Unit 19 · Alternating Current

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Electricity and Magnetism (Units 14–19 together): 21 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** long (8): graphs of e.m.f. and current in an LRC circuit: is the phase constant positive or negative; sketch the phasor diagram; more inductive or capacitive; to deliver more power to the resistance, raise or lower L (towards resonance)
- **MCQ (1 mark)**
  - coil on d.c. and a.c.: find L
  - LCR with $X_L = X_C$: $I = V/R$
  - an inductor's $X_L \gg R$
  - $C \times 4 \rightarrow f_0 \div 2$
  - mean ÷ rms $= 2\sqrt{2}/\pi$
  - $V = \sqrt{V_R^2 + (V_L - V_C)^2}$
  - LC from a charged capacitor: I_{max}
  - transformer currents
  - C from X_C
  - wattless current: 90°
- **Short and long (5 or 8 marks, in parts)**
  - define rms value, impedance, resonance, wattless current, power factor
  - peak, rms, mean, f, T, energy from a graph
  - phase in a pure L or C (derive); power factor 0; short or open circuit for d.c.
  - impedance of LR, CR, LCR (phasor diagram, derive); phase angle
  - prove $f_0 = 1/2\pi\sqrt{LC}$; a radio tuner's L
  - numericals: current, p.d.s, phase angle, L from two frequencies
  - bulb and coil, then a capacitor added; currents in R, L, C as f rises

Key terms

Alternating current (e.m.f.): one whose size changes continuously with time and whose direction reverses periodically.

Peak value (amplitude) I_0, V_0: the greatest value.

Period T: time for one cycle; **frequency** f: cycles per second (Hz); $\omega = 2\pi f$.

rms (virtual, effective) value: the steady current that produces the same heat in the same resistance in the same time.

Mean value: average over **half** a cycle.

Phasor: arrow, length = peak value, turning anticlockwise at ω; its vertical projection is the instantaneous value.

Inductive reactance X_L: an inductor's opposition to a.c., ωL.

Capacitive reactance X_C: a capacitor's opposition to a.c., $1/\omega C$.

Impedance Z: total opposition of a circuit to a.c., V/I; Ω.

Phase angle φ: angle by which the e.m.f. leads the current.

Resonance: $X_L = X_C$; current greatest at the circuit's natural frequency.

Quality factor Q: p.d. across L (or C) at resonance ÷ applied e.m.f.; sharpness of resonance.

Apparent power: $V_{rms}I_{rms}$, in VA.

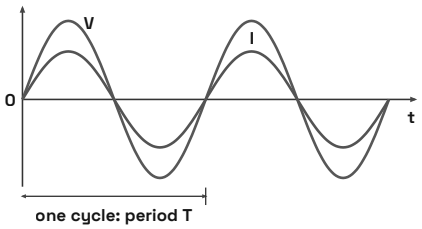
Power factor: real power ÷ apparent power, $\cos\phi$.

Wattless current: current in a pure L or C; no average power.

Formulas

$I = I_0 \sin \omega t,\ V = V_0 \sin \omega t$	$\omega = 2\pi f,\ T = 1/f$ **Use:** instantaneous
$I_{\text{rms}} = \dfrac{I_0}{\sqrt{2}} = 0.707 I_0$	same for V **Use:** rms
$I_m = \dfrac{2I_0}{\pi} = 0.637 I_0$	half a cycle **Use:** mean
$X_L = \omega L = 2\pi f L$	V leads I by 90° **Use:** inductor
$X_C = \dfrac{1}{\omega C} = \dfrac{1}{2\pi f C}$	I leads V by 90° **Use:** capacitor
$Z = \sqrt{R^2 + X_L^2},\ \tan\phi = \dfrac{X_L}{R}$	e.m.f. leads **Use:** LR
$Z = \sqrt{R^2 + X_C^2},\ \tan\phi = \dfrac{X_C}{R}$	e.m.f. lags **Use:** CR
$Z = \sqrt{R^2 + (X_L - X_C)^2}$	$\tan\phi = \dfrac{X_L - X_C}{R}$ **Use:** LCR
$f_0 = \dfrac{1}{2\pi\sqrt{LC}}$	$Z = R,\ \varphi = 0$ **Use:** resonance
$Q = \dfrac{\omega_0 L}{R} = \dfrac{1}{R}\sqrt{\dfrac{L}{C}}$	$\Delta f = f_0/Q$ **Use:** quality factor
$P_{\text{av}} = V_{\text{rms}} I_{\text{rms}} \cos\phi$	$\cos\phi = R/Z$ **Use:** average power

Alternating current



$I = I_0 \sin \omega t$: from 0 up to $+I_0$, through 0 to $-I_0$, back to 0 in one period T.

