon_0 r_n = -me^4/8\varepsilon_0^2 n^2h^2$ (negative: bound; KE = −E, PE = 2E)
6. $hf = E_{n_2} - E_{n_1} \to 1/\lambda = R(1/n_1^2 - 1/n_2^2)$, $R = me^4/8\varepsilon_0^2 ch^3$

- Rydberg found the formula in 1888 by fitting; Bohr's theory explained it

Limitations: one-electron atoms only (H, He^+, Li^{2+}); no fine structure or splitting in a magnetic field; ignores the electron's wave nature; no chemistry or bonding; exact orbits break the uncertainty principle

Spectral series and levels



Hydrogen's energy levels, $-13.6/n^2$ eV, with the jumps of the Lyman (to n = 1), Balmer (to 2) and Paschen (to 3) series.

Series	to n_1	from n_2	region
Lyman	1	2, 3, ...	ultraviolet (121.6–91.2 nm)
Balmer	2	3, 4, ...	visible (656, 486, 434, 410 nm ...)
Paschen	3	4, 5, ...	infrared
Brackett	4	5, 6, ...	infrared
Pfund	5	6, 7, ...	far infrared

- first line ($n_2 = n_1 + 1$) the **longest** λ; lines crowd to the **series limit** ($n_2 \to \infty$)
- $E_1 = -13.6$, $E_2 = -3.4$ (first excited), $E_3 = -1.51$ (second excited), $E_4 = -0.85$, $E_5 = -0.54$ eV; $E_\infty = 0$
- from level n: $n(n-1)/2$ possible lines (from −0.85 eV, $n = 4$: 6); shortest λ = biggest gap
- **first excitation** 10.2 eV; second 12.09 eV; **ionization** $0 - (-13.6) = 13.6$ eV; potentials 10.2 V, 13.6 V (energies in eV, potentials in V)
- from $n = 3$: 1.51 eV to ionize; He^+ (Z = 2): 54.4 eV

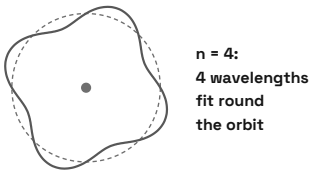
Spectra

- emission: **continuous** (hot solids, liquids, dense gases), **line** (separate atoms of a gas), **band** (molecules)
- absorption: white light through a **cooler gas** → dark lines at the gas's own wavelengths
- cold hydrogen: atoms all in $n = 1$ → absorb only jumps **from 1**: Lyman lines

Matter waves and uncertainty

- radiation: waves (interference, diffraction, polarization) and particles (photoelectric, Compton) → **dual nature**

- **de Broglie** (1924): matter too; photon $p = E/c = hf/c = h/\lambda \rightarrow$ for a particle $\lambda = h/mv$
- confirmed: electrons diffract through crystals; **electron microscope**
- electron through V: $\frac{1}{2}mv^2 = eV \rightarrow \lambda = h/\sqrt{2meV} = 12.27/\sqrt{V}$ Å
- same KE: $\lambda = h/\sqrt{2m\,\mathrm{KE}} \rightarrow$ lighter **electron** longer
- Bohr's orbit holds a whole number of waves: $2\pi r = n\lambda \Leftrightarrow mvr = nh/2\pi$
- everyday bodies: λ far too small to notice

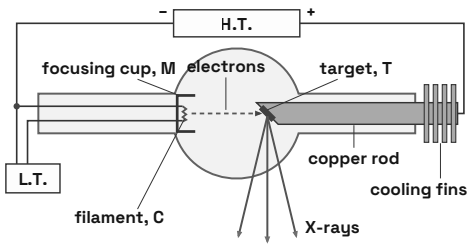


A whole number of de Broglie waves fits round an allowed orbit.

