Unit 18 · Electromagnetic Induction

STUDY NOTES

WHAT NEB'S GRADE 12 EXAM ASKS

- **CDC's grid (2078):** Electricity and Magnetism (Units 14–19 together): 21 of 75 marks
- **The paper:** 11 MCQs × 1 · 8 short × 5 · 3 long × 8; 3 hours; 1 mark per expected point
- **CDC's model paper:** short (5): $\varepsilon = -N\, d\phi/dt$: what each symbol means and why the minus sign (Lenz's law); a copper ball dropped through a strong field falls more slowly than one outside it: eddy currents in the ball oppose its motion
- **MCQ (1 mark)**
  - $\varepsilon = Blv$ for a moving wire
  - Lenz's law: conservation of energy
  - inductor opposes a change of current
  - peak e.m.f. $NBA\omega$
  - power in a loop as B falls
  - M from ε and dI/dt; from flux and current
  - ε from $\Phi(t)$ at an instant
  - $L \propto N^2$
- **Short and long (5 or 8 marks, in parts)**
  - state Faraday's laws (qualitative, quantitative), Lenz's law; Lenz and energy
  - direction of the current in a loop beside a wire whose current grows
  - aluminium ring follows a receding magnet; is pushed by an approaching one
  - electrical inertia; L of a solenoid; of a straight wire (≈ 0)
  - spokes of a turning wheel: $\frac{1}{2}B\omega r^2$
  - a coil and a reversed current; average e.m.f. when a coil turns 90°
  - a.c. generator: why an e.m.f., slip rings, ways to raise it
  - transformer: why a current in the secondary; turns and currents; transmission at high voltage

Key terms

Electromagnetic induction: an e.m.f. (and a current in a closed circuit) produced when the magnetic flux linked with a conductor changes.

Motional e.m.f.: e.m.f. in a conductor because it moves through a field.

Eddy currents: induced currents swirling in a solid piece of metal in a changing flux.

Self-induction: e.m.f. induced in a coil by a change in its **own** current; the **back e.m.f.**

Self-inductance L: $N\Phi/I$; a coil's **electrical inertia**.

Mutual induction: e.m.f. induced in a coil by a change in the current in a **neighboring** coil.

Mutual inductance M: $N_S\Phi_S/I_P$.

One henry: the inductance in which a current changing at 1 A/s induces 1 V (1 H = 1 V s/A = 1 Wb/A).

Transformation (turns) ratio k: N_S/N_P; > 1 step-up, < 1 step-down.

Formulas

$\varepsilon = -N\dfrac{d\Phi}{dt}$	also $N(\Phi_2 - \Phi_1)/t$ **Use:** Faraday
$I = \dfrac{\varepsilon}{R}, q = \dfrac{N\Delta\Phi}{R}$	q independent of time **Use:** induced current, charge
$\varepsilon = Blv\sin\theta$	θ between v and B **Use:** moving rod
$\varepsilon = \frac{1}{2}B\omega r^2 = BAf$	rod turning about one end **Use:** spoke, disc
$\varepsilon = NBA\omega\sin\omega t, \varepsilon_0 = NBA\omega$	$\theta = \omega$t from the normal **Use:** rotating coil
$\varepsilon = -L\dfrac{dI}{dt}, N\Phi = LI$	H **Use:** self-induction
$L = \dfrac{\mu_0 N^2 A}{l}$	$\times\,\mu_r$ with a core **Use:** long solenoid
$\varepsilon_S = -M\dfrac{dI_P}{dt}, N_S\Phi_S = MI_P$	H **Use:** mutual induction
$E = \frac{1}{2}LI^2, u = \dfrac{B^2}{2\mu_0}$	J, J/m^3 **Use:** energy in an inductor
$\dfrac{E_S}{E_P} = \dfrac{N_S}{N_P} = \dfrac{I_P}{I_S}$	ideal **Use:** transformer
$\eta = \dfrac{E_S I_S}{E_P I_P} \times 100\%$	output ÷ input **Use:** efficiency

Faraday's laws

- Faraday: magnet moved towards a coil with a galvanometer: sudden deflection; only **while moving**; faster → larger; away or other pole → reversed; coil moved instead: the same (**relative motion**)
- **first law** (qualitative): whenever the flux linked with a closed circuit changes, an e.m.f. is induced; lasts only while the flux changes
- **second law** (quantitative): induced e.m.f. ∝ **rate of change of flux linkage**

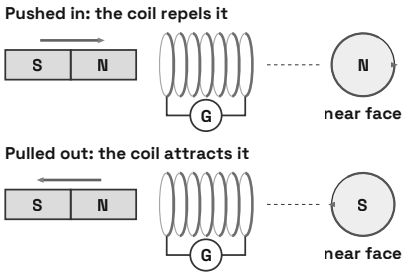
Derivation of $\varepsilon = -N\,d\Phi/dt$