- from a coil turning in a field; Nepal's mains 50 Hz, about 230 V rms
- a.c. because **transformers** can change its voltage (they need a changing current) → high-voltage transmission
- a.c. → d.c.: **rectifier**; d.c. → a.c.: **inverter**

rms and mean values

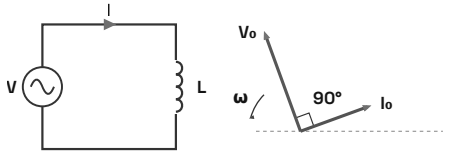
Derivation of $I_{\text{rms}} = I_0/\sqrt{2}$

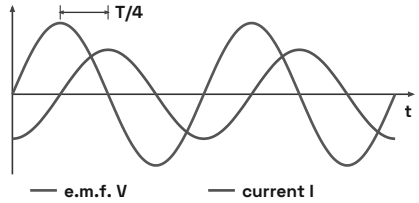
1. average current over a cycle: $I_0 \langle \sin \omega t \rangle = 0$
2. heating I^2R never negative → $\langle P \rangle = \langle I^2 \rangle R$
3. same heat as a steady I_{rms}: $I_{\text{rms}}^2 R = \langle I^2 \rangle R \rightarrow I_{\text{rms}} = \sqrt{\langle I^2 \rangle}$
4. $\langle \sin^2 \omega t \rangle = \frac{1}{2}$ ($\sin^2 = \frac{1}{2}(1 - \cos 2\omega t)$)
5. $I_{\text{rms}} = I_0/\sqrt{2} = 0.707 I_0$

Mean over half a cycle:
$I_m = \dfrac{1}{T/2}\int_0^{T/2} I_0 \sin \omega t\, dt = \dfrac{2I_0}{\pi} = 0.637 I_0$ (matters for charging and electrolysis)

- 230 V is **rms**; peak $230\sqrt{2} \approx 325$ V (insulation must stand it)
- mean : rms = $0.637 : 0.707 = 2\sqrt{2}/\pi \approx 0.9$
- meters and labels read rms

R, L and C alone





Pure inductor: graphs of V and I, V a quarter cycle ahead; phasor V 90° ahead of I.

- **R**: $I = (V_0/R)\sin \omega t$; **in phase**; R not on f

Pure inductor ($I = I_0 \sin \omega t$)

1. back e.m.f. $-L\, dI/dt$; loop law, no resistance: $V = L\, dI/dt$
2. $V = \omega L I_0 \cos \omega t$
3. $X_L = \omega L\ (\Omega) \rightarrow V = V_0 \sin(\omega t + \pi/2)$
4. **V leads I by 90°**

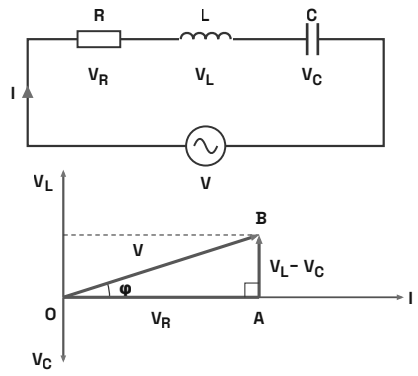
- $X_L \propto f$: d.c. → 0, a **short circuit**; high f → large; a **choke** passes d.c., holds back a.c.

Capacitor ($I = I_0 \sin \omega t$)

1. $V = q/C,\ dq = I\, dt$
2. $q = -(I_0/\omega)\cos \omega t$
3. $V = -(I_0/\omega C)\cos \omega t;\ X_C = 1/\omega C$
4. $V = V_0 \sin(\omega t - \pi/2)$: **I leads V by 90°**

- $X_C \propto 1/f$: d.c. → ∞, an **open circuit** (no charge crosses); high f passes
- **CIVIL**: in C, I before V; V before I in L

Series circuits



Series LCR: V_R along I, V_L up, V_C down; resultant V at φ to I.