- **Heisenberg** (1927): $\Delta x\,\Delta p \geq h/2\pi$; $\Delta E\,\Delta t \geq h/2\pi$ (books also write $h/4\pi$: order of size)
- uses: no electrons inside a nucleus (Δx 10^{-14} m $\rightarrow$ huge momentum); atoms stable; energy levels; spectral lines have a width (life about 10^{-8} s)

X-rays

- **Röntgen** (1895), discharge tube: an unknown radiation through opaque things: "X"
- made when **fast electrons** hit a **heavy target** and stop



Coolidge tube: heated tungsten filament (cathode) in a focusing cup; high p.d. to a tungsten target at 45° in a copper rod; X-rays out sideways; cooling fins.

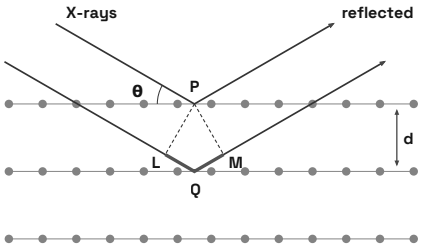
- **Coolidge tube** (1913): high **vacuum**; filament (thermionic emission), focusing cup, target (anticathode) at about 45°, 10 kV to over 100 kV, cooling
- target: high Z, high melting point, high thermal conductivity $\rightarrow$ **tungsten** (74, 3422 °C); aluminium won't do (low Z, low melting point)
- $\frac{1}{2}mv^2 = eV$; about **1%** becomes X-rays, the rest **heat** (the tube gets hot)
- **quality** (penetrating power, hard or soft): set by the **tube voltage**; **intensity**: set by the **filament current**
- whole KE in one photon: $f_{\max} = eV/h$, $\lambda_{\min} = hc/eV$ (20 kV $\rightarrow$ 0.62 Å)
- electrons a second = I/e (0.02 A $\rightarrow 1.25 \times 10^{17}$)

Properties: electromagnetic, 0.06–100 Å, speed of light; **not deflected** by fields; affect photographic plates; ionize gases; make zinc sulphide etc. fluoresce; photoelectric effect; straight lines; penetrate flesh, not bone or lead; harmful; secondary X-rays

Uses: fractures, stones, lungs; radiotherapy (cancer); cracks in castings and welds; airport and customs scanners; crystal and DNA structure; hidden paintings

Diffraction and Bragg's law

- **Laue** (1912): X-rays through zinc sulphide $\rightarrow$ spots $\rightarrow$ X-rays are **waves**, and atoms in a crystal are a regular 3-D grating (spacing ~10^{-10} m, like the X-rays' λ)



Bragg reflection: rays at glancing angle θ off planes d apart; the lower ray travels 2d sin θ farther.

1. rays reflected from planes d apart, glancing angle θ
2. extra path of the lower ray $LQ + QM = 2d \sin\theta$
3. strong reflection when it is $n\lambda$: $2d \sin\theta = n\lambda$

- e.g. $d = 3.08 \times 10^{-8}$ cm, $\theta = 12°$, $n = 1 \rightarrow \lambda = 2 \times 3.08 \times 10^{-10} \times 0.208 \approx 1.28$ Å
- $\lambda = 1$ Å, θ first order $\rightarrow d = \lambda/2 \sin\theta$

COMMON MISTAKES

✗ ground-state energy +13.6 eV
✓ **−13.6 eV**; ionization energy +13.6 eV.

✗ second excited state is $n = 2$
✓ $n = 3$; $n = 2$ is the first excited.

✗ first line of a series the shortest λ
✓ the **longest**; the series limit is the shortest.

✗ quality set by the filament current
✓ by the **tube voltage**; intensity by the filament current.

✗ X-rays bent by a magnet
✓ no charge: **not deflected**.

✗ Bragg's θ from the normal
✓ the **glancing** angle, from the planes.

✗ a photon has mass E/c^2
✓ no rest mass; $p = h/\lambda$ directly.

Science Playground, Katunje, Bhaktapur
https://science-playground.com/learn/grade-12-physics/quantization-of-energy