1. rate of change of linkage $= (N\Phi_2 - N\Phi_1)/t$
2. $\varepsilon \propto (N\Phi_2 - N\Phi_1)/t \to \varepsilon \propto d(N\Phi)/dt$
3. SI units: constant 1 $\to \varepsilon = -N\,d\Phi/dt$

- symbols: ε induced e.m.f. (V); N turns; $d\Phi/dt$ rate of change of flux (Wb/s)
- **minus sign**: Lenz's law, the e.m.f. opposes the change; flux rising → ε negative
- e.m.f. **independent of** R; current ε/R is not
- e.g. 5000 turns, 8×10^{-4} Wb in 0.1 s → 40 V

Ways to induce an e.m.f.: move a magnet or coil (B changes); change the coil's own current (self); change a neighbor's current (mutual); move a rod (A changes); turn a coil (θ changes)

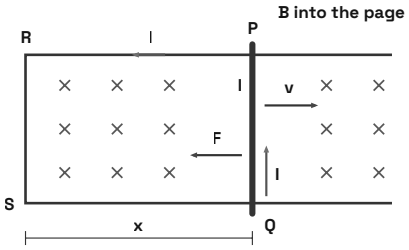
Lenz's law



N pole pushed towards a coil: the near end becomes N and repels it; pulled away: the near end becomes S and attracts it.

- **Lenz's law**: the induced current flows so as to **oppose the change** that produces it
- approaching N pole: flux grows → near end **N**, repels; receding: flux shrinks → near end **S**, attracts → the coil always **opposes the motion**
- induced field opposite to the magnet's only while flux **grows**; the **same** way while it shrinks
- **energy**: you push against the repulsion → your work = electrical energy (then heat); if it helped the change, energy from nothing
- aluminium ring: follows a receding magnet, is pushed by an approaching one
- wire with a growing current: loop beside it gets the current that makes a field **against** the wire's inside it (anticlockwise if the wire's field is into the loop)
- **Fleming's right-hand rule** (generator): First finger Field, thuMb Motion, seCond finger induced Current; left hand for motors

Motional e.m.f.

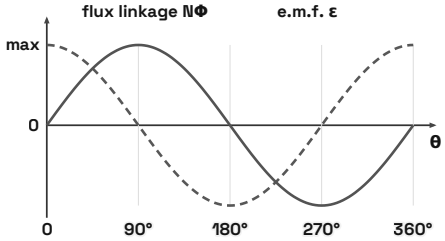


Rod PQ, length l, slides at v along rails in a field B into the page; circuit PQRS of area lx.

1. $\Phi = BA = Blx$
2. $dx/dt = v$
3. $\varepsilon = d\Phi/dt = Bl\,dx/dt = Blv$

- at an angle θ: $Blv\sin\theta$; 10 cm, 10 m/s, 0.01 T → 0.01 V
- why: each electron feels $q\vec{v} \times \vec{B}$ along the rod; one end −, the other +
- energy: current Blv/R feels BIl **against** the motion → push needed; power $B^2l^2v^2/R$
- rod turning about one end, length r: $\frac{1}{2}B\omega r^2$; wheel spokes in parallel: the same (1.2 m, 5×10^{-5} T, 10^{-2} V → $\omega = 278$ rad/s, 44 turns a second)

Rotating coil and the a.c. generator



Flux linkage NBA cos ωt and e.m.f. NBAω sin ωt against time: the e.m.f. follows the slope of the flux graph.

1. $N\Phi = NBA\cos\theta = NBA\cos\omega t$
2. $\varepsilon = -d(N\Phi)/dt = NBA\omega\sin\omega t$
3. peak $\varepsilon_0 = NBA\omega$

- $\theta = 0°$ (plane ⊥ field, most flux): $\varepsilon = \mathbf{0}$; $\theta = 90°$ (plane along the field, no flux): $\varepsilon = \varepsilon_0$; 270°: $-\varepsilon_0$
- e.m.f. greatest where flux is zero, zero where flux is greatest
- average e.m.f. turning 90°: $N\Delta\Phi/t = NBA/t$

A.c. generator (alternator): mechanical → electrical

- **principle**: electromagnetic induction; a coil turned in a field
- **construction**: **armature** (many turns on a soft iron core); **field magnet** (concave poles); **slip rings** (brass, one to each end, turn with the coil); **brushes** (carbon, still, carry current to the load)
- **working**: $\varepsilon = NBA\omega\sin\omega t$, $I = (NBA\omega/R)\sin\omega t$; each slip ring stays on the same end → current reverses each half turn: **a.c.**
- larger e.m.f.: faster, more turns, larger area, stronger field (iron core)