LCR (same current in each: draw I first)

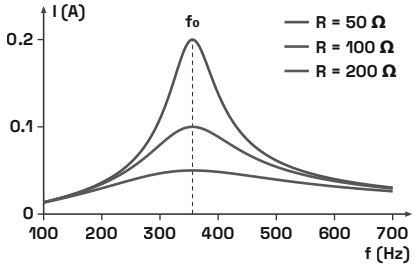
1. $V_R = IR$ along I; $V_L = IX_L$ 90° ahead; $V_C = IX_C$ 90° behind
2. V_L, V_C opposite $\rightarrow V_L - V_C$
3. $V^2 = V_R^2 + (V_L - V_C)^2 = I^2[R^2 + (X_L - X_C)^2]$
4. $Z = \sqrt{R^2 + (X_L - X_C)^2}$; $\tan\phi = (X_L - X_C)/R$

- $X_L > X_C$: φ **positive**, e.m.f. **leads**: **inductive**
- $X_L < X_C$: φ **negative**, e.m.f. **lags**: **capacitive**
- $X_L = X_C$: Z = R, $\varphi = 0$: **resonance**
- LR: put $X_C = 0$; CR: $X_L = 0$ (e.m.f. lags by $\tan^{-1}(X_C/R)$)
- p.d.s add as **phasors**: 40, 60, 30 V $\rightarrow \sqrt{40^2 + 30^2} =$ 50 V; 6 V + 8 V on a 10 V supply
- φ lies between −90° and +90°

Reading the model paper's graphs

- the wave that peaks **first** (to the left) **leads**
- e.m.f. first $\rightarrow \varphi > 0$, inductive; phasor: V at φ anticlockwise from I
- more power to R: move towards resonance; inductive $\rightarrow$ **decrease** L

Resonance



Resonance curve: current against frequency, a peak V/R at f_0; smaller R, taller and sharper peak.

1. Z least when $\omega L - 1/\omega C = 0$
2. $\omega^2 = 1/LC$
3. $f_0 = 1/2\pi\sqrt{LC}$

- at resonance: $V_L = V_C$, cancel; Z = R; $I_{max} = V/R$; in phase
- R doesn't change f_0; only the peak's height and sharpness
- $C \times 4 \rightarrow f_0 \div 2$; 33 pF at 810 kHz $\rightarrow L = 1/(4\pi^2 f^2 C) \approx 1.2$ mH
- **Q**: $V_L/V = X_L/R$ at resonance; high Q $\rightarrow$ tall, narrow peak, picks one station (**radio tuner**); $\Delta f = f_0/Q$
- L/R, CR, $\sqrt{LC}$: dimensions of **time**

Power

1. $P = V_0 I_0 \sin\omega t \sin(\omega t - \phi)$
2. $= \frac{1}{2} V_0 I_0 [\cos\phi - \cos(2\omega t - \phi)]$
3. average: second term 0 $\rightarrow$ $P_{av} = \frac{1}{2} V_0 I_0 \cos\phi = V_{rms} I_{rms} \cos\phi$

- real power = apparent power × power factor
- power factor $\cos\phi = R/Z = R/\sqrt{R^2 + (X_L - X_C)^2}$
- pure R: 1; pure L or C: 0 (energy stored and returned) $\rightarrow$ **wattless current**
- low power factor (either sign of φ) $\rightarrow$ more current for the same power $\rightarrow$ more I^2R loss in the wires
- **choke coil**: a low-resistance coil in series limits a.c. current with almost no power lost (unlike a resistor)

COMMON MISTAKES

✗ $V_{rms} = 0.637 V_0$
✓ $0.707 V_0$; 0.637 is the half-cycle mean.

✗ in a CR circuit the e.m.f. leads
✓ the **current** leads; the e.m.f. lags.

✗ power factor $R/\sqrt{R^2 + X_L^2 - X_C^2}$
✓ $R/\sqrt{R^2 + (X_L - X_C)^2}$.

✗ φ can be up to 180°
✓ between −90° and +90°.

✗ highly capacitive: high power factor
✓ **low**, as highly inductive.

✗ add the p.d.s across R, L, C as numbers
✓ add them as **phasors**.

✗ a capacitor passes d.c.
✓ blocks d.c.; a coil passes it.

✗ R changes the resonant frequency
✓ only the height and sharpness of the peak.