- d.c. generator: slip rings → a split ring (**commutator**), which reverses the connections every half turn

Eddy currents

- induced in solid metal in a changing flux, or moving through a field; oppose the cause (Lenz)
- **copper ball falling through a field**: eddy currents in it oppose its motion → falls **slower** (its KE goes to heat)
- **harm**: heat in transformer, motor and generator cores → **laminate** them (thin sheets, varnished)
- **uses**: damping in a galvanometer (coil on an aluminium frame), induction furnace, induction cooker, induction motor (rotor a little **slower** than the field), electromagnetic brakes, diathermy, speedometer, metal detectors

Self-induction

- key pressed: current grows → own flux grows → e.m.f. **opposes the growth**; released: opposes the fall (spark)
- **electrical inertia**, like mass for velocity

Derivation of $\varepsilon = -L\,dI/dt$

1. $N\Phi \propto I \rightarrow N\Phi = LI$
2. $\varepsilon = -d(N\Phi)/dt = -d(LI)/dt = -L\,dI/dt$

Long solenoid

1. $B = \mu_0 NI/l$
2. $\Phi = BA = \mu_0 NIA/l$
3. $N\Phi = \mu_0 N^2 IA/l$
4. $L = N\Phi/I = \mu_0 N^2 A/l$

- $L \propto$ permeability, $\propto A$, $\propto N^2$, $\propto 1/l$; not on the current; uses: chokes, smoothing, tuned LC circuits, ignition sparks
- 600 turns, 108 mH → 500 turns: $108 \times (5/6)^2 = 75$ mH
- straight wire: $L \approx 0$

Mutual induction

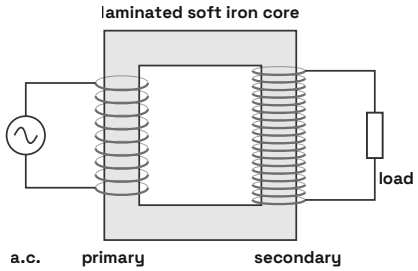
- key pressed: P's flux grows → kick in S; released: kick the other way; steady current: **nothing**
- $N_S\Phi_S = MI \rightarrow \varepsilon_S = -M\,dI/dt$; henry
- e.g. 3 A to 0 in 0.001 s, 15 000 V → $M = 5$ H; 8 mV at 3 A/s → 2.7 mH
- uses: transformers, wireless charging, induction motors, metal detectors

Energy in an inductor

1. back e.m.f. $L\,di/dt$
2. work to push $dq = i\,dt$: $dW = Li\,di$
3. $W = \int_0^I Li\,di = \frac{1}{2}LI^2$

- like $\frac{1}{2}mv^2$: L inertia, I velocity; $\propto I^2$
- stored in the field: solenoid → $(B^2/2\mu_0) \times Al$, density $B^2/2\mu_0$

Transformer



Primary P and secondary S on one laminated soft iron core; a.c. in, a.c. out.

- **principle**: mutual induction; **construction**: two coils on a laminated soft iron core
- step-up: $N_S > N_P$, voltage up, current down; step-down: the reverse

Theory

1. $E_P = -N_P\,d\Phi/dt$
2. $E_S = -N_S\,d\Phi/dt$ (same flux)
3. $E_S/E_P = N_S/N_P$
4. ideal: $E_S I_S = E_P I_P \rightarrow E_S/E_P = I_P/I_S$

- works on **a.c. only**: a steady current makes no changing flux
- secondary current: alternating current in P → alternating flux in the core → e.m.f. in S → current in the load
- e.g. 220 V to 15 W at 0.5 A (30 V), 20 turns → $N_P \approx 147$; $I_P \approx 0.068$ A
- **losses**: flux leakage (coils wound over each other); copper I^2R (thick wire); **hysteresis** (soft iron, narrow loop); **eddy currents** (laminate); hum
- **transmission**: loss I^2R; same power at high voltage → small current → small loss; step-up at the station (11 kV → 132, 220 or 400 kV), step-down near towns; a.c. because transformers change it easily

COMMON MISTAKES

✗ induced current opposes the flux
✓ opposes the **change** in flux.

✗ flux change = initial − final
✓ **final − initial**: $\Delta\Phi = \Phi_2 - \Phi_1$.

✗ a coil's e.m.f. is largest when most flux passes through it
✓ **zero** then; largest when its plane is along the field.

✗ right hand for motors
✓ **right** hand for generators, **left** for motors.

✗ $L = \mu_0 N^2 AI/l$
✓ no I: $L = \mu_0 N^2 A/l$.

✗ an induction motor turns with the field
✓ always a little **slower**.

✗ a transformer steps up power
✓ it steps up voltage; power at most the same.

✗ transformers work on d.c.
✓ only a **changing** current.